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Simulation of phase-dependent transverse focusing in dielectric laser accelerator based lattices

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Abstract. The Accelerator on a CHip International Program (ACHIP) funded by the Gordon and Betty Moore Foundation aims to demonstrate a prototype of a fully integrated accelerator on a microchip based on laser-driven dielectric structures until 2021. Such an accelerator on a chip needs all components known from classical accelerators. This includes an electron source, accelerating structures and transverse focusing arrangements. Since the period of the accelerating field is connected to the drive laser wavelength of typically a few microns, not only longitudinal but also transverse effects are strongly phase-dependent even for few femtosecond long bunches. If both the accelerating and focusing elements are DLA-based, this needs to be taken into account. In this work we study in detail the implications of a phase-dependent focusing lattice on the evolution of the transverse phase space of a transported bunch.

1. Introduction

Current room temperature and superconducting RF (Radio-Frequency) cavities are usually limited to accelerating gradients < 100 MV/m, which can be attributed to the so called breakdown phenomena caused by high surface electric fields [1]. Dielectric structures based on Si or SiO_2 on the other hand can withstand up to two orders of magnitude higher surface fields if operated at optical frequencies, allowing for accelerating gradients in the range of $\sim$GV/m [2]. Operation at optical frequencies also implies a substantial size reduction compared to conventional RF structures (six orders of magnitude smaller). This makes specifically tailored laser-driven dielectric structures an interesting candidate for the development of compact and efficient novel particle accelerators.

The Accelerator on a CHip International Program (ACHIP) aims to demonstrate a prototype of a fully integrated, all-optical accelerator on a microchip based on laser-driven dielectric structures. If in the future a fully integrated DLA-based accelerator should be able to reach high energies, a need for beam transportation along the miniaturized beamline is implied. In [3] we showed that the phase-dependence of the focusing force has a significant effect on the betatron phase advance $\Psi_{x,y}$ in a DLA-based FODO-type transport lattice. In this work we use the same matrix formalism to study the implications of the phase-dependence more thoroughly. We also discuss possible mitigation techniques.

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2. Phase-Dependent Transport Matrix

For the following calculations the transport matrix approach as described in [3] is used. The well known quadrupole transfer matrix to be applied on a given phase-space coordinate (x_0, x'_0) is given by

$$\mathbf{M}_{\mathrm{QF}} = \begin{pmatrix} \cos(\Omega) & \frac{1}{\sqrt{k}}\sin(\Omega) \\ -\sqrt{k}\sin(\Omega) & \cos(\Omega) \end{pmatrix}, \qquad \mathbf{M}_{\mathrm{QD}} = \begin{pmatrix} \cosh(\Omega) & \frac{1}{\sqrt{|k|}}\sinh(\Omega) \\ \sqrt{|k|}\sinh(\Omega) & \cosh(\Omega) \end{pmatrix}, \tag{1}$$

where $\Omega = \sqrt{|k|}s$ and s is the quadrupole length. If the normalized quadrupole strength k is > 0 the quadrupole is focusing ($\mathbf{M}_{\mathrm{QF}}$), if $k < 0$ it is defocusing ($\mathbf{M}_{\mathrm{QD}}$). Otherwise it is just a drift section ($k = 0$). The phase-dependence of the focusing forces inside the DLA-channel can now be mapped onto the k value. Hence it is re-defined as

$$k(\phi_0) = 0.2998 \cdot \frac{G}{\beta E[\mathrm{GeV}]} \cdot \cos(\phi_0), \tag{2}$$

where βc is the particle velocity, E its energy, ϕ_0 the particle to DLA drive laser phase and G the equivalent magnetic focusing gradient inside the channel. It is now possible to construct any transport lattice by matrix multiplication of focusing, defocusing and drift matrices respectively.

In the following we assume a theoretical structure, which is specifically designed to only exert transverse forces on the particles. With this we avoid any phase slippage effects that could occur at low energies due to the velocity changes during acceleration or deceleration. The inclusion of slippage effects would excede the scope of this work and needs to be included in further studies. See [4, 5] for studies on how to mitigate phase slippage with the help of so-called tapered or chirped DLA structures.

