

Quantum Toolbox for Neurobiology Sensory Systems

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Abstract. The quantum-like paradigm has emerged over the last decade to describe non-linear, dynamical, complex phenomena using quantum mechanics as a tool. In essence, it takes advantage of the linearity of quantum information processing, allowing for complex correlations through entanglement. In a quantum- and neuroscience truly interdisciplinary research, we found that an open quantum spin network, mapping a neural system, can successfully simulate the human sense of number as a global dynamical property, in contrast with the poor performance of conventional Artificial Neural Networks. Here, we discuss how the simulation can be extended to other important complex perceptual phenomena like the perception of space, time, and numbers, known to be inter-dependent with each other, suggesting that a shared neuronal mechanism is operating in the brain. Here we present a research program that aims at creating a quantum toolbox to simulate this integrated space-time-number sensory ability of our brain, with open-quantum systems methods. We will explore the implications of more general quantum-matter paradigms, and their possible coding into a quantum technology.

1. Introduction

Perception is a task which our brain continuously performs by solving complex problems through the processing of external inputs. In the visual system, the brain reconstructs the sense of space by building a spatiotopic representation of our external world via dynamical, predictive live compensation of the continuous changes received from rapid eye movements, known as saccades, so to merge corresponding retinal images across them [1]. These operations are so many and so complex, involving a huge amount of data storage, processing, and retrieval, that in fact saccades are one sustainability trick played for highest resolution, otherwise possible only in the central part of the fovea. Surprisingly, besides a “where” in space, this mechanism provides also a “when” in time via the gaze shift: our brain works as if it were equipped with many different clocks, ticking all together and possibly even in a chaotic manner, each of them attached to the object position in our external world [1]. Even more strikingly, the neuronal remapping process, necessary to transform the retinal coordinate of space to the external-world space coordinates, induces localization errors of transient stimuli, that result into a space and time compression [2,3], occurring in correlated manner during saccades, that - as a rule of thumb - is valid for any sensorial network and their combination [4].

Similar problems arise in the perception of numbers. Humans and animals, in fact, possess the ability to estimate how many objects are present in a given space or time interval, that is their numerosity. Humans perform this task remarkably well, with an error rate of about 15% over a very large range of items, up to 200 [5-8]. While determining the number of elements in a set is traditionally associated with counting, it emerges as a perceptual function taking place even



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without attention and with higher precision than related perceptual functions (such as area and density estimation) [9,10]. It is significant that this is a ubiquitous trait of sensory systems. Several neurophysiological studies have demonstrated that, in the primate, brain neurons selective to numerosity do exist and their tuning is proportional to the preferred numerosity [11]. Remarkably, it is known that the task-associated uncertainty increases linearly with the numerosity itself. Modeling this law, named after Weber [5], is significantly challenging. Weber's law is very general: it occurs regardless of the modality of presentation, no matter if the items sequence involves the visual, auditory or tactile system, or whether the items are in same or in different positions. All these observations point to Weber's law as a hallmark of the central mechanism which estimates numerosity from different sensory formats. Indeed, Weber's law fails only under circumstances [7], such as when items are too close together to be segregated, and mechanisms for estimation are based on the visual-texture grain and follow a square-root law. Interestingly, individual performance with sparse displays is predictive of mathematical literacy of the individual, whereas performance with high clutter display does not [12], suggesting that these sparse perceptual mechanisms are relevant to mathematical knowledge, whereas those enabling perception in higher-density displays are not. It is as well interesting that Weber's law cannot be attained by the simplest model of numerosity perception, i.e. one that counts the number of items in a region of space (or events in a period of time). In the latter case, the statistics indeed follows a Poissonian distribution, the variance scaling with numerosity (the standard deviation with its square root). This is a well-known problem in time perception where a central assumption is Poisson pacemaker, with the Weber's law modeled only resorting to ad-hoc assumptions [13]. Yet, the neural mechanisms allowing Weber's law over such a large range of numerosity, and conditions are unknown.

