# Mass spectrum of heavy quarkonium for harmonic plus screened Kratzer potential (HSKP) using series expansion method

Kaushal Purohit [1],* Ajay Kumar Rai [1], and Rajendrasinh H. Parmar [2]
[1] *Department of Physics , Sardar Vallabhbhai National Institute of Technology, Surat, Gujarat-395007, India and*
[2] *Sir P T Science College, Modasa, Gujarat-385315, India*

## Introduction

The Schrodinger equation (SE) portrays numerous problems in various areas of physics and chemistry. The exact solution of Schrodinger equation with some solvable potential assumes an essential part in nuclei, atoms, molecules, and spectroscopy and in many fields of modern Physics. There are areas in which the solution to SE can be applied, for the solution of the SE with screened Kratzer potential in studying thermodynamic properties of the system. Other application of the solution SE include investigation of the mass spectra of quarkonium system, which is the main focus of this work.

The HSKP is one of the successful potential models for such system because it produces its mass spectra in agreement with experimental data.The radial Schrodinger equation is solved with the HSKP using the series expansion method. The bound state energy spectra are obtained. The energy eigenvalues for any state have been determined. The present results are applied to calculate the mass spectra of heavy quarkonium systems such as charmonium and bottomonium. A comparison is discussed with the experimental data and recent other works. The present results are improved in comparison with other recent studies and are in a good agreement with the experimental data. Our results will have possible applications in high energy physics, molecular physics, etc.

Here,we proposed harmonic plus screened Kratzer potential. The proposed HSKP stated as,

$$V(r) = -2D_e\left(\frac{a}{r} - \frac{b}{2r^2}\right)e^{-\alpha r} + cr^2 \tag{1}$$

where $a = r_e$ and $b = r_e^2$, $D_e$ is the dissociation energy, $r_e$ is the equilibrium bond length, $r$ is the interatomic distance and $\alpha$ is the screening parameter.

## Bound state solution with radial Schrodinger equation

We consider the radial Schrodinger equation as

$$\frac{d^2\psi(r)}{d^2} + \frac{2}{r}\frac{d\psi(r)}{dr} + \left[\frac{2\mu}{\hbar^2}(E - V) - \frac{l(l+1)}{r^2}\right]\psi(r) = 0 \tag{2}$$

Taylor expanding the exponential term of the potential and ignoring terms greater than $r^3$ our Eq.(1) becomes

$$V(r) = \left(\frac{aD_e\alpha^3}{3} + c\right)r^2 + \left(-a\alpha^2 - \frac{bD_e\alpha^3}{6}\right)r - (2D_ea + D_eb\alpha)\frac{1}{r} + \frac{D_eb}{r^2} + 2D_ea\alpha + \frac{D_eb\alpha^2}{2} \tag{3}$$

Now we substitute Eq.(3) into Eq.(2) and we obtain

$$\left[\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{L(L+1)}{r^2}\right]\psi(r) + \left(\varepsilon - A_1r^2 - A_2r + A_3\frac{1}{r} + A_4\frac{1}{r^2}\right)\psi(r) = 0 \tag{4}$$

*Electronic address: kaushalsep1996@gmail.com

Where

$$\varepsilon = \frac{2\mu}{\hbar^2}\left(E - \left(2D_e a\alpha + \frac{D_e b\alpha^2}{2}\right)\right)$$

We obtaining the energy eigenvalue expression as

$$E_{nl} = \sqrt{\frac{\hbar^2 A_1}{2\mu}}\left(4n + 2 + \sqrt{(2l+1)^2 + \frac{8\mu A_4}{\hbar^2}}\right)$$

$$-\frac{2\mu A_3^2}{\hbar^2}\left(4n + 1 + \sqrt{(2l+1)^2 + \frac{8\mu A_4}{\hbar^2}}\right)^{-2} + \delta \tag{5}$$

Where

$$A_1 = \frac{aD_e\alpha^3}{3} + c, A_3 = 2D_e a + D_e b\alpha, A_4 = D_e b$$

$$A_2 = \left(-a\alpha^2 - \frac{bD_e\alpha^3}{6}\right), \delta = 2D_e a\alpha + \frac{D_e b\alpha^2}{2}$$