2.1. Matched Beta Function

In order to show the effect of the phase-dependence of the transverse forces on the properties of a DLA-based transport lattice in a demonstrative way, the matched beta-function of a FODO-type lattice can be considered. It is given by

$$\hat{\beta}(\phi_0) = L_{\mathrm{cell}} \cdot \frac{1 + \sin(\Psi(\phi_0)/2)}{\sin(\Psi(\phi_0))}, \tag{3}$$

where L_{cell} is the length of a single cell of the FODO arrangement and Ψ the betatron phase advance. It can be seen that $\hat{\beta}(\phi_0)$ is now phase-dependent, since $\Psi \equiv \Psi(\phi_0)$ (as already shown in [3] via particle tracking). The phase advance per cell is given analytically as

$$\Psi(\phi_0) = 2 \cdot \arcsin(1/\kappa(\phi_0)), \tag{4}$$

where $\kappa(\phi_0) = 1/(k(\phi_0)l)$ and l is the drift space between the focusing and the defocusing element of the FODO cell. Figure 1 shows the matched beta function for a phase-dependent FODO lattice defined by the parameters shown in Table 1 for different values of G. In addition to that the derived emittance acceptance $\hat{\epsilon} = \sigma^2/\hat{\beta}$ for a fixed $\sigma = 0.5 \cdot \lambda_{\mathrm{S}}$, where λ_{S} is the DLA periodicity and σ the rms beam size, is plotted. It can be seen that $\hat{\beta}(\phi_0)$ diverges at $\phi_0 = (2n+1) \cdot \pi/2$, with $n \in \mathbb{N}$. Considering that $k(\phi_0) \propto \cos(\phi_0)$, this corresponds to the cases where $k(\phi_0) = 0$, or in other words where the lattice is just a drift. As expected the emittance acceptance is highest at the maximum of $k(\phi_0)$. Furthermore it can be seen that the convergence towards the drift case is steeper as G increases. In the following, the effect of this is studied using particle tracking through multiple FODO cells.

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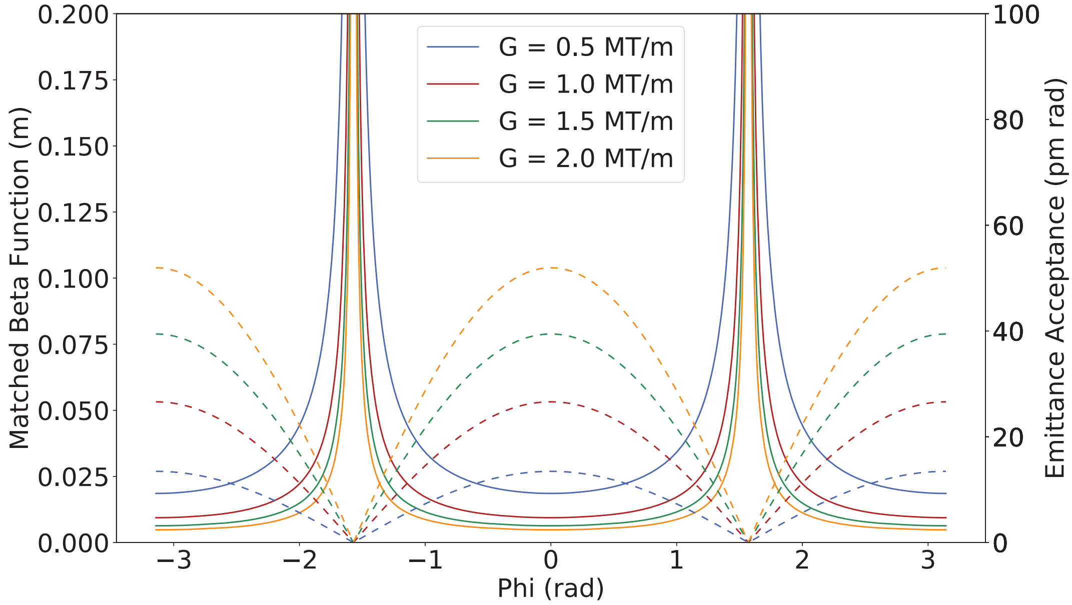

Figure 1. Matched beta function $\hat{\beta}$ vs. the injection phase (**solid**) and the emittance acceptance of the transport lattice for fixed rms beam size $\sigma = 0.5 \cdot \lambda_S$ (**dashed**) for different values of G, i.e. the focusing strength.

3. Particle Tracking

In this section the evolution of the transverse phase-space of a particle distribution is studied. We simulate the transport of a matched Gaussian electron bunch (both transversally and longitudinally) with $\sigma_t = 5\,\mathrm{fs}$ through a long DLA-based FODO lattice using the aforementioned transfer matrices. For now only one plane is taken into account. The parameters of the lattice are summarized in Table 1 and satisfy – in the on-crest case – the usual stability criterion $f > L_{\mathrm{cell}}/4$ for FODO cells of length L_{cell} and focal length $f = 1/kl_{\mathrm{Q}}$, where in our case $l_{\mathrm{Q}} = N_{\lambda_S,F} \cdot \lambda_S$.

