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# COIL SHAPES TOWARDS PURE MULTIPOLES IN CIRCULAR REGIONS


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Abstract

Coil shapes to produce approximately pure multipole fields in circular regions have been studied. The proposed coil shapes are functions of a parameter $\Delta$ where $\Delta < 1.0$. The design multipole field is found to be of the order of $\Delta$ and the multipole impurities are found to be of the order of $\Delta^2$ or higher.

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## 1. INTRODUCTION

Accelerator design involves the design of magnets with pure multipole fields; dipoles are used for bending and steering the beam, quadrupoles are used for focussing and defocussing the beam, sextupoles are used for correction of chromaticity etc. Presence of multipole components in the magnetic field other than the desired component constitutes impurity in the field and will compromise the design purpose of the magnet through errors. Therefore, it is of interest to produce a pure multipole field within a circular beam pipe, or keep the impurities to a minimum.

It is possible to produce a pure dipole field by overlapping two cylindrical regions with the same diameter and with uniform current densities ($j$) and ($-j$) respectively. In the region of overlap (which will not be circular), the currents cancel and it can be shown that a pure dipole field is obtained in this region. Schmuser (Ref. 1) has shown that pure multipole fields can be produced in circular regions by varying the current in coils of constant thicknesses surrounding circular regions, as folows:

$$I = I_0 \cos(m\phi) \tag{1}$$

Pure dipole field results for m=1; quad-field for m=2; sextupole field for m=3 etc. In practice approximations to the $\cos(m\phi)$ current variations have been achieved by introducing zero current wedges at appropriate places in coils of constant thicknesses (Ref. 1). Generation of approximately pure multipole fields in circular regions with current densities of constant magnitude, but with coils of varying thicknesses , is investigated in this report. Follow primarily the notation in Ref. 1.

## 2. MULTIPOLE EXPANSION

With a current source $I = ja(da)(d\phi)$, located at the source co-ordinates $(a)$, the magnetic potential for 2-d field at the field co-ordinates $(r, \theta)$, for $r < a$ is given (Ref.1) by:

$$A_z(r,\theta) = \frac{\mu_o}{2\pi} \sum_{n=1}^{\infty} \frac{I}{n} (\frac{r}{a})^n \cos\ n(\phi - \theta) \tag{2}$$

Using the expression for the current I, Eq.(2) can be written as

$$A_z(r,\theta) = \frac{\mu_o}{2\pi}\sum_{n=1}^{\infty}\frac{j}{n}(\frac{r}{a})^n \ a \ \cos \ n(\phi-\theta) \ d\phi \ da \tag{3}$$

Let the coil be bounded by the constant radius $a_1$ of the circular field region on the inside and a radius $a_2(\phi)$ on the outside. Let the magnitude of the current density be constant equal to $j$. The potential $A_z(r,\theta)$ for $r < a$ due to such a coil is given by the following equation:

$$A_z(r,\theta) = \frac{\mu_o}{2\pi}\sum_{n=1}^{\infty}\frac{1}{n}\int_0^{2\pi}\int_{a_1}^{a_2} j(\frac{r}{a})^n a \ \cos \ n(\phi-\theta) \ d\phi \ da$$

Carrying out the integration over $a$, $A_z(r,\theta)$ is found to be

$$A_z(r,\theta) = \frac{\mu_o}{2\pi}\sum_{n=1}^{\infty}\frac{r^n}{n(n-2)}\frac{1}{a_1^{n-2}}\int_0^{2\pi} j \ [1-(\frac{a_1}{a_2})^{(n-2)}] \ \cos \ n(\phi-\theta) \ d\phi \tag{4}$$

Expanding the summation,

$$\begin{aligned}
A_z = \frac{\mu_o}{2\pi}\int_0^{2\pi} j \Bigg[ & -r \ a_1 \ [1-(\frac{a_1}{a_2})^{-1}] \ \cos(\phi-\theta) \\
& + \lim_{n\to 2}\frac{r^2}{n(n-2)} \ [1-(\frac{a_1}{a_2})^{(n-2)}] \ \cos \ 2(\phi-\theta) \\
& + \frac{r^3}{3a_1}[1-\frac{a_1}{a_2}] \ \cos \ 3(\phi-\theta) \\
& + \frac{r^4}{8{a_1}^2}[1-(\frac{a_1}{a_2})^2] \ \cos \ 4(\phi-\theta) \\
& + \frac{r^5}{15{a_1}^3}[1-(\frac{a_1}{a_2})^3] \ \cos \ 5(\phi-\theta) \\
& + \frac{r^6}{24{a_1}^4}[1-(\frac{a_1}{a_2})^4] \ \cos \ 6(\phi-\theta) \\
& + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \Bigg] \ d\phi
\end{aligned}$$

