INELASTIC ELECTRON-POSITRON SCATTERING

M. Gourdin
Laboratoire de Physique Théorique et Particules Elémentaires, Orsay
and
Laboratoire de Physique Théorique et Hautes Energies, Paris

Abstract : The two-photon exchange in the inelastic Bhabha scattering $e^+e^- \to e^+e^-$ + anything is studied without the quasi real photon approximation.

Résumé : L'échange de deux photons dans la diffusion inélastique $e^+e^- \to e^+e^- + \Gamma$ est étudié sans l'approximation des photons quasi réels.

I wish to report very briefly on a recent work done in Paris on the two-photon contributions to inelastic Bhabha scattering

$$e^+ + e^- \Rightarrow e^+ + e^- + \Gamma \tag{1}$$

where Γ is a one or multiparticle system not detected.

1°) More precisely we have computed the dominant diagram at high energy

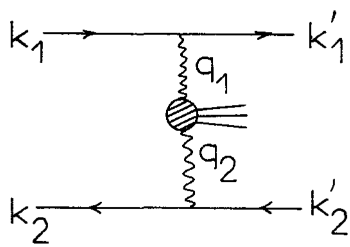

Fig. 1

The knowledge of the cross section for the process (1) will give information on the two virtual photon annihilation reaction

$$\gamma + \gamma \Rightarrow \Gamma \tag{2}$$

When the particles entering the system Γ of effective mass W are not detected, we can define 10 structure functions depending on q_1^2, q_2^2 and W^2 only. If time reversal holds, this number is reduced to 8.

An experiment with unpolarized electrons and positrons is able to measure only 6 structure functions, and we get

$$\frac{E_1'}{d_3k_1'} \frac{E_2'}{d_3k_2'} d\sigma = \frac{\alpha^2}{2\pi^4} \frac{1}{s} \frac{KW}{q_1^2 q_2^2} \Big\{ \sigma_{LL} K_{LL} + \sigma_{LT} K_{LT} + \sigma_{TL} K_{TL} + \sigma_{LL} K_{LL} + (\sigma_{\parallel} - \sigma_{\perp}) K_{TT}^{ex} + (\tau_0 + \tau_1) K_{LT}^{ex} \Big\}$$

where K is the c.m. momentum of the $\gamma\gamma$, S the square of the total incident energy $S = (E_1 + E_2)^2$.

The function $K_{\lambda\lambda'}$ are known functions of $\vec{k}'_1$ and $\vec{k}'_2$ but obviously they depend only on 5 variables because of an overall symmetry around the incident axis of the beams. The structure functions depending only on 3 variables, the dependence of the cross section on the remaining 2 variables is known as a consequence of the two-photon exchange approximation and is completely contained in the functions $K_{\lambda\lambda'}$.

The two main features of this process is the smallness of the electron mass and the presence of the photon propagators. The poles existing at $q_1^2 = 0$ and $q_2^2 = 0$ dominate the cross section and suggests the use of a quasi-real photon approximation. Our calculations are free of such an approximation.

2°) The calculations have been made for a state Γ free of any e^+e^- pair.

(a) The differential cross section $\frac{d^3\sigma}{dq_1^2\,dq_2^2\,dW^2}$ has been computed exactly, integrating analytically over the two kinematical variables [1].

(b) The dominant contribution of $\frac{d\sigma}{dW^2}$ is associated to the total cross section $\sigma_{TT}(0,0,W^2)$ for process (2) with real photons

$$\frac{d\sigma^{TT}}{dW^2} = \left(\frac{\alpha}{\pi}\right)^2 \frac{\sigma_{TT}(0,0,W^2)}{W^2}\, F_{TT}(W^2,s) \tag{3}$$

The function $F_{TT}(W^2, s)$ can be exactly computed and is independent of any model. It exhibits a logarithmic dependence in the energy we can order when W^2 is small as compared to s, according to the number of logarithms involved as

$$3\text{-}\mathrm{Log} + 2\text{-}\mathrm{Log} + 1\text{-}\mathrm{log} + C^{te} + O\left(\frac{W^2}{s}\right)$$

For instance we get, for the 3-Log terms [1]

$$\left(\mathrm{Log}\,\frac{s}{m^2}\right)^2 \mathrm{Log}\,\frac{s}{W^2} - \frac{1}{3}\left(\mathrm{Log}\,\frac{s}{W^2}\right)^3 \tag{4}$$

(c) The other contributions of $\frac{d\sigma}{dW^2}$ which are model dependent and where enter the other helicity states and the dependence in q_1^2 and q_2^2 of the structure functions [2].