Table 1. Parameters of the DLA-based FODO lattice.

Parameter	Value
β	0.9999 ($\rightarrow$ 50 MeV)
λ_S	$\beta \cdot 2\,\mu\mathrm{m}$
N_{λ_S} per F,D	10
N_{λ_S} per O	100
G	2.0 MT/m

3.1. Results

The result of the transport through the lattice is shown in Figure 2. It can be seen that due to the phase dependent phase advance, and hence acceptance of the lattice, the beam quality is degraded as the beam traverses more and more cells. Some parts of the bunch are kept stable and others become mismatched according to Fig 1. This leads to charge loss as microbunches are eventually formed at twice the periodicity of the DLA, i.e. $0.5 \cdot \lambda_S$.

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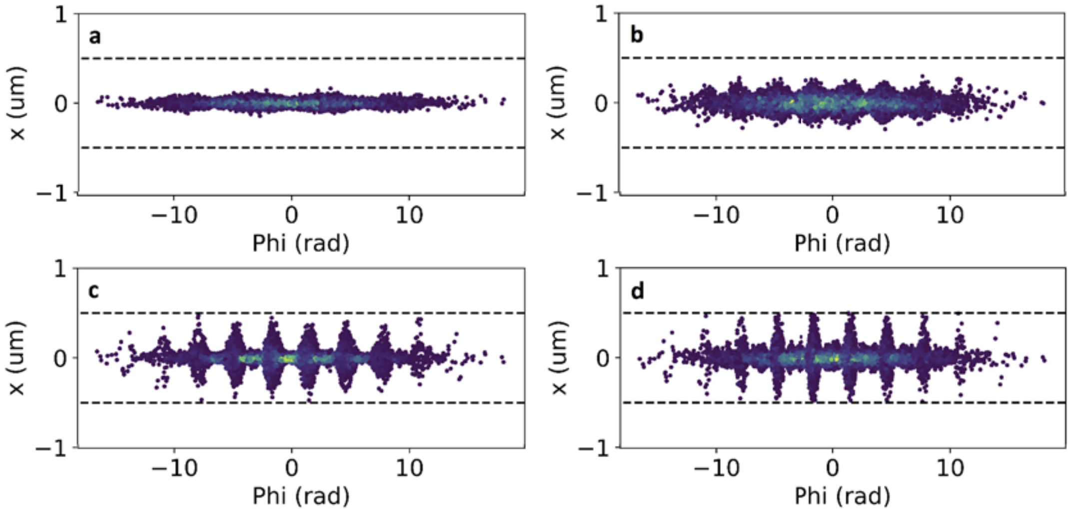

Figure 2. Particle distribution after the traversal of (**a**) 10, (**b**) 40, (**c**) 70, (**d**) 100 DLA-FODO cells defined by the parameters shown in Table 1.

4. Mitigation Techniques

In this section we present different ways how to work around the strong phase dependence in the case of a long DLA-based transport lattice. Considering the well established technique of *Alternating Phase Focusing* (APF) [6, 7, 8], the ideal solution to the problem would be to alternate the DLA drive laser phase by $\pi/2$ with the periodicity $\lambda_{\mathrm{AP}} = \lambda_{\mathrm{S}}$. This situation is very challenging, if not impossible to realize experimentally, because it would involve sudden phase shifts on the wavelength scale. Hence, APF approaches for DLAs have been proposed with larger λ_{AP}. In [4] the phase shift is induced by introducing small drifts after a given number of DLA periods. Another approach would be to phase modulate the drive laser pulse in a way that the desired phase configuration (albeit with $\lambda_{\mathrm{AP}} \gg \lambda_{\mathrm{S}}$) is met (see [9]). In the following, two different mitigation schemes based on alternating the DLA phase are compared:

- FODO lattice with (FODO) and w.o. alternating phase (FODO+AP),
- Continuous DLA (F) with and w.o. AP (F+AP).

4.1. Comparison

In order to compare the different schemes, the emittance growth of a bunch, which is transported through a lattice with a high number of cells is recorded for all of the different schemes. Figure 3 shows the result of the simulation. It can be seen that in the case of the scheme which combines both the classical FODO lattice with the alternating phase focusing the transverse phase space is transported best. Note that here the data is multiplied by 10 in order to make it visible in the plot. The F+AP scheme shows emittance oscillations with a periodicity, which depends on the lattice configuration. This behavior can also be seen in the slice plot of the transverse phase space shown in Figure 4. Different slices along the bunch rotate differently leading to an increase in projected emittance.