The summation has been expanded showing the first six terms. The second term in the integral is undefined as $n$ approaches 2. Redefine this term using L'Hospital's rule.

$$\lim_{n\to 2} \frac{1}{n(n-2)}[1-(\frac{a_1}{a_2})^{n-2}] = -\frac{1}{2}\ ln(\frac{a_1}{a_2}) \tag{5}$$

Using Eq.(5) , the expression for $A_z$ can be re-written as follows:

$$\begin{aligned} A_z = \frac{\mu_o}{2\pi} \int_0^{2\pi} j \Bigg[ & -r\ a_1\ [1-(\frac{a_1}{a_2})^{-1}]\ \cos(\phi-\theta) \\ & \frac{-r^2}{2}\ ln\frac{a_1}{a_2}\ \cos\ 2(\phi-\theta) \\ & +\frac{r^3}{3a_1}[1-\frac{a_1}{a_2}]\ \cos\ 3(\phi-\theta) \\ & +\frac{r^4}{8{a_1}^2}[1-(\frac{a_1}{a_2})^2]\ \cos\ 4(\phi-\theta) \\ & +\frac{r^5}{15{a_1}^3}[1-(\frac{a_1}{a_2})^3]\ \cos\ 5(\phi-\theta) \\ & +\frac{r^6}{24{a_1}^4}[1-(\frac{a_1}{a_2})^4]\ \cos\ 6(\phi-\theta) \\ & + \cdots\cdots \Bigg] d\phi \end{aligned} \tag{6}$$

Each term in the above integral will be labeled as $I_1, I_2$.. etc.

The two-dimensional magnetic field is given by

$$\vec{B} = \nabla\ X\ A \tag{7}$$

The radial and azimuthal components of $\vec{B}$ can be expressed as follows.

$$\begin{aligned} B_r &= \frac{1}{r}\frac{\partial A_z}{\partial \theta} \\ B_\theta &= -\frac{\partial A_z}{\partial r} \end{aligned} \tag{8}$$

The components of the magnetic field in the cartesian co-ordinate system are given by

$$B_x = B_r \cos(\theta) - B_\theta \sin(\theta)$$
$$B_y = B_r \sin(\theta) + B_\theta \cos(\theta) \tag{9}$$

The following series expansion formulae and trigonometric integral relations will be used.

$$(1 \pm x)^n = 1 \pm nx + \frac{n(n-1)x^2}{2!} \pm \frac{n(n-1)(n-2)}{3!} + ...etc$$
$$(1 \pm x)^{-n} = 1 \mp nx + \frac{n(n+1)x^2}{2!} \mp \frac{n(n+1)(n+2)}{3!} + ...etc$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...$$
$$ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ... \tag{10}$$

$$\int_0^{2\pi} \cos(m\phi)\ \cos(n\phi)\ d\phi = \pi\delta_{m,n}$$
$$\int_0^{2\pi} \cos(m\phi)\ \sin(n\phi)\ d\phi = 0 \tag{11}$$

## 3. COIL SHAPES

### 3.1 Case I, Dipole.

Assume a coil shape and current distribution as follows:

$$a_2 = a_1(1 + \Delta\ |\cos\phi|)$$
$$j = j_o \frac{\cos\phi}{|\cos\phi|} \tag{12}$$

In the above equation, $\Delta$ is a constant input parameter; it is equal to the maximum thickness of the coil divided by $a_1$. The current density $j$ has a constant magnitude $j_o$

and its sign changes according to Eq.(12). Substitute the coil shape and current density given by Eq.(12) into the integral for the potential given by Eq.(6) and evaluate it term by term. Omit the constant $\frac{\mu_0}{2\pi}$ for convenience.

