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Spin-Tracking Simulations in an idealized COSY Model using Bmad

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The matter-antimatter asymmetry in our Universe might be understood by investigating the EDM (Electric Dipole Moment) of elementary charged particles. A permanent EDM of a subatomic particle violates time reversal and parity symmetry at the same time and would be an indication for further CP violation than established in the SM (Standard Model). The JEDI-Collaboration (Jülich Electric Dipole moment Investigations) in Jülich has performed a first direct EDM measurement with deuterons in the so-called precurser experiments at the storage ring COSY (COoler SYnchrotron) at Forschungszentrum Jülich in Germany. In order to understand the measured data and to disentangle a potential EDM signal from systematic effects, spin tracking simulations in an accurate computer model of COSY are needed. Therefore, a COSY model was implemented using the software library Bmad. As a first step, the effect of the EDM on the invariant spin axis was benchmarked in an idealized COSY model. The results of the benchmarking procedure and the methods used to determine the invariant spin axis are presented in this paper.

KEYWORDS: edm, cosy, bmad, spin-tracking

1. Introduction

In order to explain the matter-antimatter asymmetry in the Universe, $\mathcal{CP}$-violating processes beyond the ones already known are needed [1]. A non-vanishing EDM of a subatomic particle is a candidate for such a process, since it is a source of $\mathcal{P}$ and $\mathcal{T}$ violation leading to $\mathcal{CP}$ violation, assuming the $\mathcal{CPT}$-theorem holds. An EDM is similar to the MDM (Magnetic Dipole Moment) and is predicted by the SM. Its magnitude, however, is expected to be unobservably small with current techniques. Therefore, the measurement of an EDM at a higher magnitude would be an indication for further CP violation than explained by the SM. The so-called precurser experiments were performed by the JEDI-Collaboration to perform an EDM measurement for deuterons at the storage ring COSY. A storage ring allows a direct measurement of an EDM, as the interaction of particle's spin with electromagnetic field results in spin rotations defined by EDM and MDM contribution [2, 3]. In order to separate systematic effects caused by misaligned elements, steerer contributions, etc., from a potential EDM signal, spin tracking simulations in a simulation model of COSY are required [4]. The software tool used to study and benchmark the deuertons EDM effect in a simulation is the Fortran based library Bmad [5].

2. Spin Dynamics in Storage Rings

The impact of electromagnetic fields in a storage ring on the spin $\vec{S}$ is described by the Thomas-BMT equation [2, 3]. Since COSY is a pure magnetic ring, only magnetic fields $\vec{B}$, pointing in the vertical direction, act on the particle's spin. Therefore the Thomas-BMT equation is reduced to equation (1).

$$\frac{d\vec{S}}{dt} = (\vec{\Omega}_{MDM} + \vec{\Omega}_{EDM}) \times \vec{S} = -\frac{q}{m}\left(G\vec{B} + \frac{\eta}{2}\vec{\beta} \times \vec{B}\right) \times \vec{S} \quad (1)$$

In equation (1), $\vec{\Omega}_{MDM}$ and $\vec{\Omega}_{EDM}$ indicate the angular frequency induced by the MDM and the EDM. The quantities q, m, G are the particle's electric charge, its mass and the gyromagnetic anomaly, while $\vec{\beta}$ gives its velocity. The dimensionless proportionality factor η contains the EDM's magnitude. As shown by the Thomas-BMT equation, a permanent EDM rotates the spin vertically n_y, while the MDM rotates the spin horizontally n_x under the assumption of $\vec{\beta} \perp \vec{B}$ and $\vec{\beta}=(0, 0, \beta_z)^T$. For this reason, a characterization of the spin motion can be cone using the invariant spin axis, the vector that is perpendicular to the spin's precession plane. Assuming no EDM contribution and an ideal ring, the invariant spin axis should always point in vertical direction n_y. However, in presence of an EDM, the invariant spin axis is tilted in the horizontal direction n_x by the angle ξ as indicated in figure (1). A theoretical prediction of ξ is given by equation (2):

