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$X(1835)$ Decay in the Fock-Tani Formalism

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In this work we have obtained an effective Hamiltonian to describe the strong decay of pseudoscalar glueball, where the microscopic interaction between the constituent gluons have been taking into account. The pseudoscalar glueball candidate we have considered here is the $X(1835)$ resonance. In our calculation the $X(1835)$ was a pure glueball.

Keywords: Glueball, Strong Decay, Fock-Tani.

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1. Introduction

The $X(1835)$ was subject of several works which have tried to set its internal structure. The most investigated possibility is the baryonium (bound state $p\bar{p}$) (see Ref. [1,2]). The pseudoscalar glueball is another possibility that was investigated in the literature [3]. On this way we shall calculate the $X(1835)$ decay width considering this as a pseudoscalar glueball. For this purpose we shall use the Fock-Tani formalism that is an effective non relativistic field theory which consider the constituent quarks and gluons inside the hadrons [4].

2. The Fock-Tani Formalism

In the Fock-Tani formalism, the starting point are the definition of the composite glueball and meson creation operators. First, an operator that creates quark-antiquark bound-state meson can be written as

$$M^{\dagger}_{\alpha} = \Phi^{\mu\nu}_{\alpha} q^{\dagger}_{\mu} \bar{q}^{\dagger}_{\nu}, \tag{1}$$

Φ is the bound-state wave-function and $q^{\dagger}(\bar{q}^{\dagger})$ is the u, d, s quark (antiquark) creation operator. The next step is the definition of the glueball creation operator, written as a two-gluon bound-state

$$G_\alpha^\dagger = \frac{1}{\sqrt{2}} \Psi_\alpha^{\mu\nu} a_\mu^\dagger a_\nu^\dagger, \tag{2}$$

Ψ is the bound-state wave-function and $a^\dagger$ is the gluon creation operator. The gluon, quark and antiquark operators in the former equations satisfy the following canonical relations $[a_\mu, a_\nu^\dagger] = \delta_{\mu\nu}$ and $\{q_\mu, q_\nu^\dagger\} = \{\bar{q}_\mu, \bar{q}_\nu^\dagger\} = \delta_{\mu\nu}$, all other (anti)commutators are zero. The composite operators M and G in (1) and (2) have non-canonical commutators

$$[M_\alpha, M_\beta^\dagger] = \delta_{\alpha\beta} - \mathcal{M}_{\alpha\beta} \quad ; \quad [G_\alpha, G_\beta^\dagger] = \delta_{\alpha\beta} + \mathcal{G}_{\alpha\beta}\,, \tag{3}$$

where

$$\mathcal{M}_{\alpha\beta} = \Phi_\alpha^{*\mu\nu} \Phi_\beta^{\mu\sigma} \bar{q}_\sigma^\dagger \bar{q}_\nu + \Phi_\alpha^{*\mu\nu} \Phi_\beta^{\rho\nu} q_\rho^\dagger q_\mu \quad ; \quad \mathcal{G}_{\alpha\beta} = 2\Psi_\alpha^{\bar{s}t\mu\gamma} \Psi_\beta^{\gamma\rho} a_\rho^\dagger a_\mu. \tag{4}$$

The presence of $\mathcal{M}_{\alpha\beta}$ and $\mathcal{G}_{\alpha\beta}$ in (3) reflects the composite nature of the meson. In the FTf, the physical particles $M^\dagger$ and $G^\dagger$ are replaced by "ideal particles" $m^\dagger$ and $g^\dagger$, where canonical relations are satisfied:

$$[m_\alpha, m_\beta^\dagger] = \delta_{\alpha\beta} \quad ; \quad [g_\alpha, g_\beta^\dagger] = \delta_{\alpha\beta}. \tag{5}$$

For more details on this calculation see Ref. [5].

Applying the Fock-Tani formalism to a microscopic Hamiltonian H gives rise to an effective interaction $\mathcal{H}_G$,

$$\mathcal{H}_G = U^{-1} H U\,. \tag{6}$$

For the glueball decay we shall use the following phenomenological microscopic Hamiltonian

$$H_{aq\bar{q}} = \frac{g_G^2}{\Lambda} \int d^3x\, d^3y\, \psi^\dagger(\vec{x})\psi(\vec{x}) V(\vec{x},\vec{y}) \psi^\dagger(\vec{y})\psi(\vec{y})\,, \tag{7}$$

where

$$V(\vec{x},\vec{y}) = \left[\vec{\alpha}\cdot\vec{A}(\vec{x})\right]\left[\vec{\alpha}\cdot\vec{A}(\vec{y})\right],$$

$$A_i(\vec{x}) = A_i^b(\vec{x})\lambda^b/2\,,$$

and Λ is a scale parameter, set to 1 GeV in our calculation. An effective second order amplitude with a four quark-antiquark and two gluon operator structure

$\left(q^\dagger \bar{q}^\dagger q^\dagger \bar{q}^\dagger\right)(a\,a)$ can be obtained from (7). The effective Hamiltonian $\mathcal{H}_G$ obtained is

$$\mathcal{H}_G = V_I\ \Phi_\beta^{\bar{s}t\mu\rho}\,\Phi_\delta^{\bar{s}t\nu\eta}\,\Psi_\alpha^{\tau\xi}\,m_\beta^\dagger\,m_\delta^\dagger\,g_\alpha\,, \tag{8}$$

with

$$V_I \equiv \bar{\delta}^2 \frac{\alpha_s}{\Lambda\,8\sqrt{2}\pi^2}\frac{\lambda^{b_\tau}\lambda^{b_\xi}}{\sqrt{\omega_{\vec{p}_\tau}\omega_{\vec{p}_\xi}}}\,\Pi_{\mu\,\nu}(\mathcal{P}_\tau)\,\Pi_{\eta\,\rho}(\mathcal{P}_\xi)\,, \tag{9}$$

where

$$\alpha_s = g_G^2/(4\pi)\,,$$

$$\Pi_{\mu\,\nu}(\mathcal{P}_\tau) = \vec{\sigma}_{\mu\nu}\cdot\vec{\epsilon}(\mathcal{P}_\tau)\,,$$