$$I_1 = -\int_0^{2\pi} r\ j_0 \frac{\cos\phi}{|\cos\phi|}\ a_1[1-(1+\Delta|\cos\phi|)]\ \cos(\phi-\theta)\ d\phi$$

$$= j_0 a_1 r \int_0^{2\pi} \Delta\cos\phi(\cos\phi\cos\theta + Sin\phi Sin\theta)\ d\phi$$

Using Eq.(11), $I_1$ reduces to

$$I_1 = r a_1 j_0 \Delta\pi\cos\theta$$

Using Eqs.(8) and (9), this is found to lead to a pure dipole field (Ref.1). Further the dipole field will be of the order $\Delta$. Evaluate the next term $I_2$.

$$I_2 = \int_0^{2\pi} -r^2 j_0 \frac{\cos\phi}{|\cos\phi|}\frac{1}{2}\ ln(\frac{a_1}{a_2})\ \cos\ 2(\phi-\theta)\ d\phi$$

Using Eq.(10), $I_2$ reduces to,

$$I_2 = \int_0^{2\pi} \frac{r^2}{2} j_0 \frac{\cos\phi}{|\cos\phi|}\ [\Delta|\cos\phi| - \frac{\Delta^2|\cos\phi|^2}{2} + ...]\cos\ 2(\phi-\theta)\ d\phi$$

$$= \int_0^{2\pi} \frac{r^2}{2} j_0[\Delta\cos\phi - \frac{\Delta^2}{2}\cos\phi|\cos\phi| + ...]\ \cos\ 2(\phi-\theta)\ d\phi$$

Using Eq.(11), the first term in the above integral, which is of the order $\Delta$ reduces to zero; the higher order terms of the order of $\Delta^2$ and higher may not integrate to zero. The terms $I_3.....I_n$ can be studied by the generic term $I_n$.

$$I_n = \int_0^{2\pi} \frac{jr^n}{n(n-2)}\ \frac{1}{{a_1}^{n-2}}[1-(\frac{a_1}{a_2})^{n-2}]\ \cos\ n(\phi-\theta)\ d\phi$$

$$= \int_0^{2\pi} \frac{r^n}{n(n-2)} \frac{1}{a_1{}^{n-2}} j_0 \frac{\cos\phi}{|\cos\phi|} [1 - (1 - (n-2)\Delta|\cos\phi| + \frac{(n-2)(n-1)}{2}\Delta^2\ |\cos\phi|^2$$

$$-....)]\ \cos\ n(\phi - \theta)\ d\phi$$

$$= \frac{r^n}{n(n-2)} \frac{j_0}{a_1{}^{n-2}} \int_0^{2\pi} [(n-2)\Delta\cos\phi - \frac{(n-2)(n-1)}{2}\Delta^2 \cos\phi|\cos\phi| -$$

$$...]\ \cos\ n(\phi - \theta)\ d\phi$$

As before, the first term of the order of $\Delta$ integrates out to zero; the higher order terms survive. Thus, use of the coil shape and current density given by Eqn.(12), a dipole field of the order $\Delta$ results; the multipole components will be of the order $\Delta^2$ or higher. We can approximate the dipole field by keeping $\Delta < 1$ and as small as possible.

3.2 Case II, Quadrupole.

Assume a coil shape and current as follows:

$$\frac{a_2}{a_1} = e^{\Delta|\cos\ 2\phi|}$$

$$j = j_0 \frac{\cos\ 2\phi}{|\cos\ 2\phi|} \tag{13}$$

The thickness of the coil in the center is not $\Delta$ in this case; but, it can be obtained from Eq.(13). Substitute Eq.(13) into the expression for the potential $A_z$ given by Eq.(6) and evaluate the terms $I_2, I_1, ....I_n$.

$$I_2 = -\int_0^{2\pi} r_2 j \frac{1}{2} ln(\frac{a_1}{a_2}) \cos\ 2(\phi - \theta)\ d\phi$$

$$= \int_0^{2\pi} r_2 j_0 \frac{1}{2} \frac{\cos 2\phi}{|\cos 2\phi|} \Delta|\cos 2\phi|\ \cos 2(\phi - \theta)\ d\phi$$

Using Eq.(11).

$$I_2 = \frac{r_2}{2} j_0 \Delta\pi \cos 2\theta$$

Using Eqs.((8) and (9), this is found to lead to a pure quadrupole field (Ref.1). $I_1$ can be reduce using Eq.(13) as follows.