This leads to one more question, about how this local-scale space, time, and number perception ability, and error processing, connects to the way neural circuits oscillate and integrate the information. It is known that neural oscillations, possibly chaotic, phase reset under external sensory inputs, and then phase restart. Synchronization can occur by motor action, with direct consequences on low-level visual and auditory processing, in fact mediating motor and sensory processing [14]. Electro-encephalography, blood oxygenation level-dependent imaging, and psychophysical studies [15-16] demonstrate that the memory trace of past experience oscillates in synchrony with action or other multisensory signals acting as a phase reset. Therefore, oscillations synchronization maybe a generalized brain mechanism, the frequency identification possibly working as decoding strategy consistent with temporal and serial dependence of visual and auditory stimuli and suggesting that the transmission of the memory trace of prior information occurs through endogenous oscillations of phase resetting mechanisms [17], though the benefit of oscillatory memory trace and of a representation of a prior, remain open questions to be addressed.

All in all, the bottom line of a vast consolidated literature is that our brain represents space, time, and numbers in a correlated space, where perturbing one of this dimension necessarily will alter the other two dimension [3], raising the possibility that these phenomena, particularly the compression errors, are consequence of the concerted oscillatory behavior of the brain. In the rest, we will refer to this concept as: integrated space-time-number sensory circuitry. Therefore, describing numerosity has profound implications.

One may then ask whether these observations can be described by Artificial Neural Network (ANN) computations, which have demonstrated to be a valid approach to the simulation of many aspects of perception. Pioneering work by John Hopfield, employing statistical mechanics, demonstrated that it was possible to study the behavior of unsupervised networks comprised of simple processing units, based on spin-glass models in physics [18]. Interestingly, with such a network Stoianov and Zorzi successfully modelled human processing of numerosity [19]. The network, comprising one input and 2 hidden layers, and trained in classifying 2 dimensional

stimuli containing various numbers of objects, spontaneously built a representation of numerosity, which could then be decoded by a simple linear classifier. If, in this model, the representations read-out of the classifier were used to perform a numerosity comparison, the behaviour would comply with Weber's Law. More recently, Nasr et al. [20] have found that some units of a deep convolutional neural network (CNN) using 8 convolutional layers for local filtering and 5 pooling layers for aggregated response, designed and trained for objects recognition, could also perform number discrimination on static displays. However, this CNN recognizes numerosity not as a global property, in fact appearing only in around 10% of its units and with an approximate Weber's law behaviour. Similar conclusions have been drawn with untrained networks [21], again with a limited number of units.

In a more recent interdisciplinary work by Yago Malo et al. [22], we have proposed a different approach based on a mapping of the neural system with an open quantum network. After recalling from Ref. [22] how this approach can describe in a conceptually simple manner the numerosity perception, we here discuss how the underlying research program can be extended to describe the integrated time-space-number sensory system.

The paper is organized as follows. We introduce in Sec. 2 the quantum-like paradigm as the main theoretical framework supporting the possibility of describing the behavior of complex systems that do not need to be quantum, as with our brain information processing functions, by exploiting as a toolbox the quantum mechanics and statistics of quantum network dynamics. In this section, we also briefly describe the methodology that we plan to adopt, along with the underlying quantum many-body paradigms of open quantum systems dynamics, that can be exploited to map the considered neuroscience phenomena considered. We finally illustrate in Sec. 3 how the theoretical framework and the methodology of Sec. 2 will be implemented in our research program to investigate the considered neuroscience phenomena. We close the paper with a discussion of the research program potential and perspectives.

2. Theoretical and simulational framework for the research program

2.1 Quantum-like paradigm

Using quantum mechanics as a toolbox to tackle non-linear complex dynamics [25,26] can be strategic for a number of reasons. First, it fully exploits the inherent linearity of information processing, while accommodating complex correlations in the form of entangled quantum states. Second, biological functions are represented as open information systems interacting with environments: information processing using superposition states can save computational resources, given that a biological function does not need to resolve uncertainties encoded in superpositions of all compatible variables involved with its performance. At the same time, biological functions developed the ability to perform self-measurements that are needed to determine their functioning: in the quantum-like paradigm, this decision making is performed through decoherence (without involving any notion of quantum state collapse) [27]. Third, quantumness can emerge even for just the pragmatic concept of quantum annealing, meaning that adding some quantumness via e.g. tunneling can speed up optimization processes while self-trapping in one metastable state is prevented. Emergent quantumness can also be argued to be a generic macroscopic prediction of any learning system with coupled dynamics between trainable (microscopic) (in discrete or curved spaces) and hidden variables described as statistical ensembles [28].

