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Exotic Particle Production in Heavy Ion Collision

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In this talk, I will first summarize the recent interests in heavy multiquark configurations and then discuss why there could be compact multiquark states within the constituent quark model. I will then discuss few most probable compact pentaquark and tetraquark candidates. Finally, I will discuss particle production in heavy collision and summarize the result of ExHIC collaboration, which showed that the production of compact multiquark states are expected to be smaller than when the states were assumed to be loosely bound molecular configurations with the same total quark content.

KEYWORDS: Multiquark, Exotics, Heavy Ion Collision

1. Introduction

The possible existence of mutiquark hadrons beyond the ordinary hadrons were first discussed for the tetraquark states in Ref. [1,2] and for the H-dibaryon in Ref. [3]. Later, possible stable pentaquark configurations $\bar{Q}sqqq$ were proposed in Ref. [4] and in Ref. [5]. The long experimental search for the H-dibaryon was not successful so far but is still planned at J-PARC [6]. The search by Fermilab E791 [7] for the proposed pentaquark state also failed to find any significant signal for the exotic configurations.

On the other hand, starting from the $X(3872)$ [8], possible exotic meson configurations XYZ and the pentaquark P_c [9] were recently found. These states are not flavor exotic but are known to contain $\bar{c}c$ quarks. Heavy quarks were for many years considered to be stable color sources that would allow for a stable multiquark configuration that does not fall into usual hadrons. In particular, with the recent experimental confirmation of the doubly charmed baryon [10–12], there is a new excitement in the physics of exotics in general and in hiterto unobserved flavor exotic states with more than one heavy quarks [13–17].

However, it is not clear if an exotic candidate is an compact multiquark configuration or a loosely bound molecular configuration. In a series of papers [18–20], together with the ExHIC collaboration, we have shown that the yields of a hadron can be used to discriminate its structure between a multiquark configuration and a normal or molecular configuration. In this talk, I will present the origin why certain multiquark states with heavy quarks could be stable, and then argue why measuring them in an heavy ion experiment is interesting.

2. Quark model and compact multiquark configurations

A compact multiquark state can be stable if there is an attraction at short distance between two separated hadrons with the same total quark content. The short distance interaction between hadrons are known to be interpolated via vector meson exchange. On the quark level, it was shown within the quark cluster model that the repulsion between nucleons at short distance originates from the Pauli principle and color-spin interaction between quarks [21]. Recently, we have shown that latest lattice

κ	κ'	a_0	D	α	β	$m_{u,d}$	m_s	m_c	m_b
0.59	0.5	5.386 GeV^{-2}	0.96 GeV	2.1 fm^{-1}	0.552	0.343 GeV	0.632 GeV	1.93 GeV	5.3 GeV

Table I. The parameters of CQM fitted to light and heavy baryon masses [25].

result on the nucleon nucleon potential can be indeed qualitatively understood using the constituent quark model (CQM) [22,23]. This result pawed the way to search for all configuration that allows for a short distance attraction compared to its lowest threshold hadrons, which is necessary ingredient for having a compact multiquark configuration in theat channel. Here, we will summarize the basic ingredients for the result and discuss candidates for possible compact configurations.

2.1 Origin of short range interaction

Let us consider a compact six-quark configuration from the CQM point of view. We will assume that the spatial wave function of all the six quarks are in the lowest energy s-wave state. Because the quarks have color, flavor and spin, only specific total quantum numbers are allowed for the six quark configurations made of two octet baryons. In this subsection, we will show the recent work reported in Ref. [22], which shows that the short range part of the lattice calculated nucleon nucleon potential can be well understood in terms of constituent quark model.

The interaction of two baryons at short distances in CQM [21] are governed by the quark dynamics with the following Hamiltonian,

$$H = \sum_{i=1}^{N}(m_i + \frac{\mathbf{p}_i^2}{2m_i}) - \frac{3}{16}\sum_{i<j}^{N}(V_{ij}^C + V_{ij}^{CS}), \tag{1}$$

where N is the total number of constituent quarks and m_i's are the constituent quark masses. The spin-independent (spin-dependent) color interaction denoted as V_{ij}^C (V_{ij}^{CS}) is given by [24,25].