$$I_1 = \int_0^{2\pi} -ra_1 j[1 - \frac{a_2}{a_1}] \cos(\phi - \theta)\ d\phi$$

$$= \int_0^{2\pi} -ra_1 j_0 \frac{\cos 2\phi}{|\cos 2\phi|} \left[ -\Delta |\cos(2\phi)| - \frac{\Delta^2}{2!} |\cos(2\phi)|^2 - \frac{\Delta^3}{3!} |\cos(3\phi)|^3 \right.$$

$$\left. - ... \right] \cos(\phi - \theta)\ d\phi$$

Using Eq.(11), the first term of the order of $\Delta$ reduces to zero; higher order terms of the order of $\Delta^2$ or, higher may survive. Evaluate terms $I_3 ... I_n$ in a similar way, by considering the generic term $I_n$.

$$I_n = \int_0^{2\pi} \frac{jr^n}{n(n-2)} \frac{1}{a_1^{(n-2)}} \left[ 1 - (\frac{a_1}{a_2})^{(n-2)} \right] \cos\ n(\phi - \theta)\ d\phi$$

$$(\frac{a_1}{a_2})^{(n-2)} = e^{-(n-2)\Delta |\cos 2\phi|}$$

$$= \left[ 1 - (n-2)\Delta |\cos 2\phi| + \frac{(n-2)^2 \Delta^2 |\cos 2\phi|^2}{2} - \frac{(n-2)^3 \Delta^3 |\cos 2\phi|^3}{6} + ... \right]$$

therefore,

$$I_n = \int_0^{2\pi} \frac{j_0 r^n}{n(n-2)} \frac{1}{a_1^{(n-2)}} \frac{\cos 2\phi}{|\cos 2\phi|} \left[ (n-2)\Delta |\cos 2\phi| \right.$$

$$\left. - \frac{(n-2)^2}{2} \Delta^2 |\cos 2\phi|^2 + \frac{(n-2)^3}{6} \Delta^3 |\cos 2\phi|^3 - ... \right] \cos\ n(\phi - \theta)\ d\phi$$

Using Eq.(11), the first term reduces to zero; terms involving higher powers of $\Delta$ may survive. Thus, if a coil profile and current distribuition given by Eq.(13), a pure quadrupole field of the order of $\Delta$ can be achieved and the multipole impurities can be kept to a minimum by keeping $\Delta$ as small possible and less than one.

### 3.3 Case III, Sextupole And Higher Order Poles.

Assume a coil shape and current as follows:

$$\left[1-(\frac{a_1}{a_2})^{(m-2)}\right] = \Delta|cosm\ \phi|$$

Or,

$$\frac{a_2}{a_1} = \left[1-\Delta|\cos\ m\phi|\right]^{-\frac{1}{(m-2)}}$$

$$j = j_0\frac{\cos\ m\phi}{|\cos\ m\phi|} \tag{14}$$

In the above equation, $m = 3$ leads to a sextupole, $m = 4$ leads to an octupole etc. Substitute the above coil profile and current distribution in Eq.(6) and evaluate $I_m, I_1, I_2, I_n$ etc.

$$I_n = \int_0^{2\pi} \frac{j_0 r^n}{n(n-2)}\frac{1}{a_1^{(n-2)}}\frac{\cos\ m\phi}{|\cos\ m\phi|}\left[1-(1-\Delta|\cos\ m\phi|)^{\frac{(n-2)}{(m-2)}}\right]\cos\ n(\phi-\theta)\ d\phi$$

When m=n,

$$I_m = \frac{j_0 r_m}{m(m-2)}\frac{1}{a_1^{(m-2)}}\int_0^{2\pi}\frac{\cos\ m\phi}{|\cos\ m\phi|}\left[1-(1-\Delta|\cos\ m\phi|+...)\right]$$

$$\cos\ m(\phi-\theta)\ d\phi$$

$$= \frac{j_0 r^n}{m(m-2)a_1^{(m-2)}}\pi\Delta\ \cos\ m\theta$$

Eq.(11) was used in evaluating the above integral. Using Eqs.(8) and (9), this is found to a lead to a pure sextupole of the order $\Delta$ for m=3, and to a pure octupole of the same order for m=4 etc. When $m \neq n$, $I_1, I_2$ and $I_n$ will be evaluated separately.

