FRACTAL STRUCTURES AND INTERMITTENCY IN QCD

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ABSTRACT

New results are presented for fractal structures and intermittency in QCD parton showers. The particle momenta define a fractal curve in energy-momentum space with a dimension $1+\sqrt{3\alpha_s/2\pi}$, i.e one plus the anomalous dimension of QCD. For the distributions in small y-regions the property with jets within jets corresponds to a fractal structure with a dimension $\sqrt{3\alpha_s/2\pi}$ (for high moments). At lower energies in the PEP-PETRA range the soft hadronization is important. The experimental data can be understood if the properties of the directly produced pions are carefully taken into account.

1. INTRODUCTION

The results presented in this talk are obtained in collaboration with A. Nilsson and C. Sjögren in Lund. A more extensive discussion is presented in refs. [1,2].

e^+e^--annihilation into hadrons is often described in terms of two phases, a hard perturbative phase, described in terms of quarks and gluons, and a soft phase in which the energy of these partons is transformed into the observable hadrons. The latter phase can be described in terms of clusters or in terms of strings.

To study the hard phase we use two important tools:

- The dipole formulation of QCD cascades [3]
- An infrared stable measure on parton states [4].

Dipole formulation

A high energy $q\bar{q}$-system radiates gluons according to the dipole formula

$$dn = \frac{3\alpha_s}{4\pi^2}\frac{dk_\perp^2}{k_\perp^2}dy d\varphi \tag{1}$$

The phase space available is given by the relation $2|y| \le \ln(s/k_\perp^2)$, which is a triangular region in a $y-\ln k_\perp^2$ diagram. If two gluons are emitted, then the distribution of the hardest gluon is described by eq. (1), while the distribution of the second, softer, gluon corresponds to two dipoles, one between the quark and the first gluon, and the second between this gluon and the antiquark [5].

The total phase space available in the two dipoles corresponds to adding a fold to the triangular region in the $y-\ln k_\perp^2$-plane (see fig. 1), with the constraint $k_{\perp 2}^2 < k_{\perp 1}^2$. This procedure can be generalized so that the distribution of a third, still softer, gluon corresponds to three dipoles, etc. Thus, with many gluons the gluonic phase space can be represented by the multifaceted surface in fig. 1. Each gluon adds a fold to the surface, which increases the phase space for softer gluons.

In this process the recoils are neglected, as is normal in the leading log approximation. Recoil effects and kinematical constraints can be taken into account in a MC simulation program. Such a program called ARIADNE is developed by U. Pettersson and L. Lönnblad [6].

Multiplicity measure

In string fragmentation the hadronic multiplicity for a simple $q\bar{q}$-system is proportional to $\ln s$. The hadrons are evenly distributed in rapidity, which means that their energy-momentum four-vectors are distributed around a hyperbola (cf fig. 2a).

For a $q\bar{q}g$-system, we obtain in the Lund string model a bent string with two straight segments. If the energy in the segments is s_{12} and s_{23}, the average multiplicity, n, is given by the relation

$$\langle n \rangle \sim \ln s_{12} + \ln s_{23} \approx \ln s + \ln k_\perp^2 \tag{2}$$

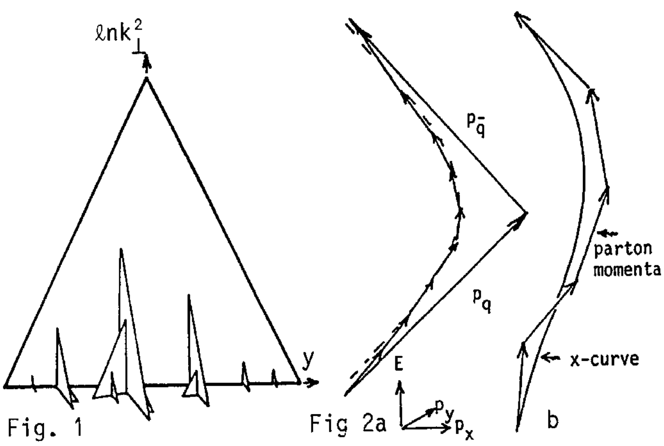

Fig. 1. The phase space available for a gluon emitted by a high energy $q\bar{q}$-system is a triangular region in the $y-\ln k_\perp^2$-plane.