(d) The total cross section σ (s) for the multiparticle states Γ [2].

3°) Some applications of these calculations have been investigated [2].

(a) The first one concerns the $\mu^+\mu^-$ production where the quantum electrodynamics gives us a model for an exact and complete calculation. In particular, the differential $\frac{d}{dW^2}\sigma(s, W^2)$ and total σ (s) cross sections have been computed. The comparison of σ(s) with previous calculations is as follows, see fig. 2.

1.- With formula (3) of Bonneau-Gourdin-Martin, exactly computed, the agreement is better than 2%.

2.- With the Baier-Fadin [3] calculation, the 3-Log, 2-Log, and 1-Log terms are identical, but in that paper the C^k and $O(\frac{m_\mu^2}{s})$ terms have not been evaluated so that we have a nice agreement asymptotically, but at intermediate energy, Baier and Fadin make too small an evaluation, having an incorrect threshold.

3.- With Brodsky-Kinoshita-Terazawa [4] 's calculation where the pole term $(\mathrm{Log}\,\frac{E}{m})^2$ only is taken into account, giving systematically too high a cross section. Of course the threshold is there correct and the disagreement is about 15-20% at intermediate energy where the future machine will operate and of course 50% at infinite energy where the second term of equation (4) plays a role.

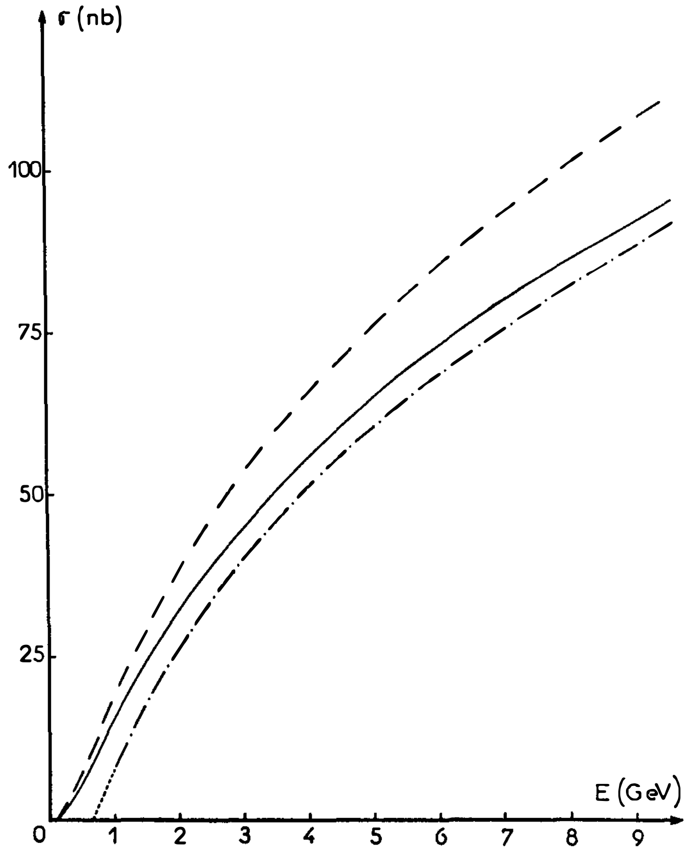

Fig. 2 - The total cross section for the process ee → ee $\mu^+\mu^-$
as a function of E : ———— our complete result
— — — — result of ref. [4]
—·—·—·— result of ref. [3]

(b) The second application of the general calculations concerns the production of pseudoscalar meson $\Gamma = \pi^\circ, \eta, X^\circ$. As in reference [4] a vector meson dominance type model has been used with one parameter M in order to describe the variation with q_1^2 and q_2^2 of the amplitude

For π° production, the model independent part gives 3/4 of the cross section and the model dependent part 1/4 for M of the order of the ρ meson mass. If M varies between 5 and 1.5 GeV, then the model dependent part can change by 30% so that the computed cross section varies by approximatively 5%-10%.

For η and X production, the situation is better in the sense that the model independent part is relatively more important so that if the model dependent part is more sensible to the mass parameter M as in the π° case, the total prediction is not too sensible to that parameter.

Of course these computations agree with similar ones of Brodsky, Kinoshita, Terazawa [4] corresponding to specific values of M.

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