$$I_1 = -\int_0^{2\pi} rj_0\frac{\cos\ m\phi}{|\cos\ m\phi|}a_1[1-\frac{a_2}{a_1}]\ \cos(\phi-\theta)\ d\phi$$

$$= -\int_0^{2\pi} rj_0a_1\frac{\cos\ m\phi}{|\cos\ m\phi|}[1-(1-\Delta\ |\cos\ m\phi|)^{\frac{-1}{m-2}}]\ \cos(\phi-\theta)\ d\phi$$

$$= -\int_0^{2\pi} rj_0a_1\frac{\cos\ m\phi}{|\cos\ m\phi|}[-\frac{\Delta}{m-2}|\cos\ m\phi|+....]\ \cos(\phi-\theta)\ d\phi$$

Since $m \neq 1$, the first term of the order of $\Delta$ integrates out to zero; higher order terms may survive.

$$I_2 = -\int_0^{2\pi} r^2 j_0 \frac{\cos\ m\phi}{|\cos\ m\phi|}\frac{1}{2}\ ln(\frac{a_1}{a_2}) \cos\ 2(\phi-\theta)\ d\phi$$

$$= -\int_0^{2\pi} \frac{r^2 j_0}{2}\frac{\cos\ m\phi}{|\cos\ m\phi|}\frac{1}{m-2}\ ln[1-\Delta|\cos\ m\phi|]\ \cos\ 2(\phi-\theta)\ d\phi$$

$$= -\int_0^{2\pi} \frac{r^2 j_0}{2(m-2)}\frac{\cos\ m\phi}{|\cos\ m\phi|}[-\Delta|\cos\ m\phi| - \frac{\Delta^2|\cos\ m\phi|^2}{2} - ...]\ \cos\ 2(\phi-\theta)\ d\phi$$

Using Eq.(11), the first term of the order of $\Delta$ integrates to zero, since $m \neq 2$; higher order terms may survive. Terms $I_3 ... I_n$ excepting $I_m$ can be studied with the generic term $I_n$.

$$I_n = -\int_0^{2\pi} \frac{r^n j_0}{n(n-2)}\frac{\cos\ m\phi}{|\cos\ m\phi|}\frac{1}{n-2}\left[1-(\frac{a_1}{a_2})^{n-2}\right]\ \cos\ n(\phi-\theta)\ d\phi$$

$$= -\int_0^{2\pi} \frac{r^n}{n(n-2)}\frac{1}{a_1^{n-2}}\frac{\cos\ m\phi}{|\cos\ m\phi|}\left[1-(1-\Delta|\cos\ m\phi|)^{\frac{n-2}{m-2}}\right] \cos\ n(\phi-\theta)\ d\phi$$

$$= -\int_0^{2\pi} \frac{r^n}{n(n-2)}\frac{1}{a_1^{n-2}}\frac{\cos\ m\phi}{|\cos\ m\phi|}\left[\frac{(n-2)}{(m-2)}\Delta|\cos\ m\phi| + ....\right] \cos\ n(\phi-\theta)\ d\phi$$

Using Eq. (11) , the first term of the order of $\Delta$ integrates out to zero since $m \neq n$; higher order terms may survive.

Thus, if coil shapes used and current density distribution given by Eq.(14), pure *mth* pole of order $\Delta$ is derived, mixed with impurities of other multipole components of the order $\Delta^2$ or higher. The required multipole can be approximated by suitably choosing the parameter $\Delta$ and keeping it less than one.

## 4. RESULTS AND DISCUSSION

Numerical computations were done using the finite element code PE2D (Ref.2), for case (a) a dipole and case (b) a quadrupole and the results are shown in Figures. 1 and 2 and Tables 1 and 2.

