Bianchi type V viscous fluid dark energy cosmological model with gauge function

B. Mishra
Department of Mathematics,Birla Institute of Technology and Science-Pilani, Hyderabad Campus,Hyderabad-500078, INDIA, [a]

We have investigated Bianchi type V space-time in scale invariant theory with dark energy. The matter field is considered in the form of viscous fluid. The field equations for scale invariant theory has been solved by applying a variation law for generalized Hubble's parameter [Berman [7]]. The gauge function depends on time coordinate only (Dirac gauge). The cosmological model is constructed, its physical and kinematical properties are discussed.

1 Introduction

Another modification of Einstein's theory of gravitation is the scale invariant theory of gravitation proposed by Wesson [28,29]. This is one of the prominent alternative theory. In this theory, the Einstein field equations have been written in a scale dependent way by using the conformal or scale transformation as:

$$\bar{g_{ij}} = \beta^2(x^k) g_{ij}. \tag{1}$$

where the gauge function β, in its most general formulation, is a function of all space-time coordinates. Thus, using the conformal transformation of the type given by (1), Wesson [28,29] transforms the usual Einstein field equations into

$$G_{ij} + 2\frac{\beta_{;ij}}{\beta} - 4\frac{\beta_{,i}\beta_{,j}}{\beta^2} + (g^{ab}\frac{\beta_{,a}\beta_{,b}}{\beta^2} - 2g^{ab}\frac{\beta_{;ij}}{\beta})g_{ij} + \Lambda_0\beta^2 g_{ij} = -T_{ij}. \tag{2}$$

where $G_{ij} \equiv R_{ij} - \frac{1}{2}Rg_{ij}$. Semicolon and comma respectively denote covariant differentiation with respect to g_{ij} and partial differentiation with respect to coordinates. G_{ij} is the conventional Einstein tensor involving g_{ij}. R_{ij} is the Ricci tensor, and R is the Ricci scalar. The cosmological term Λg_{ij} of Einstein theory is now transformed to $\Lambda_0\beta^2 g_{ij}$ in scale invariant theory with dimensionless cosmological constant Λ_0. G is the Newtonian gravitational parameter. T_{ij} is the energy momentum tensor of the matter field. It is worthy to note that no independent equation for β exists in this theory. In this theory, Beesham [4,5,6], Mohanty and Daud [16], Mohanty and Mishra [17,18], Mishra [11,12], Mishra and Sahoo [13,14,15] have investigated several aspects of scale invariant theory.

Cosmic observations from supernovae [Riess et al.[23]; Perlmutter et al.[22]], cosmic microwave background (CMB) radiation [Spergel et al.[26]; Komatsu et al.[10]], large scale structure (LSS) [Tegmark et al.[27]; Seljak et al.[25]], baryon acoustic oscillations (BAO) [Eisenstein et al.[8]] and weak lensing [Jain and Taylor[9]] have implied that the expansion of the universe is accelerating at the present stage. The latest data sets coming from astrophysics and cosmological observations such as CMB and supernovae survey indicate that the energy budget of the universe is the following:

[a] bivudutta@yahoo.com

74 percent dark energy, 22 percent dark matter and 4 percent ordinary baryonic matter [Riess et al.[24]; Eisenstein et al.[8]; Astier et al.[2]; Spergel et al.[26]]. Nojiri and Odintsov[19,20,21], Bamba et al.[3] have developed the cosmological reconstruction method in terms of cosmological time.

2 Field equations for Bianchi Type V metric

Here we consider Bianchi type V space-time with a Dirac gauge function $\beta = \beta(ct)$ of the form

$$ds_W^2 = \beta^2(-dt^2 + A^2dx^2 + e^{2\alpha x}(B^2dy^2 + C^2dz^2)). \tag{3}$$