We derive the mass spectra of heavy quarkonium system such as charmonium and bottomonium. To determine the mass spectra we use the following relation

$$M = 2m_b + E_{nl} \tag{6}$$

Substituting Eq.(5) into Eq.(6) we obtain mass spectra as

$$= 2m + \sqrt{\frac{\hbar^2 A_1}{2\mu}}\left(4n + 2 + \sqrt{(2l+1)^2 + \frac{8\mu A_4}{\hbar^2}}\right)$$

$$-\frac{2\mu A_3^2}{\hbar^2}\left(4n + 1 + \sqrt{(2l+1)^2 + \frac{8\mu A_4}{\hbar^2}}\right)^{-2} + \delta \tag{7}$$

## Discussion of Results

Table (I) and (II) present the numerical values of mass spectra for some quarkonium systems charmonium and bottomonium, respectively.The results of mass spectra are in consonance with experimental values and other theoretical works of similar investigation. The details of the formula will be presented in the conference.

TABLE I: Mass spectra of charmonium in (GeV) with the mass $m_c = 1.209GeV$, $A_4 = 0.023$, $\delta = 0.7264$, $A_3 = 3.9422$ and $A_1 = 0.00724$

| State | Our | Ref.[2] | Ref.[4] | Exp. |
|---|---|---|---|---|
| 1S | 3.0961 | 3.0959 | 3.098 | 3.096 |
| 2S | 3.6862 | 3.6858 | 3.689 | 3.686 |
| 2P | 3.7634 | 3.7565 | 2.784 | - |
| 3S | 4.0403 | 4.3228 | 4.041 | 4.040 |
| 4S | 4.3411 | 4.9894 | 4.266 | 4.263 |

TABLE II: Mass spectra of bottomonium in (GeV) with the mass $m_b = 4.822GeV$, $A_4 = 0.52$, $\delta = 0.00836$, $A_3 = 3.0884$ and $A_1 = 0.0102$

| State | Our | Ref.[1] | Ref.[2] | Ref.[4] | Exp. |
|---|---|---|---|---|---|
| 1S | 9.460 | 9.470 | 9.515 | 9.461 | 9.460 |
| 2S | 10.024 | 10.033 | 10.018 | 10.023 | 10.023 |
| 2P | 10.0944 | 10.279 | 10.094 | 10.110 | 10.260 |
| 3S | 10.3563 | 10.302 | 10.441 | 10.365 | 10.355 |
| 4S | 10.6196 | 10.548 | 10.857 | 10.588 | 10.580 |

## References

[1] R.Chaturvedi et al. J. Phys. G: Nucl. Part. Phys.**47** 115003 (2020)
[2] E.E. Ibekwe et al. Iran J. Sci. Technol. Trans. Sci. **44**, 1191 (2020).
[3] R.Kumar and C.Fakir,Phys. Scr. **85**, 055008 (2012).
[4] E.Omugbe et al. Adv.High Energy Phys. 5901464 (2020).
[5] R.Chaturvedi, A.K.Rai, Int.J.Theo.Phys. **59**, 3508 (2020).
[6] V.Kher, A.K.Rai, Chin.Phy. C **42**, 8 (2018).
[7] A.K.Rai et al Phys. Rev. C **78**, 055202 (2008).
[8] N.V.Maksimenko,S.M.Kuchin, Russ. Phys. J **54**, 57 (2011).
[9] K.R.Purohit et al. Annals of Physics **424** , 168335 (2021).
[10] A.K.Rai et al. J. Phys. G: Nucl. Part. Phys. **28(8)**,2275 (2002).
[11] G.Bhanot and S.Rudaz, Phys.Lett. **78**, 119 (1978)
[12] K.R.Purohit et al. Eur.Phy.J.Plus **135**, 3 (2020)
[13] J.N.Pandya et al. Chin. Phys. C **39**, 123101 (2015)
[14] A.K.Rai et al. Eur.Phys.J.A **38**,77 (2008)