$$V_{ij}^C = -\lambda_i^c\lambda_j^c\left(-\frac{\kappa}{r_{ij}} + \frac{r_{ij}}{a_0} - D\right), \tag{2}$$

$$V_{ij}^{CS} = \frac{\kappa'}{m_i m_j r_{0ij}^2}\frac{1}{r_{ij}}e^{-(r_{ij}/r_{0ij})^2}\lambda_i^c\lambda_j^c\,\sigma_i\cdot\sigma_j, \tag{3}$$

where λ_i^c are the Gell-Mann matrices of the i'th quark for the color SU(3). Here, r_{ij} is the distance between quarks, while r_{0ij} is chosen to depend on the constituent quark masses as

$$r_{0ij} = (\alpha + \beta\mu_{ij})^{-1}. \tag{4}$$

with $\mu_{ij} = m_i m_j/(m_i + m_j)$ being the reduced mass. Throughout this paper, we assume isospin symmetry, $m_u = m_d$. In Table I, we show the values of the parameters fitted to low-lying baryon masses with S=1/2 and 3/2 including those with charm and bottom quarks [25]. Table II shows the calculated masses of light baryons relevant to this work.

The static interaction energy $V_{\rm CQM}$ between two baryons located on top of each other can be estimated by looking at the interaction in terms of the six-quark configuration relative to the two-baryon threshold:

$$V_{\rm CQM} = \langle H\rangle_{6q} - E_{BB'}, \tag{5}$$

(I,S)	$(\frac{1}{2},\frac{1}{2})$ N, P	$(\frac{1}{2},\frac{3}{2})$ Δ	$(0,\frac{1}{2})$ Λ	$(1,\frac{1}{2})$ Σ	$(1,\frac{3}{2})$ Σ^*	$(\frac{1}{2},\frac{1}{2})$ Ξ	$(\frac{1}{2},\frac{3}{2})$ Ξ^*
M_B	0.977	1.23	1.12	1.2	1.38	1.324	1.52
Expt.	0.938	1.232	1.115	1.189	1.382	1.315	1.532

Table II. Masses of light baryons relevant to the present work in the unit of GeV.

$$E_{BB'} \quad = \quad M_B + M_{B'} + K_{\mathrm{rel},BB'}. \tag{6}$$

Here $\langle H\rangle_{6q}$ is the expectation value of the Hamiltonian with $N = 6$ with respect to the six-quark in the s-wave, M_B and $M_{B'}$ are the single baryon energies obtained by the Hamiltonian with $N = 3$, and $K_{\mathrm{rel},BB'}$ is the relative kinetic energy between two baryons. This formula will be used to estimate the interaction energies for both the flavor SU(3) symmetric and non-symmetric cases.

The total energy of the six-quark system in CQM can also be decomposed as

$$\langle H\rangle_{6q} = \sum_{i=1}^{6} m_i + K + E_C + E_{CS}, \tag{7}$$

where K stands for the total kinetic energy, E_C is obtained from V_C and E_{CS} is obtained from V_{CS}.

The average matrix elements for the quark pairs contributing to the spin-independent color interaction V_C will not contribute in appreciable strength in Eq.(5) as long as the spatial size of a single baryon and that of the 6-quark system similar. This is so because the color factor in V_C taken with respect to the color-singlet state is proportional to N as

$$\sum_{i<j} \lambda_i^c \lambda_j^c \quad = \quad -\frac{8}{3}N. \tag{8}$$

This is also true for f-type or d-type three-quark interactions in the color singlet state [26]

$$\begin{aligned} \sum_{i\neq j\neq k} f^{abc}\lambda_i^a\lambda_j^b\lambda_k^c \quad &= \quad 0 \\ \sum_{i\neq j\neq k} d^{abc}\lambda_i^a\lambda_j^b\lambda_k^c \quad &= \quad \frac{160}{9}N, \end{aligned} \tag{9}$$

where f and d are the antisymmetric and symmetric structure constants for color SU(3), respectively.

On the other hand, the sum of the color-spin factor in the color-spin interaction V_{CS} depends non-linearly on N so that it induces non-vanishing contribution to Eq.(5). To estimate its strength, let us now consider the flavor SU(3) symmetric case where the matrix element shown below is a basic quantity to determine V_{CQM}:

$$\begin{aligned} X &\equiv -\sum_{i<j}^{N} \langle \lambda_i^c \lambda_j^c\, \sigma_i \cdot \sigma_j \rangle \\ &= N(N-10) + \frac{4}{3}S(S+1) + 4C_{\mathrm{f}} + 2C_{\mathrm{c}}. \end{aligned} \tag{10}$$

Here C_{f} and C_{c} are the first kind of Casimir operators of flavor and color for the N-quark system, respectively [27]. Assuming that the size of the quark wave-function of the six-quark system and that for each 3-quark system are the same for simplicity, we obtain, with a common constant γ,

$$V_{\mathrm{CQM}}(F_\ell) = \gamma(X_{6q} - (X_B + X_{B'})). \tag{11}$$

F_ℓ	F_1	F_{27}	F_{10}	$F_{\overline{10}}$	F_8
$\mathcal{R}_\ell^{\mathrm{CQM}}$	−0.33	1	0.78	0.78	0.28
$\mathcal{R}_\ell^{\mathrm{LQCD}}$	−0.53(3)	1	0.93(1)	0.81(1)	0.20(1)

Table III. Comparison of ratios of the color-spin interactions at short distance R_ℓ between the constituent quark model (CQM) and the lattice QCD data (LQCD) in the flavor SU(3) symmetric case [28].