4.2. Phase-Dependent Focusing Characteristic

As the comparison of the different lattice designs showed, the DLA phase dependence of the focusing lattice can be minimized by combining a classical FODO arrangement with the AP scheme. This characteristic can also be shown by considering the thin lense matrix element M_{21} for a FODO arrangement with and without AP. Figure 5 shows $M_{21}(\phi_0)$ for both cases. It can

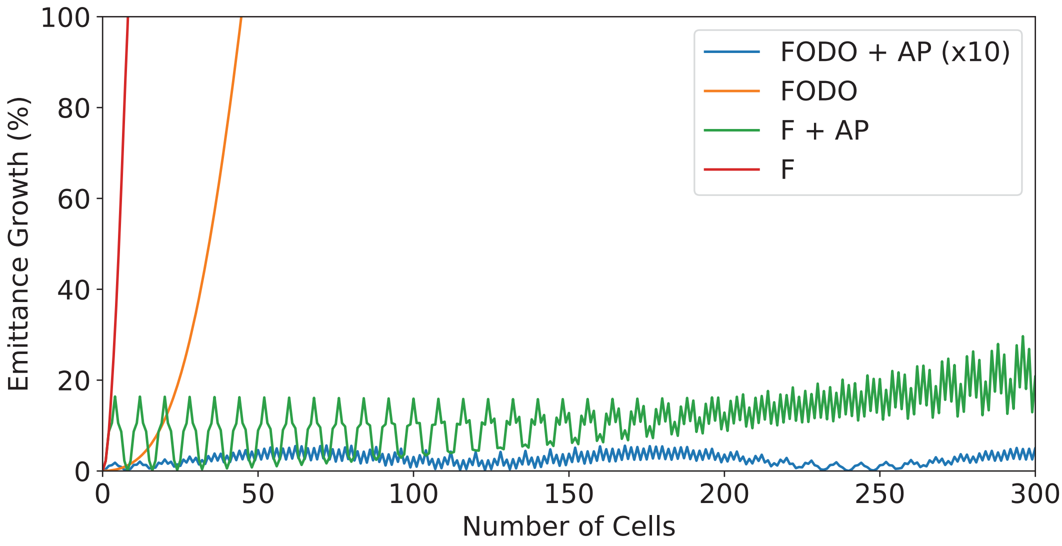

Figure 3. Emittance growth for four different transport lattices. **NB**: FODO+AP is given x10.

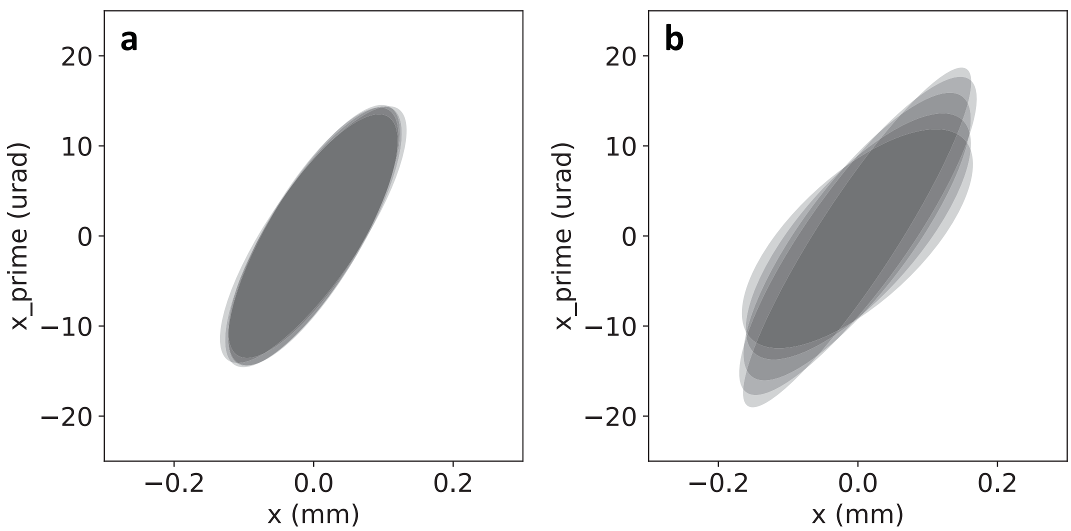

Figure 4. Transverse phase space ellipses (3σ) for multiple longitudinal slices along the bunch after having traversed 500 cells. **a**: FODO+AP, **b**: F+AP.

immediately be seen that the phase-dependence is strong for the normal arrangement, wheras it is significantly reduced using an AP scheme.