The metric potentials A, B and C are functions of t only. c is the velocity of light. ds_W^2 represents the intervals in Wesson theory. Further, $x^i, i = 1, 2, 3, 4$ respectively denote for x,y,z and t only. The energy momentum tensor T_{ij} in eqn. (2) in gravitational units $G = c = 1$ is the combination of viscous fluid $T_{ij}^{(vis)}$ and dark energy fluid $T_{ij}^{(de)}$, which can be expressed as

$$T_{ij}^{(vis)} = (\rho + \bar{p})u_iu_j + \bar{p}g_{ij} = diag[-\rho, p, p, p]. \tag{4}$$

where $\bar{p} = p - \xi u^i_{;i}$, and

$$\begin{aligned} T_{ij}^{(de)} &= diag[-\rho^{de)}, p_x^{(de)}, p_y^{(de)}, p_z^{(de)}] \\ &= diag[-1, \omega_x, \omega_y, \omega_z]\rho^{(de)} \\ &= diag[-1, (\omega+\delta), (\omega+\gamma), (\omega+\eta)]\rho^{(de)}. \end{aligned} \tag{5}$$

The skewness parameters δ, γ, and η are the deviations from ω on x, y and z axes respectively. Hence, for the metric (3) and energy momentum tensor (4) and (5), the field equations for scale invariant theory (2) yield the following equations:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{\alpha^2}{A^2} + 2\frac{\dot{\beta}}{\beta}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + 2\frac{\ddot{\beta}}{\beta} - \frac{\dot{\beta}^2}{\beta^2} + \Lambda_0\beta^2 = -p + 3\Xi H - (\omega+\delta)\rho^{(de)}. \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{\alpha^2}{A^2} + 2\frac{\dot{\beta}}{\beta}\left(\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\right) + 2\frac{\ddot{\beta}}{\beta} - \frac{\dot{\beta}^2}{\beta^2} + \Lambda_0\beta^2 = -p + 3\Xi H - (\omega+\gamma)\rho^{(de)}. \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2} + 2\frac{\dot{\beta}}{\beta}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) + 2\frac{\ddot{\beta}}{\beta} - \frac{\dot{\beta}}{\beta^2} + \Lambda_0\beta^2 = -p + 3\Xi H - (\omega+\eta)\rho^{(de)}. \tag{8}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - 3\frac{\alpha^2}{A^2} + 2\frac{\dot{\beta}}{\beta}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + 3\frac{\dot{\beta}^2}{\beta^2} + \Lambda_0\beta^2 = \rho + \rho^{(de)}. \tag{9}$$

$$2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0. \tag{10}$$

The over dot on a field variable denotes exact differentiation with respect to time t. We conserve the energy momentum tensors of the two sources separately. The energy conservation equation for the viscous fluid $T^{(vis)ij}_{;j} = 0$ and dark energy $T^{(de)ij}_{;j} = 0$ components respectively defined as $\dot{\rho} + 3(\bar{p} + \rho)H = 0$ and $\dot{\rho}^{(de)} + 3\rho^{(de)}(\omega+1)H + \rho^{(de)}(\delta H_1 + \gamma H_2 + \eta H_3) = 0$. Then we split the conservation of energy momentum tensor of the dark energy into two parts [Akarsu and Kilinc [1]]. One corresponds to the deviations of equation of state (EoS) parameter and the other is the deviation free part. This can be expressed as:

$$\dot{\rho}^{(de)} + 3\rho^{(de)}(\omega+1)H = 0 \tag{11}$$

$$\rho^{(de)}(\delta H_x + \gamma H_y + \eta H_z) = 0 \tag{12}$$

Now, the behaviour of ρ^{de} is controlled by deviation free part of EoS parameter of dark energy but deviations will affect ρ^{de} indirectly. For a physically viable model of the universe consistent with observations, the choice of skewness parameters are quite arbitrary. However, we consider the skewness parameters δ, γ and η be functions of cosmic time.