Coarse-grained quantum models have been used e.g. to study the generation of persistent quantum-chaotic patterns at a microscopic scale and the amplification of quantum effects to a macroscopic scale [26,29]; propose quantum versions of the IIT theory for consciousness [30,31]

and information processing [32]; explore entanglement-driven features through dissipative quantum models of the brain and in a quantum field-theory model of brain functional activity [33]; describe information processing in biosystems by using quantum measurement theory, and then applied to model combinations of cognitive effects and gene regulation [27]. Quantum-inspired techniques in machine learning have also been introduced in psychology [34], giving birth so-called quantum cognition [35], hinging the mathematical structure of quantum probability. For example, incompatibility produces superposition states of uncertainty which eventually result in violations of classical/probability laws [36]. Quantum cognition has been used to address reinforcement learning during human decision-making [34], known to be hardly described via classical probability theory [37]. On the same page, it has been highlighted how the brain could work as a not only classical but also quantum Bayesian-inference machine [38,39]. Finally, several proposals for the use of quantum spin models and neuronal activity have been discussed [40]. While classical and quantum models may use similar spin Hamiltonians [18,41], the approaches are substantially different. The classical models focus on using adaptable statistical structures [41,42] and aim at defining a flexible network connectivity suited for learning models to encode the input information.

Table 1. Minimal elements for information processing and their quantum counterpart.

Neural systems	Quantum physics model
Neuron/unit: activity vs no activity	Spin: orientation (up/down)
Propagation of activity	Excitation transfer via tunneling/exchange
Built-in connectivity with excitatory+inhibitory mechanisms	Interaction potential between nodes
Flexibility in connectivity for different functions	From nearest-neighbor to all-to-all coupling
System decays to a resting state	Dissipation mechanisms

In a more recent truly interdisciplinary work between quantum scientists and neuroscientists [22], we have used the nature of quantum-mechanics as a formal toolbox to describe the highly dynamical complexity underlying the encoding of numerosity perception. We work out a coarse-grained quantum model as a mathematical toolbox to map information processing of a network of neurons (see Table 1).

2.2 The prototype

The quantum mapping in Table 1 of the neural-system excitations and decay to a resting state necessarily leads to a driven-dissipative quantum system out of equilibrium. In fact, Open quantum systems (OQS) represent a paradigm not only to characterize driven-dissipative systems, but – even more interestingly – to engineer quantum platforms and program them with high accuracy [43-47], allowing for the preparation of topological phases-of-matter [48], environment-assisted quantum transport [49-50], and dynamical phase transitions [51-52]. The coupling between system and environment can lead to both dissipation [53] and coherence loss (decoherence), a phenomenon related to the fundamental measurement problem [54] in quantum information. While the description of OQS is often simplified by relying on a series of

approximations known as the Markovian limit [55], there are numerous examples where this limit can be overcome [56-57], leading to the possibility of incorporating memory-based coupling with the environment. The question arises whether these tools can be helpful in the description of the complex-system dynamics within the quantum-like paradigm.