4.3. Transport in Both Planes

So far all studies shown here were conducted for only one plane. It is clear that in reality both the x- and y-plane need to be transported simultaniously. Considering grating-type DLAs, which can only act on one transverse plane, it will be necessary to build a lattice, where every second device is physically rotated by 90 deg. Hence an interleaved FODO lattice is the only possible choice. The resulting lattice could for example be written as $\mathrm{F}_x\mathrm{F}_y\mathrm{D}_x\mathrm{D}_y$. Figure 6 shows the scheme in more detail. In [10] we show a proof-of-principle simulation of such an interleaved setup.

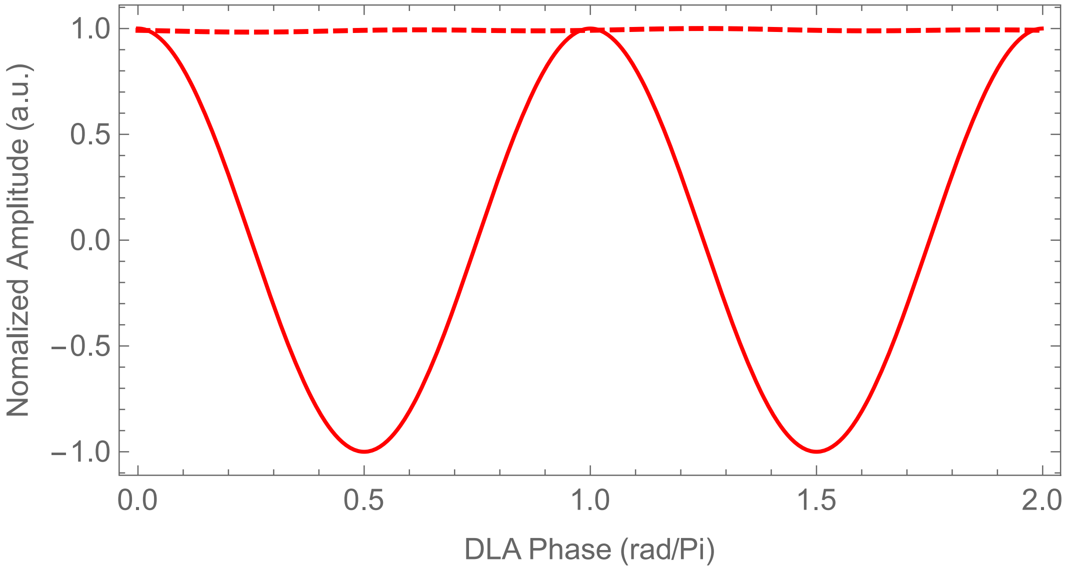

Figure 5. DLA phase dependent M_{12} matrix element of the transport matrix used for the calculations (**solid**: FODO arrangement, **dashed**: FODO arrangement plus $\pi/2$ alternating phase focusing).

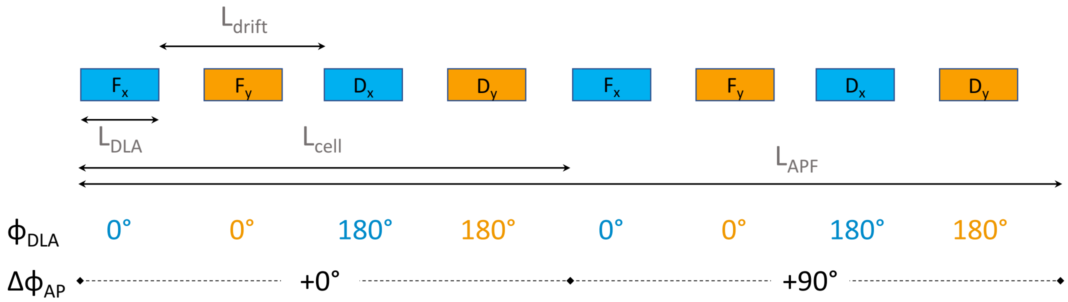

Figure 6. Schematic of the combination of a classical FODO lattice and APF. The respective elements for the x (**blue**) and y (**orange**) plane are interleaved.

5. Conclusion

We have studied the phase dependence of the transverse focusing in DLA-based transport lattices for relativistic particles using modified quadrupole transfer matrices. The comparison of two different mitigation schemes shows that a combination of a classical FODO lattice with the alternating phase focusing technique offers the best transport in one plane, as the phase dependence can be reduced substantially. In order to transport both x and y, a scheme with interleaved elements can be employed. Further studies should include phase slippage effects in the case of the simultaneous acceleration and focusing of non-relativistic particles.

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