3 Solutions of the field equations

In the field eqns.(6)-(9), number of unknowns exceeds the number of equations. In order to obtain explicit exact solution, we need additional constraints relating to the unknowns. With the help of special law of variations proposed by Berman [7], which yields constant deceleration parameter of the models of the universe, we will solve the system of equations. The constant deceleration parameter model for defined as

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = constant \tag{13}$$

where $R = (ABC)^{\frac{1}{3}}$. Hence, eqn.(13) yields the solution

$$R = \left(R_0^{(1+q)} + (1+q)(t - t_0)\right)^{\frac{1}{1+q}} \tag{14}$$

where R_0 is the integration constant and taken as 1 and $1 + q > 0$. Again using the physical condition that the shear scalar σ is proportional to scalar expansion θ, we take

$$B = C^m \tag{15}$$

where m is constant. With the help of eqns.(10), (14) and (15), we obtain the expression for the metric potentials as $A = R, B = C^m = R^{\frac{2m}{m+1}}$. We also consider $\beta = \beta(t) = \frac{1}{t}$ and express the directional Hubble parameter in terms of the mean Hubble parameter H as $H_x = H$, $H_y = \left(\frac{2m}{m+1}\right) H$, $H_z = \left(\frac{2}{m+1}\right) H$. Now,eqns. (6), (7), (8) and (12) yields the following

$$\gamma\rho^{de} = -\left(\frac{1-m}{1+m}\right)\left(\frac{m+5}{3(m+1)}\right)\left(\frac{\ddot{R}}{R} + 2\frac{\dot{R}^2}{R^2} - \frac{2}{t}\frac{\dot{R}}{R}\right) \tag{16}$$

$$\eta\rho^{de} = \left(\frac{1-m}{1+m}\right)\left(\frac{1+5m}{3(m+1}\right)\left(\frac{\ddot{R}}{R} + 2\frac{\dot{R}^2}{R^2} - \frac{2}{t}\frac{\dot{R}}{R}\right) \tag{17}$$

$$\delta\rho^{de} = \left(\frac{1-m}{1+m}\right)\left(\frac{2(m-1)}{3(m+1}\right)\left(\frac{\ddot{R}}{R} + 2\frac{\dot{R}^2}{R^2} - \frac{2}{t}\frac{\dot{R}}{R}\right) \tag{18}$$

We use the barotropic bulk viscus pressure and fluid relation $\bar{p} = \epsilon\rho$, where ϵ is a constant. Then the energy density ρ can be obtained as

$$\rho = \frac{\rho_0}{(ABC)^{1+\epsilon}} = \frac{\rho_0}{R^{3(1+\epsilon)}} \tag{19}$$

Subsequently, the energy density for dark energy component can be found as

$$\rho^{de} = \left(\frac{2(m^2+4m+1)}{(m+1)^2}\right)\frac{\dot{R}^2}{R^2} - 3\frac{\alpha^2}{R^2} - \frac{6}{t}.\frac{\dot{R}}{R} + \frac{3+\Lambda_0}{t^2} - \frac{\rho_0}{R^{3(1+\epsilon)}} \tag{20}$$

Now, Using eqn. (20), eqns. (16)-(18) reduces to

$$\gamma=-\frac{\left(\frac{1-m}{1+m}\right)\left(\frac{m+5}{3(m+1)}\right)\left(\frac{\ddot{R}}{R}+2\frac{\dot{R}^2}{R^2}-\frac{2}{t}\frac{\dot{R}}{R}\right)}{\left(\frac{2(m^2+4m+1)}{(m+1)^2}\right)\frac{\dot{R}^2}{R^2}-3\frac{\alpha^2}{R^2}-\frac{6}{t}\cdot\frac{\dot{R}}{R}+\frac{3+\Lambda_0}{t^2}-\frac{\rho_0}{R^{3(1+\epsilon)}}} \tag{21}$$

$$\eta=\frac{\left(\frac{1-m}{1+m}\right)\left(\frac{1+5m}{3(m+1)}\right)\left(\frac{\ddot{R}}{R}+2\frac{\dot{R}^2}{R^2}-\frac{2}{t}\frac{\dot{R}}{R}\right)}{\left(\frac{2(m^2+4m+1)}{(m+1)^2}\right)\frac{\dot{R}^2}{R^2}-3\frac{\alpha^2}{R^2}-\frac{6}{t}\cdot\frac{\dot{R}}{R}+\frac{3+\Lambda_0}{t^2}-\frac{\rho_0}{R^{3(1+\epsilon)}}} \tag{22}$$

