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QUANTUM DEPOLARIZATION OF ELECTRONS IN A MAGNETIC FIELD

by

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It was shown in ref. 1 that when electrons and positrons moving in a magnetic field emit radiation, they become polarized. Although the probability of emission with a change in spin direction is very small in comparison with the total emission probability, high-energy electrons and positrons may exhibit considerable polarization after prolonged orbiting in modern storage rings. The polarization time in a uniform field is given by

$$\tau_{pol}^{-1} = \frac{5\sqrt{3}}{8}\alpha m\left(\frac{\varepsilon}{m}\right)^2\left(\frac{H}{H_c}\right)^3 \qquad (1)$$

where $H_o = m^2/e = 4.4 \times 10^{13}$ Oe. In a storage ring designed for $E = 6$ Bev, and a field at the orbit of $H = 8000$ Oe, the polarization time τ_{pol} is 190 secs.

The polarization produced in this way may be preserved in a storage ring if the particle energy ε is chosen so that depolarizing resonances are avoided in the system. These resonances are due to the radial and azimuthal components of the magnetic field at the particle orbit. More precisely, the resonance condition

$$G \frac{\varepsilon}{m} = k + l Q_z + m Q_R + n Q_x \; ; \tag{2}$$

(where k, l, m and n are integers) must be satisfied for the largest possible l, m, n. In this expression $G = g - 2 \simeq \alpha/2\pi$ is the anomalous part of the electron g-factor and $Q_{z,R,x}$ is the number of vertical (z), radial (R), and phase (x) oscillations. The components lQ_z and mQ_R are due to the terms z^l and r^m ($r = R - R_o$ where R_o is the equilibrium radius) in the fields H_R, H_x, whereas the term nQ_x represents synchrotron oscillations in the energy, and corrections to the frequencies, $Q_{z,R}$, connected with these oscillations. Detailed analysis of each given storage ring with allowance for the specific nonlinearity of the magnetic field etc. is necessary for the correct choice of the energy ε.

Let us suppose that ε can be chosen so that depolarization effects due to the resonances described by (2) are unimportant. It turns out that at high enough energies, there is further depolarization due to the quantum nature of the emission of radiation. This source of depolarization will again only appear in the presence of the perturbing fields H_R and H_x, but the resonance conditions (2) need not now be necessarily satisfied. This is due to the fact that energy changes connected with the quantum nature of the emitted radiation contain harmonics which give rise to the resonance (2) when these changes are expanded into a Fourier integral. Here again, the phenomenon can be traced to a resonance which exhibits the irksome feature that it cannot be avoided by a suitable choice of ε.

We should also like to emphasize that we have not succeeded in discovering any other classical or quantum mechanical effects which would lead to depolarization of the beam, so that by removing the most dangerous harmonics of the perturbations, it is possible to reduce depolarization effects quite substantially. This applies to the effect discussed below.

Quantum depolarization can be considered approximately by assuming that the electron trajectory is classical and the particle energy undergoes discontinuous changes whenever a photon is emitted, so that

$$\overline{\frac{d(\Delta\varepsilon)^2}{dt}} = \frac{55\sqrt{3}}{48}\,\frac{1}{R}\left(\frac{\varepsilon}{m}\right)^3 \overline{W} \; ; \quad \overline{W} = \frac{2}{3}\left(\frac{\varepsilon}{m}\right)^4 \frac{r_0}{R^2} \tag{3}$$

where $\bar{W}$ is the classical rate of emission of radiation by an electron, and r_o is the classical radius of the electron.

The equation for the electron spin 4-vector S^i, which takes into account the perturbing fields H_R and H_x, are of the form[2]:

$$\frac{dS^1}{d\tau} = g\,\frac{eH}{m}\left(\frac{\varepsilon}{m}\right)^2 S^2 - \left(1+g\left(\frac{\varepsilon}{m}\right)^2\right)\frac{eH_R}{m} S^3 \tag{4}$$

$$\frac{dS^2}{d\tau} = -g\,\frac{eH}{m} S^1 + (1+g)\,\frac{eH_x}{m} S^3 \tag{5}$$

