Summary of the parallel session BH3

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We summarize the talks presented at the BH3 session (Black Holes in Alternative Theories of Gravity) of the 16th Marcel Grossmann Meeting held online on July 5-10 2021.

Keywords: Gravity; Alternative theories; Black holes.

1. Introduction

Black holes have received tremendous interest in recent years, originating on the one hand from the discovery of gravitational waves, generated in the merger of stellar black holes, by the LIGO collaboration,[1] and on the other hand from the observation of the shadow of the supermassive black hole at the center of M87 by the EHT collaboration.[2] The analysis of observational data of black holes is typically based on the *Kerr hypothesis*, namely the assumption that rotating black holes are well described by the Kerr solution of General Relativity. Just like General Relativity itself, also the *Kerr hypothesis* is still consistent with all available data.

However, there are a number of reasons suggesting that General Relativity will be superseded by a new theory of gravity, which would reduce to General Relativity in a limit. From a theoretical side these reasons include the problem of quantization of gravity and the presence of singularities in solutions of General Relativity, while from an observational side the need for *Dark Matter* and *Dark Energy* in a cosmological context seem most provoking.

All these reasons have led to a large number of suggestions for alternative theories of gravity based on deep theoretical reasoning or simply phenomenological modelling, see e.g.,[3–6] While alternative theories of gravity should be consistent with observations in the weak gravity regime, thus in particular, with observations in the solar system, the strong gravity regime is so far much less constrained. In the strong gravity regime there exist certainly high precision data from pulsar observations (see, e.g.[7]), but the unknown equation of state of matter at extreme densities and pressures may lead to degeneracies with the unknown theory of gravity. Therefore black holes may present cleaner probes of the strong gravity regime.

In General Relativity black holes are remarkably simple objects in terms of the characterization of the space-times, as expressed by the *no-hair theorem* (see, e.g.[8]). Consequently, Kerr black holes are uniquely determined by their mass and their angular momentum, and all their higher multipole moments can be fully expressed by these two lowest moments. Moreover, when considering the fields of the Standard Model of Particle Physics, no astrophysical rotating black holes other than the *bald* Kerr black holes arise. Only microscopic black holes might carry fields of the Standard Model as hair (see, e.g.[9,10]). However, if one allows for a hypothetical complex massive scalar field or Proca field, then General Relativity allows for rotating Kerr black holes with scalar or Proca hair that might be of astrophysical relevance.[11,12]

Alternative theories of gravity typically introduce further degrees of freedom, often in the form of gravitational scalar or vector fields.[3–6] Prominent theories with additional scalar fields are Horndeski and beyond Horndeski theories,[13–16] while their counterparts with vector fields are generalized Proca and beyond generalized Proca theories.[17–20] But further degrees of freedom may also be present when the tensorial part of gravity is modified, yielding, for instance, de Rham-Gabadadze-Tolley massive gravity[21] and bigravity theories.[22,23] While General Relativity can be formulated in terms of curvature, torsion or non-metricity, as expressed by the *geometrical trinity of gravity*,[24] the associated generalizations of these formulations are no longer equivalent.

Up to now most work on black holes in alternative theories of gravity has been done in metric theories. Here the recent years have witnessed the emergence of new types of black holes due to newly discovered phenomena. These are, for instance, curvature-induced spontaneously scalarized black holes that arise in certain Horndeski models with higher curvature terms, when curvature is sufficiently strong to induce a tachyonic instability in the background of a Schwarzschild or Kerr black hole solution.[25–29] Similarly, spin-induced spontaneously scalarized black holes emerge in the presence of sufficient curvature and spin.[30–33] While these solutions satisfy circularity, this is not necessarily true for rotating black holes in beyond Horndeski theories, where also so-called disformed black holes with a non-circular geometry can arise (see, e.g.[34,35]).

A very important aspect of black holes in alternative theories of gravity is of course their stability and their response to perturbations. In particular, the study of quasi-normal modes (QNMs) is very valuable here, since these are also of relevance for the analysis of the ringdown phase of black hole mergers. Whereas the QNMs of the Schwarzschild and Kerr black holes are known since quite some time (see, e.g.[36–40]), the study of the QNMs of black holes in alternative theories of gravity is still in its infancy. While some work has been done for static black holes (see, e.g.[41]), very little is known so far even for the case of slow rotation only. The presence of additional degrees of freedom in alternative theories of gravity, yields a much more intricate spectrum, though, since various modes forbidden in General Relativity will be present in alternative theories of gravity.