$$\xi = \arctan\left(\frac{\eta\beta}{2G}\right) \quad (2)$$

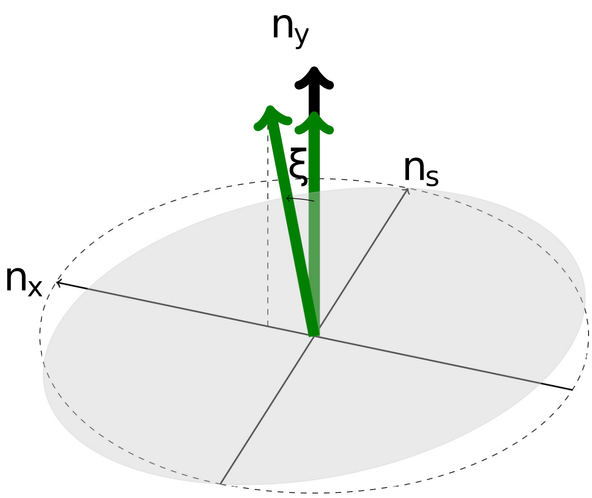

Fig. 1. A permanent EDM of magnitude η tilts the invariant spin axis in horizontal direction n_x by the angle ξ. The longitudinal direction n_s is not affected by a permanent EDM.

3. Spin-Tracking Simulations

3.1 Invariant Spin Axis

In a simulation, the invariant spin axis can be studied directly by comparing the spin vectors of two successive turns $\vec{s}_i$ and $\vec{s}_{i+1}$. Their cross product averaged over many revolutions

t, as shown in equation (3), indicates the invariant spin axis $\langle\vec{n}\rangle$, which is perpendicular to the particles' spin precession plane. The simulated precession plane for an idealized COSY lattice with and without EDM contribution is shown in figure (2):

$$\langle\vec{n}\rangle = \frac{1}{t-1}\sum_{i=1}^{t-1}\left(\frac{\vec{s}_i \times \vec{s}_{i+1}}{|\vec{s}_i \times \vec{s}_{i+1}|}\right) \tag{3}$$

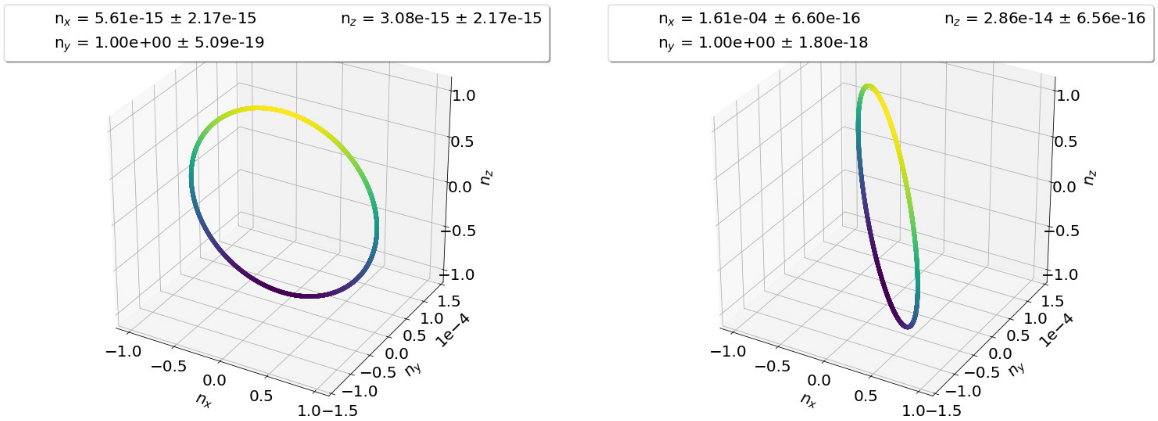

Fig. 2. Spin precession plane simulated by tracking the reference particle for 10^4 revolutions in an idealized Bmad COSY model. The left figure displays a simulation without an EDM signal, while the right figure shows a simulation with an EDM signal of $\eta = 10^{-4}$ included. The invariant spin axis $\langle\vec{n}\rangle$ is the vector perpendicular to this plane. Its orientation is displayed in the legend.

As figure (2) shows, the simulated EDM signal tilts the invariant spin axis in the horizontal n_x direction. The magnitude of the tilt angle ξ is in agreement with the theoretical prediction given by equation (2). Unfortunetely, an experiment is not capable to measure the spin vector of a particle each turn with sufficient statistics. Therefore, one has to find another way to observe the effect of the EDM on the invariant spin axis.