In the following, we consider the ratios between $V_{\mathrm{CQM}}(F_\ell)$ to get rid of the constant γ:

$$\mathcal{R}_\ell^{\mathrm{CQM}} = \frac{V_{\mathrm{CQM}}(F_\ell)}{V_{\mathrm{CQM}}(F_{27})}. \tag{12}$$

Similarly, the ratios for baryon-baryon potential at $r = 0$ can be introduced as

$$\mathcal{R}_\ell^{\mathrm{LQCD}} = \frac{V_{\mathrm{LQCD}}(F_\ell)}{V_{\mathrm{LQCD}}(F_{27})}. \tag{13}$$

As mentioned in the Introduction, the quark wave function dependence in CQM and the interpolating operator dependence in LQCD are expected to cancel independently in such ratios, so that the comparison can be made with less ambiguities.

In Table III, we compare $\mathcal{R}_\ell^{\mathrm{CQM}}$ from the color-spin interaction in the favor SU(3) case and $\mathcal{R}_\ell^{\mathrm{LQCD}}$ obtained from the currently available lattice data in the flavor SU(3) case [28] with the pseudo-scalar meson mass $M\mathrm{ps} \simeq 469$ MeV and the octet baryon mass $M_B \simeq 1161$ MeV. The errors in the parentheses for $\mathcal{R}_\ell^{\mathrm{LQCD}}$ reflect the combined statistical and systematic errors estimated from the LQCD data at the Euclidean time $t = 11$ and 12. One finds that the sign and magnitude of these ratios are qualitatively consistent with each other between CQM and LQCD. This result is reported in Ref. [22].

3. Compact multiquark candidates

Using the arguments from the previous section, we can search for configuration that gives large color spin attraction. It should be noted that for baryon octet the factor in Eq. (10) is $\mathcal{X}_N = -8$, while for delta $\mathcal{X}_\Delta = 8$. The famous H-dibaryon has a factor of $\mathcal{X}_H = -24$ so that subtracting the lowest threshold baryons it gives $\Delta\mathcal{X}_H = -8$. However, despite extensive search, the H dibaryon is not found so far. It is believed that the symmetry breaking and light quark system makes the H dibaryon unstable. On the other hand, states with heavy quarks are expected to be more stable and allow for possible compact stable configurations. Let us consider few examples.

3.1 Pentaquarks

The pentaquark $P_{\bar{c}s}$ introduced in Ref [5], assumed that the charm antiquark is infinitely heavy. In such a case, the $\mathcal{X}$ factor is −16. As the lowest threshold is a baryon and a heavy meson, the final factor after subtracting the lowest threshold hadrons is $\Delta\mathcal{X}_{P_{\bar{c}s}} = -8$. Unfortunately, the charm quark is not infinitely heavy so that the attraction is reduced for realistic charm quark mass. However, starting from a more general set of configuration, we have recently found that there is a set of pentaquark states in the (Flavor, Spin)=(3, 1/2) that have large attractive color spin interaction above the lowest hadron threshold even for realistic quark masses [29]. The previously discussed pentaquark state, where the four light quarks belong to the SU(6) [31] representation, also belongs to these sets but among the least stable configurations with realistic masses.

3.1.1 Pentaquark candidates

There are two compact pentaquark candidates found in Ref. [29]. The first one is (Isospin, Spin)=$(0, 1/2)$ $P_{cc\bar{s}} = udcc\bar{s}$ pentaquark. Its lowest threshold hadron states are $\Xi_{cc}^{+} + K^{+}$ states with color spin attraction of around 135 MeV. Depending on the binding energy its important decay modes will be the following

(1) If $P_{cc\bar{s}} = udcc\bar{s}$ is strongly bound, one of the c quarks will weakly decay, leading to the following decay mode.

$$P_{cc\bar{s}}^{++}(udcc\bar{s}) \to \Lambda_c K^- K^+ \pi^+. \tag{14}$$

It should be noted that this decay mode is similar to those used to initially identify $\Xi^{cc} \to \Lambda_c K^- K^+ \pi^+$