Fig. 2a. In a $q\bar{q}$-system the hadron momenta are distributed around a hyperbola in energy-momentum space. **b.** For a multi-gluon state the hadron momenta are distributed around a curve (the x-curve) which smoothly follows the parton momenta.

Here $k_\perp$ is the transverse momentum of the gluon. The momentum distribution of the hadrons is in this case given by two hyperbolae.

For a multigluon state we find in the same way

$$\langle n\rangle \sim \sum \ln\left(s_{i,i+1}/m_0^2\right) \approx$$
$$\approx \ln\left(s/m_0^2\right) + \sum \ln\left(k_{\perp i}^2/m_0^2\right) = \lambda \qquad (3)$$

This expression, which we call λ, is an "effective rapidity range". It is possible to calculate the distribution $P(\lambda,s)$ in λ for fixed s [4]. Thus we find e.g.

$$\bar{\lambda} = \sqrt{\frac{L}{\alpha_0}} I_1\left(2\sqrt{\alpha_0 L}\right) \sim (\ln s)^{1/4} \exp\left(2\sqrt{\alpha_0 \ln s}\right)$$

where $\alpha_0 / L = 3\alpha_s / 2\pi$ (4)

The quantity λ can be generalized in an infrared stable way. For a fixed energy W the perturbative cascade gives a parton state with definite parton momenta, and thus a definite value of λ. The soft hadronization mechanism then gives a certain hadronic state with a hadron multiplicity n which depends only on λ and not on W [7].

For the momentum distribution of the hadrons it turns out that it is possible to generalize the hyperbolae in the $q\bar{q}$- and $q\bar{q}g$-cases (fig. 2a) and define a timelike curve in energy-momentum space [7]. This curve (called the x-curve) follows smoothly the (colour ordered) parton momenta (see fig. 2b). The length of the x-curve is given by λ and if we just cut it into equal pieces, then we obtain an average momentum distribution of the hadrons. The soft hadronization just adds limited fluctuations around this average. These features give a quantitative meaning to the notion of local parton-hadron duality.

2. FRACTAL STRUCTURES AND INTERMITTENCY IN THE PERT. PHASE [1]

The x-curve has a fractal structure

The x-curve is "longer" if it is studied with a higher resolution (cf Koch's snowflake curve). The invariant length is shorter, and is thus not a suitable measure. We divide the curve in pieces with a certain invariant length $\hat{s}$ corresponding to a given resolution and we define the length of each piece as the length of a hyperbola passing through the endpoints, which is ~ $\ln\left(\hat{s}/m_0^2\right)$. Thus we define the length S obtained with the resolution $\hat{s}$.

$$S \sim (\text{no of pieces}) \cdot \ln\left(\hat{s}/m_0^2\right) \qquad (5)$$

Within the analytic approximation discussed above we find for asymptotic energies $\left(s >> \hat{s} >> m_0^2\right)$

$$S \sim (\ln s)^{1/4} \exp\left(2\sqrt{\alpha_0 \ln s}\right)(\ln \hat{s})^{3/4} \exp\left(-2\sqrt{\alpha_0 \ln \hat{s}}\right) \qquad (6)$$

Thus we see that the fractal dimension D is given by the following expression

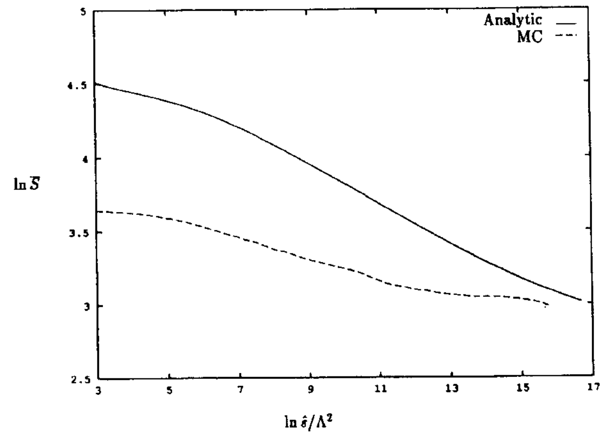

Fig. 3. The length S of the x-curve, as a function of the resolution $\hat{s}$, in e^+e^--annihilation at 1000 GeV. The dashed line is the analytic approximation and the solid line shows the MC results.