$$\frac{dS^3}{d\tau} = (1+g)\,\frac{eH_R}{m} S^1 + \left[g\,\frac{eH}{m}\frac{\rho}{m}\frac{dz}{d\tau} - (1+g)\frac{eH_x}{m}\right] S^2 \tag{6}$$

$$\frac{dS^4}{d\tau} = g\,\frac{eH}{m}\,\frac{\varepsilon\rho}{m^2} S^2 - g\,\frac{eH_R}{m}\,\frac{\varepsilon\rho}{m^2} S^3 \tag{7}$$

where $\tau = t\;\; m/\varepsilon$, and H is the equilibrium field. It can readily be verified that neither space nor time variations in H will lead to depolarization, which is of course obvious a priori. The terms with H_R and H_x may give rise to a resonance spin flip, but we shall assume that (2) is not satisfied. Since the effect under consideration is largely only indirectly dependent on the type of focusing in the storage device, we shall consider for the sake of simplicity an azimuthally symmetric, weakly-focusing field in which the k-th harmonic of the perturbation $H/R = h \cos [(keH/m)(\tau-\tau_0)]$ is operative (for $z = 0$), so that the field on the real vertically perturbed orbit $z = z_H$ can be written in the form

$$H_z = -\frac{k^2 h}{n-k^2} \cos\left[\frac{keH}{m}(\tau-\tau_0)\right] \qquad (8)$$

$$z_H = z_k \cos\left[\frac{keH}{m}(\tau-\tau_0)\right], \quad z_k = \frac{h}{H}\frac{R}{n-k^2} \qquad (9)$$

Since the r-oscillations in the approximation which is linear in h contribute nothing to the effect, we shall assume that $r = 0$. Effects due to free z-oscillations in the inhomogeneous field, and also those due to the field H_x, can be considered in a similar way, and the corresponding results are summarized at the end of the paper.

The solution of equations (4) - (7) will be written in the particle rest system in the first approximation in h. In order to transform to this system, it is necessary to apply the Lorentz transformation and the usual rotation through a small angle:

$$A_z = \frac{m}{\rho}\frac{dz}{d\tau} = -\frac{h}{H}\frac{K}{n-K^2}\sin\left[\frac{eHK}{m}(\tau-\tau_0)\right] \tag{10}$$

In this system, which we indicate by a subscript c, we have $S_c^4 = 0$ and

$$(S_c^1)^2 + (S_c^2)^2 \equiv S_\rho^2 = \sigma_\rho^2 + \sigma_\rho\sigma_z \frac{h}{H}\frac{gK}{n-K^2} F \tag{11}$$

$$(S_c^3)^2 \equiv S_z^2 = \sigma_z^2 - \sigma_\rho\sigma_z \frac{h}{H}\frac{gK}{n-K^2} F \tag{12}$$

$$F = \frac{\left(1 + K\frac{\varepsilon}{m}\right)}{\left(K - g\frac{\varepsilon}{m}\right)} \sin\frac{eH}{m}\left[\left(K - g\frac{\varepsilon}{m}\right)\tau - K\tau_0\right] - \frac{\left(1 - K\frac{\varepsilon}{m}\right)}{\left(K + g\frac{\varepsilon}{m}\right)} \sin\frac{eH}{m}\left[\left(K + g\frac{\varepsilon}{m}\right)\tau - K\tau_0\right] \tag{13}$$

where σ_z, σ_ρ are constants at constant particle energy and

$$\sigma_\rho^2 + \sigma_z^2 = 1 \tag{14}$$

The quantities S_z and z_ρ can be interpreted as the instantaneous spin components along the z-direction and in a perpendicular direction in the electron rest system; σ_z and σ_ρ can be interpreted as the averages about which small oscillations in the components S_z and S_ρ take place.

Since the relative spin-flip probability at the time of emission is negligible in comparison with the probability of emission without spin flip, the true spin components S_z and S_ρ are unaltered at the time of emission. There is however a discontinuous change in F in equations (11) and (12) which is proportional to the energy change $\Delta\varepsilon$. Since S_z and S_ρ are unaltered, there is a discontinuous change in the mean σ_z and σ_ρ. Compensation of radiative losses by the accelerating system of the storage device subsequently returns the quantity F [or, more precisely, the amplitudes in (13)] to the initial value, but a sequence of such random changes leads to a stochastic buildup in σ_z and σ_ρ and consequently in S_z and S_ρ.