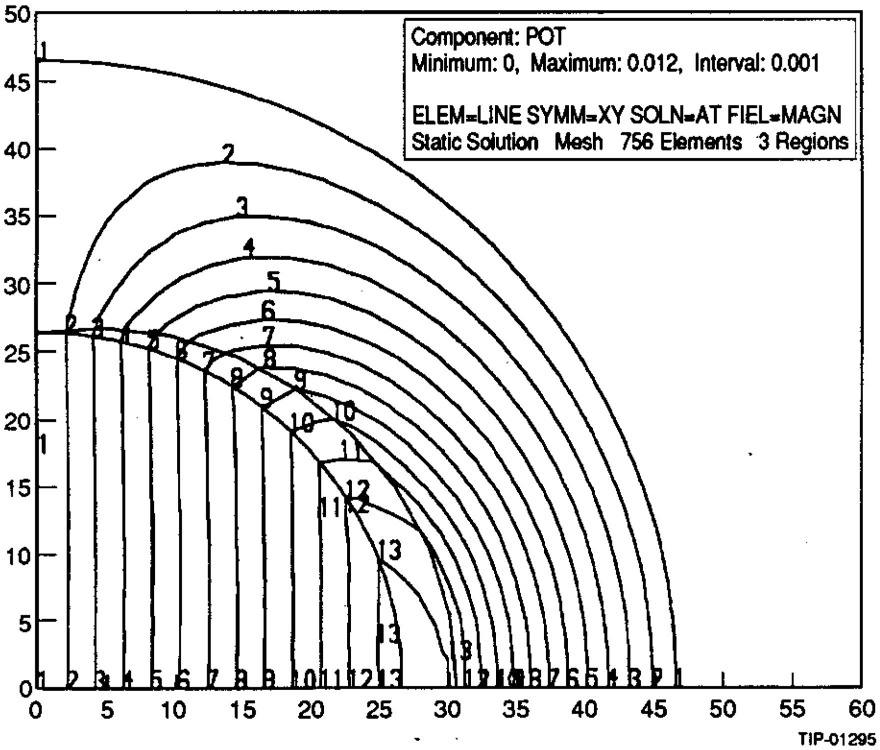


Figure 1. Numerical Computations using Finite Element Code PE2D for Dipole.

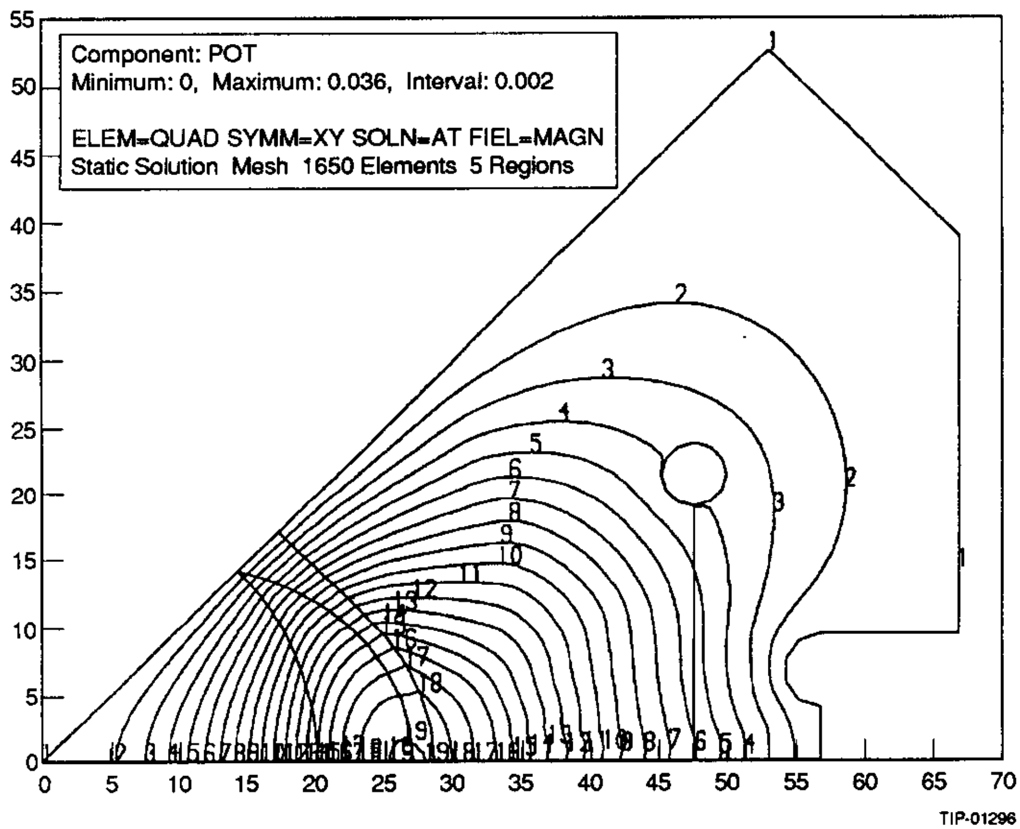


Figure 2. Numerical Computations using Finite Element Code PE2D for Quadrapole.