$$D = 1 - \frac{d(\ln S)}{d(\ln \hat{s})} \approx 1 + \sqrt{\frac{\alpha_0}{\ln \hat{s}}} = 1 + \sqrt{\frac{3\alpha_s}{2\pi}} \qquad (7)$$

In the square root we recognize the anomalous dimension of QCD. Thus the x-curve gives a geometrical interpretation of this anomalous dimension. It is possible to calculate $S(s,\hat{s})$ also with the Monte Carlo simulation program, which takes recoils and kinematical constraints better into account. The result is shown in fig. 3.

Distributions in y

A lot of interest has recently focused on the notion of intermittency. It was suggested by Bialas and Peschanski [8] to study the factorial moments of multiplicity distributions in a rapidity range δy.If we use normal moments, or scaled normal moments

$$C_q = \langle n^q \rangle / \langle n \rangle^q \qquad (8)$$

rather than the factorial moments, then it is possible to study also noninteger variables, e.g. the piece of the x-curve which has its tangent within an interval in y. This would represent the hadron distribution without the noise from the soft hadronization, and thus reveal the properties of the perturbative cascade. If C_q has a powerlike behaviour, $C_q \sim (1/\delta y)^\rho$, this can be interpreted in terms of a (multi)fractal dimension $D_q = 1 - \rho/(q-1)$ [9,10].

The feature of QCD with jets within jets within jets, similar to a Cantor dust, gives fractal properties.We see that for large q-values $\langle \lambda^q \rangle$ is dominated by few events with large values of λ. These are events with a hard jet where the tip of the jet is inside the y-range $\delta y = \delta$.

It is possible to show that the multiplicity in this tip corresponds to an e^+e^--annihilation event with a cms energy given by $W = k_\perp \delta$. Summing over all possible jet transverse momenta we then obtain

$$\langle \lambda^q \rangle_{tip} \sim q\sqrt{\frac{3\alpha_s}{2\pi}} \bar{\lambda}^q_{e^+e^-}(s)\,\delta^{1+q\sqrt{\frac{3\alpha_s}{2\pi}}} \qquad (9)$$

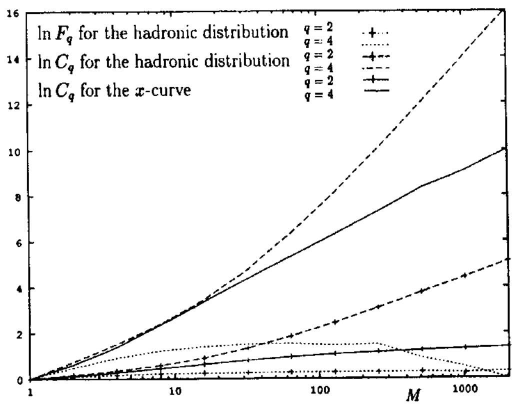

Fig. 4. Moments C_2 and C_4 for the x-curve (solid lines) compared with normal moments C_q and factorial moments F_q for the hadron distribution. $M \propto 1/\delta$ is the number of bins.

With no jets we would obtain a background contribution $\lambda = \delta$ corresponding to a straight string. This gives

$$\langle \lambda^q \rangle_{BG} = \delta^q \tag{10}$$

When q and s are large and δ is small the jet tips dominate, while when q and s are small and δ large, the background dominates. From eqs. (9,10) we get the dimension

$$D_q \approx \sqrt{3\alpha_s(s\delta^2)/2\pi} \qquad q,s \text{ large, } \delta \text{ small}$$
$$D_q \sim 1 \qquad q,s \text{ small, } \delta \text{ large} \tag{11}$$

If we include the rest of the jets and not only the tips, we obtain a sum of terms with dimension between $\sqrt{3\alpha_s/2\pi}$ and 1. Fig. 4 shows MC results for the scaled moments of the λ-distribution at 200 GeV. Here the slope of the curve is given by $(q-1)(1-D_q)$. We note that for δ large the curve is rather flat because $D \sim 1$. For smaller δ it becomes steeper and D is closer to $\sqrt{3\alpha_s/2\pi}$. For very small δ it flattens out again, because the running α_s becomes larger when $s\delta^2$ becomes small. It is also possible to use Monte Carlo to study the variation with q. For small q the terms with higher dimensions dominate while for larger q the jet tips dominate giving lower dimensions.