$\mathcal{P}$ is the gluon's polarization and

$$\bar{\delta}^2 = \delta(\vec{p}_\mu + \vec{p}_\nu - \vec{p}_\tau)\delta(\vec{p}_\sigma + \vec{p}_\rho - \vec{p}_\xi)\,.$$

The wave-function of the ρ meson is written as the following product

$$\Phi_\alpha^{\mu\nu} = \chi_{S_\alpha}^{s_\mu s_\nu}\,\mathcal{C}^{c_\mu c_\nu}\,\xi^{f_\mu f_\nu}\,\varphi_{\vec{P}_\alpha}^{\vec{p}_\mu\vec{p}_\nu}, \tag{10}$$

χ is the spin contribution (S_α is the meson's spin); $\mathcal{C}$ is the color component; ξ is the flavor part and

$$\varphi_{\vec{P}_\alpha}^{\vec{p}_\mu\vec{p}_\nu} = \delta^{(3)}(\vec{P}_\alpha - \vec{p}_\mu - \vec{p}_\nu)\,\phi_{nl}(\vec{p}_\mu,\vec{p}_\nu)\,; \tag{11}$$

the spacial wave function is [6]

$$\phi_{nl}(\vec{p}_\mu,\vec{p}_\nu) = (\frac{1}{2\beta})^l N_{nl}|\vec{p}_\mu - \vec{p}_\nu|^l \exp\left[-\frac{(\vec{p}_\mu-\vec{p}_\nu)^2}{8\beta^2}\right]\mathcal{L}_n^{l+\frac{1}{2}}\left[\frac{(\vec{p}_\mu-\vec{p}_\nu)^2}{4\beta^2}\right]Y_{lm}, \tag{12}$$

and

$$N_{nl} = \left[\frac{2(n!)}{\beta^3\,\Gamma(n+l+3/2)}\right]^{\frac{1}{2}}. \tag{13}$$

$\mathcal{L}_n^{l+\frac{1}{2}}(p)$ are the Laguerre polynomials. The glueball wave-function Ψ has a similar structure to (10 - 13) with the parameter β_q replaced by β_g and with the flavor part absent in (10).

To determine the decay rate, we define the initial and final states by $|i\rangle = X_\alpha^\dagger|0\rangle$ and $|f\rangle = m_\beta^\dagger m_\gamma^\dagger|0\rangle$. The matrix element between these states is

$$\langle f \mid \mathcal{H}_{FT} \mid i\rangle = \delta(\vec{p}_\alpha - \vec{p}_\beta - \vec{p}_\gamma)h_{fi}. \tag{14}$$

In our example we shall study the following decay channel $X(1835) \to \rho\rho$. The amplitude obtained is

$$h_{fi} = \frac{4\sqrt{2}\sqrt{\frac{1}{\beta_g^5}}g^2}{9\pi^{7/4}\left(\frac{1}{\beta^2}+\frac{2}{\beta_g^2}\right)}. \tag{15}$$

3. Results

The h_{fi} decay amplitude can be combined with a relativistic phase space to give the differential decay rate [7]

$$\frac{d\Gamma_{\alpha\to\beta\gamma}}{d\Omega} = 2\pi \frac{PE_\beta E_\gamma}{M_\alpha} |h_{fi}|^2 = 2\pi \frac{PE_\pi^2}{M_{f_0}} |h_{fi}|^2, \tag{16}$$

which after integration in the solid angle Ω, a usual choice for the meson momenta is made: $\vec{P}_X = 0$ ($P = |\vec{P}_\rho|$). The meson masses assumed in the numerical calculation have standard values of $M_\rho = 770$ MeV and $M_X = 1835$ MeV. There are other sets of parameters, the coupling constants $\alpha_s = 0.6$, the wave-function widths β_q and β_g. The quark sector widths are in the range of 0.3 - 0.5 GeV (see Ref. [7]). The free parameter is the glueball's width β_g, that should be adjusted (see Fig. 1).

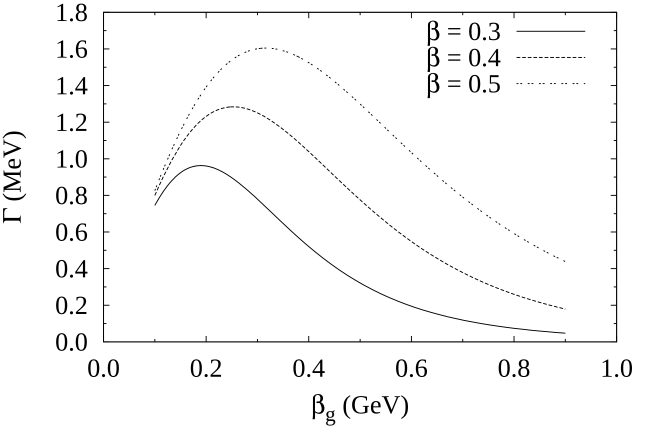

Fig. 1. Decay width of $X(1835) \to \rho\rho$ as a function of β_g for three values of β_q .

In conclusion, we have showed that the Fock-Tani formalism applied to the glueball decay seems to be promising. We need to calculate other decay channels to have more conclusive results. We also need to investigate the possibility of mixing with the quark sector. And finally use the arguments in Ref. [2] to estimate the channels observed in experiments.

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