3.2 Experiment at COSY

As the vertical component of the spin due to the EDM has only a tiny amplitude, it is better to look for a way to get a macroscopic build-up of the vertical polarization. Therefore, the so-called RF (Radio-Frequency) Wien filter, a RF device with horizontal electric E_x and vertical magnetic fields B_y, was implemented into COSY. A particle beam travelling through the center of this device is not perturbated as the fields are set up so that the Lorentz force is zero. This way, the EDM signal accumulates over time, resulting in a vertical build-up of polarization [6, 7] if the Wien filter runs at resonance. In combination with a solenoid providing a longitudinal magnetic field, it allows the invariant spin axis to be determined experimentally. This experimental set-up was implemented in the Bmad simulation. The Wien filter is changing its fields on one of the harmonics k of the spin precession frequency $\nu_{s,0}$ so that a particle passing through the device receives a spin kick in the same direction each turn. This is indicated by equation (4):

$$E_x = E_0 \cdot A_0 \cos(2\pi f_{rev}|k + \nu_{s,0}| + \phi_{rel}) \;\text{ and }\; B_y = B_0 \cdot A_0 \cdot \cos(2\pi f_{rev}|k + \nu_{s,0}| + \phi_{rel}) \tag{4}$$

In this equation, the quantity f_{rev} denotes the revolution frequency of a particle, while A_0 is a dimensionless scaling factor used to reduce simulation time. A relative phase ϕ_{rel} has to be chosen for the RF-Wien filter. The magnitude of the build-up of the vertical polarization over time depends on the relative phase. The build-up of vertical polarization for a given phase is sketched in the left graph of figure (3). The magnitude of the build-up in dependency of the relative phase is shown in the right graph of figure (3).

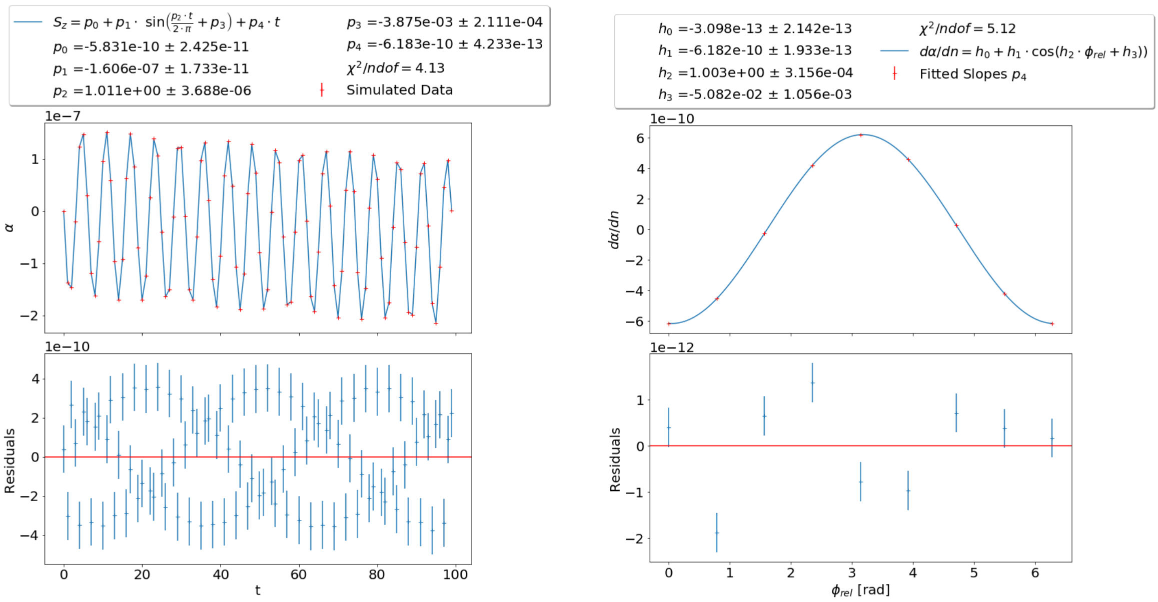

Fig. 3. The left graph shows the build-up of vertical polarization $\alpha = \arctan(P_V/P_H)$ as a function of revolutions t in COSY, using a RF-Wien filter scaling factor of $A_0 = 1000$ and a relative RF-Wien filter phase of $\phi_{rel} = 2\pi$. The fit parameter p_4 indicates the vertical build-up. This fit parameter was simulated for a variaty of relative phases and plotted against the different relative phases in the right graph. The fitparameter h_1 of a cosinus fit to these data shows the so-called EDM resonance strength ϵ_{EDM}. The error bars in the left graph originate from the fact that the Wien filter changes the spin tune each turn. The variation of the spin tune times the amplitude of the oscillation of the individual spin axis defines the magnitude of the error bars. The error bars in the right graph originate from the error on the fitparameter p_4.