Hadron distributions

The perturbative QCD cascade gives a partonic state with a certain value of λ within a given rapidity window. The soft hadroniztaion gives a definite hadronic multiplicity n. If the distribution in n for fixed λ is Poissonian, then the factorial moments of $n, C_q(n)$, are equal to the normal moments of the λ-distributions $C_q(\lambda)$. In this case the normal multiplicity moments $C_q(n)$ blow up for small rapidity windows. However, in string fragmentation the fluctuations are smaller than in a Poisson distribution. Furthermore, the production of particles in neighbouring bins are correlated.

Fig. 4 shows both normal moments and factorial moments for the hadron distribution, compared with the moments from the x-curve. We see that indeed the normal moments shoot up above the x-curve for small δy but that they nevertheless are closer to the x-curve than the factorial moments, which are much further below. It is not clear if this also implies that the normal moments are more related to the underlying dynamics of perturbative QCD.

3. INTERMITTENT EFFECTS IN SOFT HADRONIZATION [2]

In many experiments [11] $\ln F_q$ rises with $\ln(1/\delta y)$ with a steeper slope than Monte Carlo calculations based on Lund string [12] or the Webber cluster model [13].

These models are tuned to fit other variables, and are not retuned to reproduce the intermittency slopes. We have found that in the Lund model a large part of the effect is due to directly produced pions which are neighbours in rank (i.e. pions which are not decay products of ρ's and ω's and other resonances). This implies that the result is very sensitive to the vector to pseudoscalar ratio. The default value in the MC is $P/(V+P) = 0.5$, and we note that the vector meson production measured by NA22 [14] and EMC [15] is smaller than the corresponding MC results.

This observation also implies that the result is very sensitive to the fragmentation $p_\perp$ for direct pions, i.e. the $p_\perp$ (with respect to the string direction) given to the pions in the rest frame of the string.

The production of $q\bar{q}$-pairs in the string can be described as a tunnelling process [16,12]. This gives a Gaussian distribution in the transverse momenta $k_\perp$. The produced quark must also fit into the bound state meson wavefunction. This effect is estimated to produce an extra factor $(m_\perp)^{-1}$, where $m_\perp$ is the transverse mass of the meson [12]. This can explain the suppression of vector mesons relative to the pseudoscalar mesons. It also favours small $p_\perp$ for pions which have a small mass, an effect which is neglected in the MC programs. Furthermore, it causes correlations so that the pions tend to come in bunches, which increases the intermittency even more.

In fig. 5 we show results obtained with $V/(V+P) = 0.35$ and where the $p_\perp$ of the direct pions is suppressed to $\langle p_\perp^2 \rangle = (0.07)^2$ (in the rest frame of the string).

We note, however, that the TASSO data shown in fig. 5 are not corrected. When the MC generated events are processed through the detector simulation, the result is

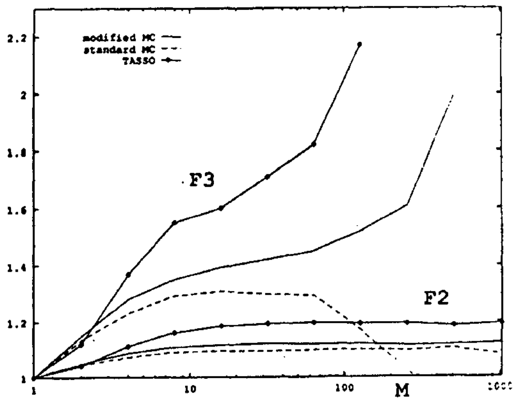

Fig. 5. Normalized factorial moments F_2 and F_3 for e^+e^- at 36 GeV. The squares are the uncorrected data from TASSO [17]. The dashed lines show results from the ARIADNE MC with default values for the parameters and the solid lines show results where $V/(V+P)=0.35$ and the directly produced pions have reduced $p_\perp$ with respect to the string.