$$\delta=\frac{\left(\frac{1-m}{1+m}\right)\left(\frac{2(m-1)}{3(m+1)}\right)\left(\frac{\ddot{R}}{R}+2\frac{\dot{R}^2}{R^2}-\frac{2}{t}\frac{\dot{R}}{R}\right)}{\left(\frac{2(m^2+4m+1)}{(m+1)^2}\right)\frac{\dot{R}^2}{R^2}-3\frac{\alpha^2}{R^2}-\frac{6}{t}\cdot\frac{\dot{R}}{R}+\frac{3+\Lambda_0}{t^2}-\frac{\rho_0}{R^{3(1+\epsilon)}}} \tag{23}$$

Moreover, the EOS parameter ω is obtained as

$$\omega=\frac{-\frac{2}{3}\left(\frac{m^2+4m+1}{(1+m)^2}\right)\left(2\frac{\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}-\frac{2}{t}2\frac{\dot{R}}{R}\right)-\frac{\alpha^2}{R^2}+\frac{3+\Lambda_0}{t^2}-\frac{\epsilon\rho_0}{R^{3(1+\epsilon)}}}{\left(\frac{2(m^2+4m+1)}{(m+1)^2}\right)\frac{\dot{R}^2}{R^2}-3\frac{\alpha^2}{R^2}-\frac{6}{t}\cdot\frac{\dot{R}}{R}+\frac{3+\Lambda_0}{t^2}-\frac{\rho_0}{R^{3(1+\epsilon)}}} \tag{24}$$

4 Some Physical and Kinematical properties of the Model

In this section, we have investigated some physical behaviour of the constructed model. The scalar expansion of the model, $\theta=\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}=3\left[1+(1+q)(t-t_0)\right]^{-1}$, which indicates that the scalar expansion remains constant for $t=0$; however for large value of t, the expansion decreases. The spatial volume found to be, $V=R^3=ABC=\left[1+(1+q)(t-t_0)\right]^{\frac{1}{1+q}}$. It is observed that the spatial volume is unity at $t=t_0$ and it increases as t increases. Thus, the universe starts evolving with unit volume at $t=t_0$ and expands with cosmic time t. Also, for $1+q>0$, the universe is expanding. The shear scalar, $\sigma^2=\frac{1}{2}\left(\Sigma H_i^2-\frac{1}{3}\theta^2\right)=\left(\frac{m-1}{m+1}\right)^2\left[1+(1+q)(t-t_0)\right]^{-2}$ becomes constant for large value of t. Therefore the shape of the universe remains unchanged during evolution. Moreover, $\frac{\sigma^2}{\theta^2}$ turns out to be a constant, the model does not approach isotropy for large value of t. However, for $m=1$, the model becomes isotropic.

The generalized mean Hubble's parameter H is $H=\frac{1}{3}(H_x+H_y+H_z)=\left[1+(1+q)(t-t_0)\right]^{-1}$. The Hubble's parameter is unity at $t=t_0$. The rate of expansion is accelerated or decelerated depends on the signature of the parameter. However, as $1+q>0$, the model indicates acceleration. The average anisotropy parameter is calculated as $A_m=\frac{4}{3}\Sigma\left(\frac{\Delta H_i}{H}\right)^2=\frac{2}{3}\left(\frac{m-1}{m+1}\right)^2$. Now, the mean anisotropic parameter is uniform throughout the evolution of the universe since A_m is constant. The energy density, $\rho(t)=\frac{\rho_0}{R^{3(1+\epsilon)}}=\frac{\rho_0}{[Q(t)]^{\frac{3(1+\epsilon)}{1+q}}}$, vanishes for large t,where $Q(t)=\left[1+(1+q)(t-t_0)\right]$. The EoS parameter ω, obtained as

$$\omega=\frac{-\frac{2}{3}\left(\frac{m^2+4m+1}{(1+m)^2}\right)\left(\frac{1-2q}{[Q(t)]^2}-\frac{4}{t[Q(t)]}\right)-\frac{\alpha^2}{[Q(t)]^{\frac{2}{1+q}}}+\frac{3+\Lambda_0}{t^2}-\frac{\epsilon\rho_0}{[Q(t)]^{\frac{3(1+\epsilon)}{1+q}}}}{\left(\frac{2(m^2+4m+1)}{(m+1)^2}\right)\frac{1}{[Q(t)]^2}-3\frac{\alpha^2}{[Q(t)]^{\frac{2}{1+q}}}-\frac{6}{t[Q(t)]}+\frac{3+\Lambda_0}{t^2}-\frac{\rho_0}{[Q(t)]^{\frac{3(1+\epsilon)}{1+q}}}}$$

It is observed that δ and γ are functions of time t. Moreover, δ and γ never diverges as t vanishes.