Table 1. Harmonic Coefficients For Case (a) Dipole

| $N$ | $B(N)$ | $A(N)$ |
|---|---|---|
| 0 | -0.4893 | 0.4278E-07 |
| 1 | -0.7212E-07 | 0.1263E-13 |
| 2 | 0.1791E-02 | -0.4697E-09 |
| 3 | -0.1114E-06 | 0.3897E-13 |
| 4 | 0.1574E-04 | -0.6882E-11 |
| 5 | -0.1526E-06 | 0.8002E-13 |
| 6 | 0.7859E-05 | -0.4810E-11 |
| 7 | -0.0.1844E-06 | 0.1289E-12 |
| 8 | -0.1540E-04 | 0.1211E-10 |
| 9 | -0.2018E-06 | 0.1762E-12 |
| 10 | 0.403E-04 | -0.3875E-10 |

Table 2. Harmonic Coefficients For Case (b) Quadrupole

| $N$ | $B(N)$ | $A(N)$ |
|---|---|---|
| 0 | 0.1325E-07 | -0.1155E-14 |
| 1 | -0.1426E+01 | 0.2493E-06 |
| 2 | -0.2136E-06 | 0.5609E-13 |
| 3 | -0.2102E-06 | 0.7353E-13 |
| 4 | -0.2336E-06 | 0.1022E-12 |
| 5 | 0.5655E-02 | -0.2966E-08 |
| 6 | -0.2924E-06 | 0.1790E-12 |
| 7 | -0.3246E-06 | 0.2217E-12 |
| 8 | -0.3562E-06 | 0.2803E-12 |
| 9 | -0.7327E-04 | 0.6406E-10 |
| 10 | -0.4165E-06 | 0.4005E-12 |

Case (a): The folowing data were used for the computations: The radius of the circular region = 2.65 cms, the coil thickness in the center = 0.4 cm (this corresponds to a $\Delta = 0.1509$), the current density = 3.13 E8 amps/$m^2$. Dirichlet boundary condition was applied at a radius of 4.65 cms. The distribution of magnetic potential is shown in Figure.1. The field in the center was 0.489 Tesla.

The field at a radius of 1.0 cm was used to conduct a harmonic analysis using the following equation.

$$B_y + i\, B_x \;= B_0 \sum_{n=0}^{\infty} (b_n \;+ i\, a_n)(x \;+\; i\, y)^n \tag{15}$$

$B_0$ in the above equation was assumed to be 1.0 and the values of $x$ *and* $y$ were input in cms. The resulting coefficients are shown in Table 1. If the dipole field were pure, the coefficient $B_0$ will be non-zero and the rest of the coefficients will be zero. It is seen that $B_2, B_4 etc.$ are not zero. $B_0 = -0.4893$ and $B_2 = -0.001791$. This shows that the multipole impurities are about 0.37%.

Case (b): The folowing data were used for the computations.: The radius of the circular region = 2.0 cms, the coil thickness in the center = 0.7 cms (this corresponds to a $\Delta = 0.3001$), the current density = 4.6 E8 amps/$m^2$. An iron yoke as shown in Figure 2 was used. The presence of iron can be expected to improve the field. The distribution of magnetic potential is shown in Figure 2. The coefficients obtained from the harmonic analysis of the field at 1.0 cm radius with $B_0$ in Eq.(15) set equal to 1.0 , are shown in Table 2. If the field were pure quadrupole field the coefficient $B_1$ should be non-zero and the rest should be zero. It is seen that $B_1 = -1.426$ and $B_5 = 0.005655$. This shows that the multipole impurities are about 0.397%.

## CONCLUSION

It is possible to achieve approximately pure multipole fields in circular regions, using coil shapes and current density distributions given by Eq.(12) for dipole, Eq.(13) for quadrupole and Eq.(14) for other multipoles. The multipole impurities are found to be of the order $\Delta^2$ or higher; the impurities can be kept to a minimum by keeping $\Delta$ as small as possible, less than one.

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