increased [17]. We also note that the MC does not include the Bose-Einstein effect. This effect must increase the slopes of the factorial moments. It can not be the dominant effect, because it would give a larger signal for same sign particles, in contradiction with the data [17,18]. Direct charged pions which are neighbours in rank must have opposite charges. Thus the effect discussed above is largest for particles of opposite charges. We conclude that with about equal contributions from detector acceptance, Bose-Einstein interference and the effect from directly produced pions discussed here, it is likely that we can satisfactorily describe the difference between the MC simulations and the data in fig. 5.

We also note that the reduced $p_\perp$ for direct pions also can explain the large K/π ratio at high $p_\perp$ observed at the ISR [19].

We finally note that the effects discussed here should be noticeable not only in e^+e^--annihilation, but also in DIS and hadronic collisions if the particles also here are produced from a stringlike colour field.

4. CONCLUSIONS

A. For the **hard perturbative phase** of QCD we have found the following properties.

• The so-called **x-curve** is defined on a parton state and gives the average momentum distribution of the final hadrons. It is a **fractal curve**, embedded in four-dimensional energy-momentum space (cf Koch's snowflake curve). If suitably defined the length increases with the resolution according to a dimension $D=1+\sqrt{3\alpha_s/2\pi}$. Thus the x-curve provides a geometrical interpretation of the anomalous dimension of QCD. We also note that the dimension varies with the resolution in accordance with the running coupling constant α_s.

• For the **distributions in** y the feature with jets within jets corresponds to a fractal structure (cf Cantor dust). For large energies and high moments the multifractal dimensions D_q are given by $\sqrt{3\alpha_s/2\pi}$, i.e. we find again the anomalous dimension of QCD.

• For the **hadronic multiplicity** we note that the normal moments are closer to the x-curve results than the factorial moments.

B. For the **soft hadronization phase** we have observed that in the Lund string fragmentation model the intermittency signal is most sensitive to directly produced pions which are neighbours in rank. From the $q\bar{q}$-tunnelling mechanism we expect that direct pions have smaller $p_\perp$ than other hadrons. This effect is neglected in the MC. We expect that together with Bose-Einstein interference it can account for the discrepancy between data and MC results for the intermittency signal.

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M. Feindt *(Univ. Hamburg)*: I want to make a comment on the difference between the TASSO and CELLO results: A major difference is that we accept tracks in a much larger solid angle interval than TASSO. Thus the rapidity distribution of TASSO has a larger "hole" in the central region. This also can effect the absolute height of the factorial moments.
They also used an older version (6.3) of the LUND Monte Carlo.
In the quantitative evaluation of slopes and comparison with Monte Carlo predictions, one should keep in mind that the data points are strongly correlated with each other such that the actual χ^2 may be very different from what one would estimate by counting error bars. Thus, even the TASSO data do not really show significant deviation from the LUND Monte Carlo, and CELLO, HRS and DELPHI data are well described by LUND.

Q. **M. Markytan** *(Inst. High Energy Physics, Vienna)*: How is the fractal dimension obtained from the Lund dipole radiation of gluons model related to that from experimental analysis? Is it larger than one, or seems to be close to zero?

A. **G. Gustafson**: The fractal dimension of the x curve is larger than one as it is embedded in 3-dimensional space. It corresponds to the *Koch* curve. In rapidity, the fractal dimension is $\sqrt{3\alpha_s/2\pi}$, and must be between one and zero. (The plot of D_q shows a decrease from one to about 0.5.) In the subsequent hadronization, the neighbouring particles in the Lund string have a strong influence. Local parton-hadron duality to hold requires a Lorentz-invariant cutoff of the parton evolution cascade.

Q. **K. Sugana** *(ANL)*: I have two comments about your MC fit to the TASSO data. First, it is amazing how well the MC can reproduce the general shape of the rise of moments. Second, the steep rise of moments above $M = 32$ does not agree with the CELLO data. Therefore, it should be checked by other experiments.

A. **G. Gustafson**: The steep rise above $M = 4$ in MC may be statistical fluctuation and should not be taken too seriously at this moment.