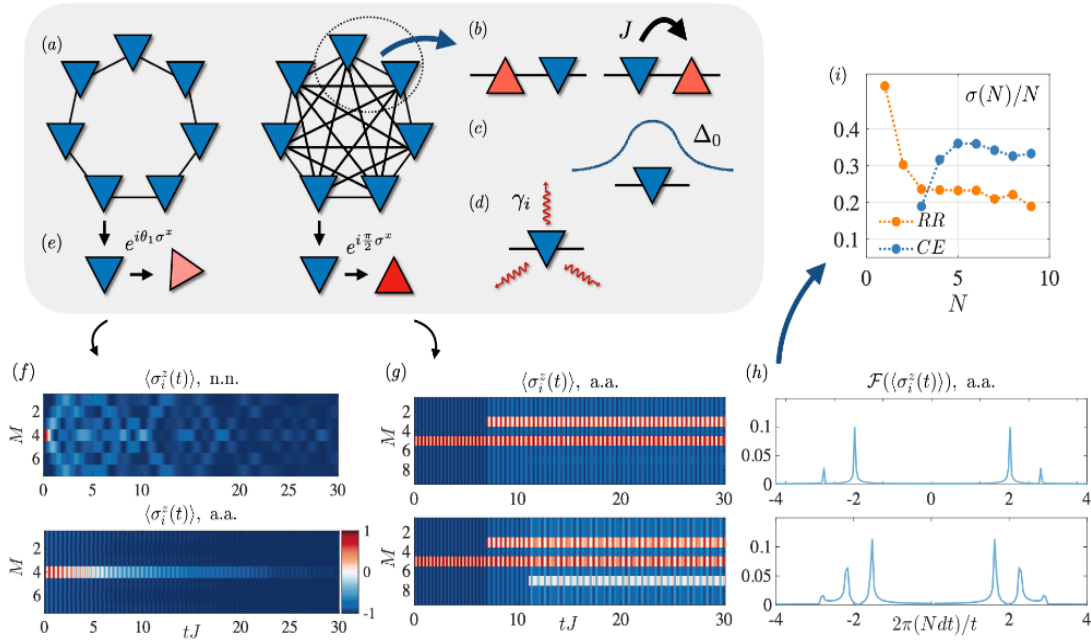


Figure 1. The prototype. Top left: schematics of the quantum spin model. Triangles represent the quantum spins. (a) Variable connectivity: nearest-neighbour (left) and all-to-all (right); (b) Hamiltonian and dissipative terms: each spin can propagate an excitation via exchange J , experiences an energy offset, modeled as a gaussian profile with amplitude Δ_0 and centered on each spin (c), and can interact with different dissipative channels (d), e.g. losses and dephasing at rate γ_i . One excitation can be injected by flipping one spin by a generic angle ϑ_1 (e)-left or by $\pi/2$ (e)-right, simulating different intensities. Bottom: counting capability. Time-evolution (horizontal axis) of the average magnetization after that $N=1$ excitation has been injected in position 4 at dimensionless time $tJ=0$ (f), in the case of nearest-neighbor (n.n., top) and all-to-all (a.a., bottom) connectivity. In a.a. interference makes the excitation neatly identifiable, and: (g) when $N=2$ (top) and $N=3$ (bottom) excitations are created in positions (3, 5) at times $tJ=(8, 0)$, and (3, 5, 7) at $tJ=(8,0,10)$, respectively, the quantum network keeps memory of the past events for long times. In the corresponding Fourier transform of the time signals (h), for each excitation one peak appears at lower frequencies with increasing N . Top right: Weber's law for numerosity. After using an ideal-observer decoding procedure [22], the quantum network complies with Weber's law independently of whether the excitation is injected with random amplitude (RR) or random-angle rotations constrained to constant total energy (CE).

Our open quantum network is a coarse-grained quantum spin model to map information processing of a network of neurons (see Table 1 and fig. 2 (a)) with different connectivity architectures from the nearest-neighbours (n.n.) to the all-to-all (a.a.) extreme (fig. 2(a)). Each neuron is associated to an on-off state of activity which, in the quantum domain, can be described in terms of a spin $\frac{1}{2}$ (blue triangles in fig.2(a)). Excitations in the system, modeled as \uparrow spins (red triangles), can be transferred to connected sites via exchange coupling J (fig. 2(b)), subjected to an energy offset Δ_0 whenever several excitations are close by (fig. 2(c)), and can decay to a resting

state due to driving from - or coupling to- the environment occurring via losses or dephasing (fig. 2(d)). The resulting Hamiltonian is the paradigmatic XXZ model for quantum magnetism [58].

We determine the time-evolution behavior of the observables characterizing the quantum network by state-of-the-art methods for treating out-of-equilibrium driven-dissipative, open quantum systems, including stochastic unraveling methods like quantum trajectories, to reduce numerical complexity at the cost of computing several trajectories [59-60].

The dynamical properties of our open quantum network are simple in the a.a. connectivity architecture (fig. 2(f)). After injecting excitations in the form of spin flips, fig. 2(e), the quantum network is seen to keep memory of these past events after long times, see fig. 2(g). The corresponding Fourier transforms of the time signals (fig. 2(h)) reveal that each injection manifests with the appearance of one additional peak in the frequency spectrum, at lower frequency. Remarkably, this feature does not depend on where and when, nor on the manner in which the excitations are injected, e.g. with random amplitude and random-angle rotations with constant total energy constrain. The quality of this behavior remains robust against a wide range of the model parameters in fig. 2(b)-(d) and with increasing system size. Using an ideal-observer decoding [22] the uncertainty $\sigma(N)$ of number estimation is extracted, which turns out to follow Weber's law, that is $\sigma(N)/N \approx \text{const}$ (fig. 2(i)) as a global dynamical property.