At sufficiently high energies ($G\,\varepsilon/m \gg 1$), for which the effect under consideration is of particular interest, the main contribution to the buildup process is due to changes in the denominator in the first term of (13). This is governed by harmonics for which $G\,\varepsilon/m \approx k$. The resulting change in the angle Θ between the spin direction and the z-axis is then given by

$$1/\tau_{depol} \approx \frac{d\overline{\vartheta^2}}{dt} \approx \frac{1}{8}\sum_k \left(\frac{z_k}{R}\right)^2 \left(\frac{k\,G\frac{\varepsilon}{m}}{k-G\frac{\varepsilon}{m}}\right)^4 \frac{1}{\varepsilon^2}\,\frac{d\overline{(\Delta\varepsilon)^2}}{dt} =$$

$$= \sum_k \frac{55}{192\sqrt{3}} \left(\frac{k\,G\frac{\varepsilon}{m}}{k-G\frac{\varepsilon}{m}}\right)^4 \frac{r_0}{mR^3}\left(\frac{z_k}{R}\right)^2\left(\frac{\varepsilon}{m}\right)^5 \tag{15}$$

where z_k is the amplitude of the forced z-oscillations excited by the perturbation H_R.

It is evident from (16)* that the effect is very dependent on the particle energy, the number of the nearest resonance harmonic k, the distance from the resonance ($|k - G\ \varepsilon/m|$), and the magnitude of z_k. Let us estimate the effect numerically for reasonable values of the various parameters: E = 6 BeV, H = 8000 Oe, R = 3000 cm, $|k-G\ \varepsilon/m| \simeq 1/2$ for k = 14, 15, z_k = 0.1 cm. Under such conditions the characteristic time for depolarization (change in Θ by unity) is τ_{depol} = 25 sec. This is smaller than τ_{pol} by an order of magnitude. It follows that in this case the beam is not polarized.

This means that special measures must be introduced in order to conserve polarization of the beam. Nevertheless, it is worth emphasizing once again the surprising persistence of the polarization.

The formulas for S_ρ and S_z with allowance for the perturbation H_x and the presence of free z-oscillations read as follows (these effects give rise to a much smaller contribution than those discussed above) :

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* Translator's note: No (16)

$$\mathcal{S}_{g,z}^2 = \sigma_{g,z}^2 \pm \sigma_g\sigma_z \frac{h_{xk}}{H}\left\{\frac{1+G}{k+G\frac{\varepsilon}{m}} \sin\frac{eH}{m}\left[\left(k+G\frac{\varepsilon}{m}\right)\tau - k\tau_1\right]\right. \tag{17}$$

$$\left. - \frac{1+G}{k-G\varepsilon/m}\sin\frac{eH}{m}\left[\left(k-G\frac{\varepsilon}{m}\right)\tau-k\tau_1\right]\right\}, \quad H_k = h_{xk}\sin\frac{keH}{m}(\tau-\tau_1)$$

$$\mathcal{S}_{g,z} = \sigma_{g,z}^2 \pm \sigma_g\sigma_z \frac{z_{max}}{R} G \left\{ \frac{1+\frac{\varepsilon}{m}\sqrt{n}}{G\frac{\varepsilon}{m}\frac{1}{\sqrt{n}}-1}\sin\frac{eH}{m}\left[\left(G\frac{\varepsilon}{m}-\sqrt{n}\right)\tau - \sqrt{n}\,\tau_2\right]\right.$$

$$\left. + \frac{\frac{\varepsilon}{m}\sqrt{n} - 1}{G\frac{\varepsilon}{m}\frac{1}{\sqrt{n}}+1}\sin\frac{eH}{m}\left[\left(G\frac{\varepsilon}{m}+\sqrt{n}\right)\tau-\sqrt{n}\,\tau_2\right]\right\} \tag{18}$$

As was to be expected, analysis of linear z-oscillations shows that the effect depends on the difference $(G\ \varepsilon/m)-Q_z$; $Q_z = \sqrt{n}$ (see equation (2) for $l = 1$).

R e f e r e n c e s

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