(2) If the state is above certain hadron state, it could be directly reconstructed through the following final states.

$$P_{cc\bar{s}}^{++}(udcc\bar{s}) \to \Xi_{cc}^{+} K^{+} \quad \text{or} \quad \Lambda_c D_s^{+}. \tag{15}$$

Another possible candidate is the $P_{sc\bar{c}} = udsc\bar{c}$ again with (Isospin, Spin)=$(0, 1/2)$. This state has hidden charm but if slightly above threshold could be reconstructed through the following process.

$$P_{sc\bar{c}}(udsc\bar{u}) \to \Lambda_c + J/\psi. \tag{16}$$

3.1.2 Tetraquark candidates

There is certainly great interest in observing the recently discovered exotic candidate such as the $X(3872)$ or the Z_c states in heavy ion collision as their transverse momentum dependence production will reveal their inner structure of these states [30]. At the same time, it would be of great interest to observe for the first time an compact multiquark candidate with multiple heavy quarks, which would be difficult to produce in an elementary production processes. Such a possible multiquark configuration is the $T_{cc}(ud\bar{c}\bar{c})$ [31–33].

Here we will list the possible final states that could be measured to reconstruct the T_{cc} from heavy ion collisions. The model calculations at present vary on the exact value of the binding energy. Therefore, we will probe all possibilities [17, 34]. It should be noted that one could also look at the charge conjugate final states and search for $T_{\bar{c}\bar{c}}$ mesons.

(1) $m_{T_{cc}} \geq m_D + m_{D^*}$: In this case,

$$T_{cc} \quad \to \quad \text{(a)}\, D^0 + D^{*+} \quad \text{or} \quad \text{(b)}\, D^+ + D^{*0} \quad \text{or} \quad \text{(c)}\, D^+ + D^+ + \pi^-. \tag{17}$$

As $D^{*+} \to D^0 + \pi^+$ and $D^0 \to K^- + \pi^+$, (a) can be reconstructed with vertex detectors. D^{*0} in (b) may not be easy to detect directly.

(2) $m_D + m_{D^*} \geq m_{T_{cc}} \geq m_D + m_D + m_\pi$: This would be the most likely case for a compact multiquark state. Then, the virtual D^{*+} component can decay into $D^0 + \pi^+$ so that a detectable final state would be

$$T_{cc} \quad \to \quad D^0 + D^0 + \pi^+. \tag{18}$$

The final state involving $T_{cc} \to D^0 + D^+ + \pi^0$ would be harder to identify. We note that the final state of Eq. (18) is not distinguishable with that of Eq. (17) (a).

(3) $m_{T_{cc}} \leq m_D + m_D + m_\pi$: In this case, the virtual D^* component should also decay into $D + \pi$ so that a detectable final state would be

$$T_{cc} \;\rightarrow\; D^0 + K^- + \pi^+ + \pi^+ \quad \text{or} \quad D^+ + K^- + \pi^+ + \pi^+ + \pi^-. \tag{19}$$

Among all the above cases, Eqs. (17) (c) ($D^+ + D^+ + \pi^-$) and (18) ($D^0 + D^0 + \pi^+$) seem to be the most probable cases to reconstruct the T_{cc}.

4. Exotics production in Heavy ion Collision

4.1 Freeze-out condition

In general, production yields of hadronic molecules, light nuclei or resonances not only depend on the conditions at the chemical freeze-out but also on their interactions with other hadrons during the hadronic stage. Specifically, for the light nuclei or hadronic molecules that are bound, the yields are affected by the freeze-out condition of the respective particle in the hadronic matter. For resonances that decay into daughter particles, the freeze-out point of the daughter particles will be important as the resonances are reconstructed from the observed daughter particles. In other words, investigation on the yields of hadronic molecules or resonances may result in understanding of how long the hadronic stage lasts in heavy ion collisions, or of when the kinetic freeze-out of various particles takes place.

Kinetic freeze-out of a particle species i from the matter occurs when its scattering rate τ^i_{scatt} becomes larger than the expansion rate of the system τ_{exp} [35]. Hence, in an expanding system of interacting particles, freeze-out takes place when

$$\tau_{exp} = \frac{1}{\partial \cdot u} = \tau^i_{scatt} = \frac{1}{\sum_j \langle \sigma_{ij} v_{ij} \rangle n_j}, \tag{20}$$

with $\langle \sigma_{ij} v_{ij} \rangle$ being the thermally averaged cross section times the relative velocity between particle species i and j, n_j the density of particle j, and u the expansion velocity of the system.