Box - Neuroscience phenomena considered in the research program

A. Integrated space-time-number perception. Our brain performs perceptual tasks continuously, by solving very complex problems through the processing of external inputs. In the visual system, the brain reconstructs the sense of space by building a spatiotopic representation of our external world via dynamical, predictive live remapping of the continuous spatial shifts of the retinæ due to the continuous eye movements [3]. Surprisingly, besides a “where” in space, this mechanism provides also a “when” in time: our brain works as if it were equipped with many different clocks, ticking all together and possibly even in a chaotic manner, each of them attached to the object position in our external world [1].

B. Correlated space and time compression. Even more strikingly, the neuronal remapping process, necessary to transform the retinal coordinate of space to the external world space coordinate, induces localization errors of transient stimuli, that result into a consistent space, time and number compression [2,23].

C. Oscillations synchronization vs. chaos. Endogenous oscillations are ubiquitous phenomenon of brain excitation. It is known that neural oscillations, possibly chaotic, phase reset under external sensory inputs, and then restart. Synchronization can occur by motor action, or by low-level visual/auditory input excitation [24]. Psycho- and bio-physical studies [15,16] demonstrate that the memory trace of past experience oscillates in synchrony with action or other multisensory signals, acting as a phase reset. Oscillation synchronization may be a generalized brain mechanism useful for behavior, where, for example, frequency identification acts as a decoding strategy to transmit the memory trace of prior information [17]. This raises the possibility that many complex phenomena of the integrated space-time-number sensory mechanism, particularly the compression errors, are consequence of the concerted oscillatory behavior of the brain.

D. Information storage. All these operations are highly complex, involving huge amounts of data storage, processing, and retrieval. However, the brain continuously absolves this task with nearly constant precision, suggesting an automatic, fast and dedicated hardware.

3. The research program

In Sec. 2 we have shown that our open quantum-network prototype can simulate the numerosity perception with a simple architecture within the quantum-like paradigm. As discussed in the introduction and summarized in the Box, the numerosity perception is intimately related to time and space perception, leading to the integrated space-time-number sensory circuitry (Box A-B). This in turn connects to other highly investigated neuroscience phenomena, specifically the way neural circuits oscillate, integrate the information, and synchronize (Box C) and the huge amount of information storage and retrieval that all these operations imply (Box D). Thus, the results obtained with the prototype open wide and deep implications for studying more complex mechanisms of perception and brain functionalities. A full research program is engendered, that hinges on two main questions and subsequent research directions, as summarized below.

3.1 *Beyond the prototype*

One first question is whether the prototype can be evolved in the form of a quantum toolbox, capable of simulating the brain's integrated space-time-number sensory capacity as a global property of the open quantum-network dynamics. To this aim, the prototype [22] will have to be designed to allow for:

- Larger sizes, so to incorporate non-numerical biases, e.g. area and inputs-density dependence [8].
- Complex connectivity architectures, such as couplings between the spins that can be non-homogeneous in space or variable in time, to simulate re-wiring and learning [61].
- Multilayer quantum neural networks, to simulate structured external inputs and intertwining information processing from differently specialized neural networks.
- Dissipation engineering and environment-assisted quantum transport under Markovian and non-Markovian conditions, to explore optimal quantum coherence routes.
- A fully-quantum decoding protocol, based on quantum measurement theory and using the probabilistic concepts and methods developed for psychology analysis within quantum cognition theory [34-46,62].

3.2 *A Quantum Toolbox for Neurobiology Sensory Systems*

One second question is how the new quantum toolbox can be further equipped with additional quantum-matter paradigms to make connection with the neuroscience phenomena described in Box C-D. Three of them appear especially relevant:

1. The connection between quantum coherence and chaos, leading to counter-intuitive outcomes in the interplay between quantum effects and non-linear complex dynamics [26].
2. The concept of time crystal, an out-of-equilibrium phase of matter breaking time translational invariance [63], acting as a natural timekeeper despite being in continuous driven-dissipative conditions, and whose realization interestingly requires injections of disorder and the presence of interactions to produce localized states and the breaking of ergodic thermalization [64-66].
3. The holographic conjecture borrowed from black-hole studies, suggesting that the brain processes and stores information through a dual bulk and boundary description [67]: the dictionary developed by Dvali [68-69], parallels a black hole state to the critical state of highly excited low-threshold neurons, leading to the idea that exponentially-large amounts of information is stored in an quantum-entangled macrostate, with long coherence time and high-precision response to the soft external stimuli [68-69].