5 Conclusions

In this paper, we have investigated Bianchi type V dark energy cosmological models with variable EoS parameter in scale invariant theory of gravitation. In the constructed model, it is observed that the dark energy model in scale invariant theory is consistent with the recent observations of Type Ia supernovae. The EoS parameters and skewness parameter turn out to be functions of cosmic time t. This study is significant, because dark energy is the best candidate to explain the cosmic acceleration in the general and alternative theories of gravitation.

Acknowledgement

BM acknowledge SERB-DST, New Delhi, India for financial support to carry out the Research project[No. SR/S4/MS:815/13].

References

1. Akarsu, O., Kilinc, C.B.: Gen. Rel Grav. **42**, 119 (2010).
2. Astier, P., et al.: Astron. Astrophys. **447**, 31 (2006).
3. Bamba, K., et al.: Astrophys Space Sci. **342**, 155 (2012).
4. Beesham, A.: Clasic. Quant. Grav. **3**, 481(1986a).
5. Beesham, A.: Astophys. Space Sci. **119**, 415(1986b).
6. Beesham, A.: Clasic. Quant. Grav. **3**, 1027(1986c).
7. Bermann, M.S.: Nuovo Cimento B**74**, 182(1983).
8. Eisenstein, D.J., et al.: (SDSS) Astron. J. **633**, 560 (2005).
9. Jain, B., Taylor, A.: Phys. Rev. Lett.**91**, 141302 (2003).
10. Komatsu, E., et al.: The Astrophys J. **180**,330 (2009).
11. Mishra, B.: Chinese. Phys. Lett.**21(12)**, 2359(2004).
12. Mishra, B.: Turk. J. Phys. **32**, 357(2008).
13. Mishra, B., Sahoo, P.K.: Int. J. pure and Appl. Maths.**80(4)**, 535(2012a).
14. Mishra, B., Sahoo, P.K.: Int. J. Theo. Phys.**51(2)**, 399(2012b).
15. Mishra, B., Sahoo, P.K.: Astrophys. Space Sci.**349**, 491(2014).
16. Mohanty, G., Daud, S.M.: Theo. Appl. Mechanics**23**, 115(1997).
17. Mohanty, G., Mishra, B.: Czech. J. Phys**51**, 525(2002).
18. Mohanty, G., Mishra, B.: Astrophys. Space Sci.**283**, 67(2003).
19. Nojiri,S., Odintsov, S.D.: Phys. Rev.D **68**, 123512 (2003).
20. Nojiri, S., Odintsov, S.D.: Phys. Rev. D. **72**, 023003 (2005).
21. Nojiri, S., Odintsov, S.D.: Phys. Rep.**505**, 59 (2011).
22. Perlmutter, S., et al.: Astrophys. J. **517**, 565 (1999).
23. Riess, A.G., et al.: Astron. J. **116**, 1009 (1998) .
24. Riess, A.G., et al. :(Supernova Search Team): Astron. J. **607**, 665 (2004).
25. Seljak, U., et al.: Phys Rev. D **71**,103515 (2005).
26. Spergel, D.N., et al.: (WMAP) Astrophys. J. Suppl. Ser. **170**,3771 (2007).
27. Tegmark, M., et al.: Phys Rev. D **69**,103510 (2004).
28. Wesson, P.S.: Astron. Astrophys.**102**, 45 (1981a).
29. Wesson, P.S.: Mon. Not. R. Astro. Soc.**39**, 429 (1981b).


Ale house
QUALITY ALES
Served Daily
Lodgings