The expansion time τ_{exp} can be approximated by the ratio of the fireball volume V to its change in time, $V/(dV/dt)$, which can be further reduced to $R/(3dR/dt)$ for the spherically expanding fireball with its radius R [36]. Let us for simplicity further assume that the system is composed of one species only and that the cross section is velocity independent. The freeze-out condition in Eq. (20) then becomes

$$\frac{R}{3dR/dt} = \frac{1}{n\sigma \langle v \rangle}. \tag{21}$$

In general the relation between dR/dt and $\langle v \rangle$ is not universal; particularly so when there is a flow. However, assuming that the rate of change in the radius is close to the average velocity of the particles in the system, the condition for the kinetic freeze-out becomes

$$\frac{N}{R^2_{fo}} = \frac{4\pi}{\sigma_{fo}}, \tag{22}$$

where a subscript $_{fo}$ stands for physical quantities at kinetic freeze-out and N is the total number of particles. We see that the two dimensional density determines the condition for freeze-out, because the transverse total cross section determines whether the particle interacts with the medium when it escapes from the medium.

On the other hand, the three dimensional density at freeze-out then goes as

$$\frac{N}{R^3_{fo}} = \left(\frac{4\pi}{\sigma_{fo}}\right)^{3/2} \frac{1}{N^{1/2}}. \tag{23}$$

This suggests that for higher collision energies and/or when the initial temperature and/or the number of particles increases, the three dimensional density at which freeze-out takes places becomes smaller. Such effect will be reflected in the resonance production, such as the K^*, in heavy ion collisions.

However, when the initial hadron yield at kinetic freeze-out point is an order of magnitude smaller than statistical model expectation, the anomalously small yield might survive the hadronic phase and provide valuable information. Such is the case for compact multiquark configuration.

4.2 Exotic production rate

Most of the recently discovered exotic candidates and/or proposed multiquark states have heavy quarks. Recently we have formed a theory collaboration called ExHIC that studies the production of the exotic states, resonances and light nuclei in heavy ion collision. Studying the production through coalescence model together with statistical model, we have noted that the production of compact multiquark states were found to be systematically smaller than those expected when they were a loosely bound molecular configuration even if total quark content were the same [18–20]. Figure 1 shows the production yields of a selected set of exotic candidates from a heavy ion collision. One finds that when the hadron is assumed to be a normal hadron and/or molecule, the production rate is similar to that expected from statistical model prediction. However, when the hadron is assumed to be compact, one finds that the yield is an order of magnitude smaller than the statistical model prediction. Hence, through systematic studies of particle production as well as the transverse momentum dependence will reveal the structure of the hadron.

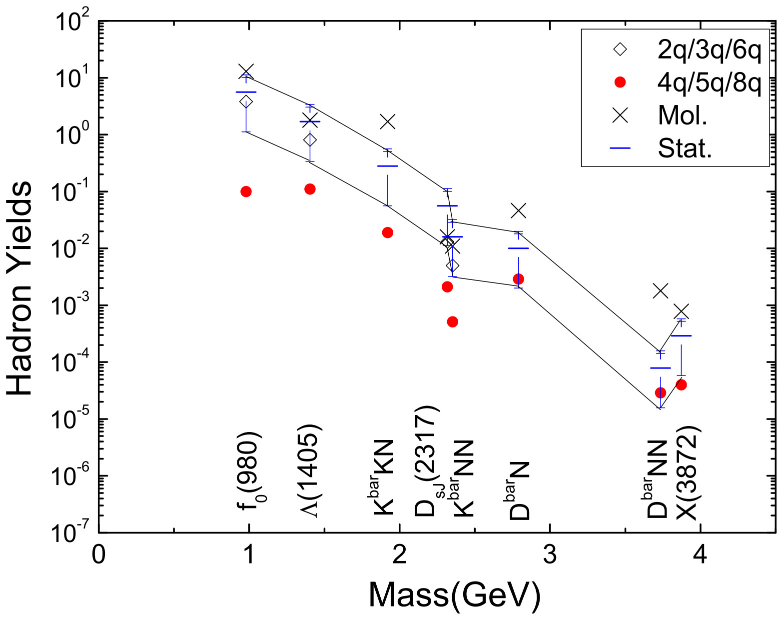

Fig. 1. Yields of candidates for multiquark configuration in an Au-Au collision at $\sqrt{s_{NN}}$ = 200 GeV. The yields are suppressed when their structure is of multiquark configurations.

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