4. Discussion

In this paper we have introduced the QoolNeSS research program, aimed at stimulating the brain's integrated space-time-number sensory capacity [23] as a global property of the dynamical behavior of a driven-dissipative open quantum network.

Using the QoolNeSS environment, we aim at three distinct studies: (i) driving the quantum network to become a quantum time-crystal state and investigate its properties as a time-keeper; (ii) inducing the system at criticality between ordered and quantum chaotic states, and characterizing the connections for the emergence of synchronized dynamical behavior; and (iii) creating a many-body entangled state of highly excitable collective neuronal modes, and test the capability of its holographic projection to store information. The developed understanding will then be used to design QoolNeSS operations that can realize either one of (i)-(iii) and hopefully all them, and envision their implementation in a quantum technology.

The designed quantum mapping can be generalized to other perception phenomena, creating a gallery of mapping quantum Hamiltonians, capable of exploiting the most counterintuitive properties of quantum systems able to capture the underlying complexity. Once established, the simulation power of the coarse-grained mapping can be potentially exploited to enable a novel form of artificial-intelligence design, with the intrinsic advantage of being implementable with quantum technology platforms.

We remark that in QoolNeSS, global network dynamics is the crucial aspect that will enable the simulation of complex perceptual processes. Interestingly, this aspect is in line with Dvali's proposal [68-69], of searching for critical extremal points of highly excited collective modes in the brain-black-hole analogy, instead than for local minimum energy states formed during learning by plastically adjusting synaptic connections. In this paradigm, the description of the perception process: is integrated in space and in time; is prototyped on Weber's law, an ubiquitous trait of all sensory systems; does not rely nor require supervised learning; does not depend on ad hoc-noise sources, nor the probability amplitude of single excitations, nor their number, though at constant overall energy; it is worked out under ideal-observer conditions with an agnostic decoder protocol. This curved trajectory can be swept in reverse direction, learning lessons that can be used to engineer quantum technologies with increasing macroscopicity and/or complexity.

Along these lines, one might also ask whether the capability for the coarse-grained quantum model of describing the information processing of the neuronal complex system, can be integrated with microscopic driving mechanisms. While the quantum-like paradigm does not necessarily imply the emergence of microscopic quantum effects in the brain, this is a timely area of research [70-74], more recently boosted by advances in experimental methods [61], and producing educated and experiment-based guesses on microscopic paradigms for quantum information processing and transport in the brain [75]. This field is progressing in tandem with quantum biology [76-78], where significant understanding has been built on, e.g., the occurrence of quantum coherence in the energy-excitation transfers in light-harvesting complexes, from the use of femtosecond two-dimensional spectroscopy [79-81,82] associated with quantum chemistry methods [83-87].

We step on commenting this crucial point. The 0.01-to-10 KHz frequencies typical of biological processes set a quantum-to-classical crossover temperature of something like 0.1 μ K, 9 orders of magnitude below room temperature. On the other hand, as highlighted by Huelga and Plenio [82], evolutionary considerations suggest that the two extremes of fully coherent or incoherent processes can be detrimental at advantage of their optimization: when dealing with intrinsically driven-dissipative, out-of-equilibrium systems, paradigms able of enhancing quantum coherence may come in help. Overall, then, this route is worth pursuing, as it also involves a shift in the

paradigm of harnessing quantum advantage in computing or complex-information transport. We can in fact ask whether the brain hosts physical objects that might serve as spin-qubits such as the Posner molecules as envisioned by Fisher [88], transport mechanisms envisioned to be effective for large-scale entanglement, i.e. biophotons as mediators of quantum information over long distances [89], and information processing mechanisms like e.g. with radical pairs [90].

Along these lines, the prototype can also inspire the multidisciplinary, highly specialized hardwiring of a fully integrated toolbox including state-of-the art methods from quantum chemistry and quantum many-body physics, open quantum systems, computational neuroscience and related data analysis, with the methods from complex (quantum) networks working to bridge micro- and macro-descriptions. On the way back, the search for optimal quantum coherence conditions, conducted during the investigation of microscopic mechanisms, can shine light and generate ideas to design quantum technologies at the edge of the quantum-to-classical crossover. While the last decade has seen a significant progress in refined and advanced experimental methods in quantum biology, significant attention has been attracted in the field and has stimulated the growing of an interdisciplinary community. In this scenario, our research program can contribute to foster associative creativity and to develop a specialized cross-discipline from individual specializations.

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