By calculating the largest possible build-up for a given RF-Wien filter rotation and solenoid strength, one obtains the EDM resonance strength ϵ_{EDM}. The resonance strength was simulated for a variety of RF-Wien filter rotations ϕ_{WF} and solenoid strengths ξ_{SN}. The data obtained by this method can be summarized in a resonance map, where the minimum $(\phi_{WF,0},\ \xi_{SN,0})$ indicates the tilt of the invariant spin axis (n_x, n_z) without Wien Filter rotation and solenoid off. A simulated resonance map with and without EDM contribution is shown in figure (4); the function to fit the minimum is displayed in equation (5) [8]:

$$\epsilon_{EDM} = \left(A_{WF}^2 (\phi_{WF} - \phi_{WF,0})^2 + A_{SN}^2 \left(\frac{\xi_{SN} - \xi_{SN,0}}{2\sin(\pi\nu_{s,0})} \right)^2 \right)^{1/2} + \epsilon_0 \qquad (5)$$

The tilt of the invariant spin axis in horizontal direction n_x due to the deuterons EDM signal becomes visible in the $\phi_{WF,0}$ parameter of equation (5). The simulation of the resonance map yields a result which is in agreement with the theoretical prediction in equation (2).

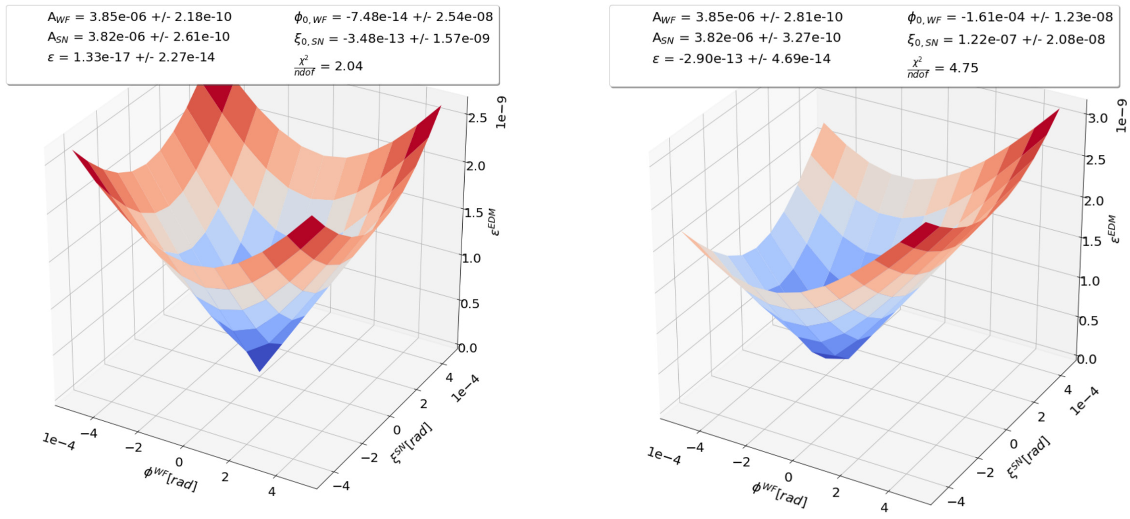

Fig. 4. The left graph shows a simulated resonance map without an EDM contribution, while the right graph shows a simulated resonance map with an EDM signal of $\eta = 10^{-4}$. The x- and y-axis indicate the RF-Wien filter rotation and the solenoid strength, while the z-axis shows the EDM resonance strength. The legend shows the fit parameter when fitting equation (5) to the simulated data.

The longitudinal direction n_z, indicated by $\xi_{SN,0}$ is unaffected by the deuterons EDM signal. Other parameter used in equation (5) are the scaling factors A_{WF} and A_{SN} and the minimum resonance strength ϵ_0.

4. Conclusion and Outlook

It could be shown that an EDM signal of a deuteron can be be successfully implemented in a Bmad COSY model simulation. In the simulation, the EDM signal can be calculated via a direct method by recording the spin vectors of successive turns and calculating the invariant spin axis this way or via a resonance map using the so-called RF-Wien filter. It was demonstrated that the simulated tilt of the invariant spin axis is in agreement with the theoretical prediction for both methods. Next steps will be the implementation of systematic effects like element misalignments in the simulation and the comparison of simulated data with the precurser data from COSY. More details about the experimental data are discussed in contributions number 79 of this conference.

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