In the following we will summarize the progress on black holes in alternative theories of gravity reported in our session of the 16th Marcel Grossmann Meeting.

2. Summary of the presentations

The 16th Marcel Grossmann Meeting was held online on July 5-10 2021. This was the first time that the meeting was held completely online, mainly because of the COVID-19 pandemic around the world. In the parallel session "Black Holes in Alternative Theories of Gravity" abbreviated as BH3, 16 physicists presented their recent research activities in two days, Tuesday 06 July 2021 09:30-12:30 and Thursday 08 July 2021 16:30-19:30 CEST. Here we recall the main points of the talks in BH3, and provide a summary of the presentations.

2.1. *Asymptotically flat hairy black holes in massive bigravity*

Presented by: Mikhail Volkov
In collaboration with: Romain Gervalle
Based on the Ref.[42].

The ghost-free theories of bigravity were first introduced by Hassan and Rosen in Ref.[43]. In these theories, there are two metrics denoted by $g_{\mu\nu}$ and $f_{\mu\nu}$. Both of the metrics have the usual Einstein-Hilbert action, and there is an interaction term between the two metrics. The metric $g_{\mu\nu}$ can be coupled to some matter, while $f_{\mu\nu}$ is not coupled to any matter. Analyzing the propagating degrees of freedom, there are two gravitons in such theories, one massive and one massless. These family of theories have been shown to be able to describe the accelerating expansion of the universe without a cosmological constant.

Any black hole as a vacuum solution in general relativity is a solution to bigravity theories by the choice of $g_{\mu\nu} = f_{\mu\nu}$. For example, the Schwarzschild metric is a solution to these theories if one chooses $g_{\mu\nu} = f_{\mu\nu}$ be equal to the Schwarzschild metric. However, this solution is not a stable solution in these theories. An interesting question arises here: if one relaxes the constraint $g_{\mu\nu} = f_{\mu\nu}$, is it possible to have spherically symmetric asymptotically flat solutions other than the Schwarzschild. The terminology "hairy black holes" has been used for such solutions in bigravity, and the "bald Schwarzschild" for the special case of $g_{\mu\nu} = f_{\mu\nu}$ to be Schwarzschild. It has been a debate to answer this question[44–46] whether there are hairy black holes in ghost-free bigravity or not. Romain Gervalle and Mikhail Volkov in Ref.[42] have tried to answer this interesting question by constructing spherically asymptotically flat black hole solutions numerically. They have found that there exist such black hole solutions (as pairs of solutions, one for $g_{\mu\nu}$ and one for $f_{\mu\nu}$) with the same horizon and surface gravity. Free parameters of the Schwarzschild hairy black holes are constrained by studying the stability of the solution. To this end, the $g_{\mu\nu}$ metric should be close to the Schwarzschild metric, the $f_{\mu\nu}$ metric should not be coupled to the matter, and the mass of the black hole should be between $0.2M_\odot$ and $0.3 \times 10^6 M_\odot$.

2.2. *An overview of quasinormal modes in modified and extended gravity*

Presented by: Aurélien Barrau
In collaboration with: Flora Moulin and Killian Martineau
Based on the Ref.[47].

Quasi-normal modes (QNMs) are dissipative perturbations around a black hole background solution. They satisfy purely outgoing and ingoing boundary conditions at infinity and at the event horizon respectively. The oscillatory time dependence of these modes is described by

$$\psi \propto e^{i\omega t} = e^{i(\omega_R + i\omega_I)t}. \tag{1}$$

The frequency ω_R characterizes the oscillatory behavior of the mode, while for the $\omega_I > 0$ and $\omega_I < 0$ the mode grows or decays exponentially, respectively. The main question in the QNM analysis is to find the behavior of the complex function ω.

In this talk, Aurélien Barrau provided an overview on QNMs, and presented their results for perturbations around a spherically symmetric black hole in some alternative theories of gravity. Their analysis is based on Wentzel-Kramers-Brillouin (WKB) approximation method.[48–50] The theories under consideration have been chosen to be the massive gravity,[21,22,51,52] Modified Scalar-Tensor-Vector (STV) Gravity,[53] Horava-Lifshitz,[54–57] $\hbar$-correction (quantum correction),[58] and loop quantum gravity (based on the model presented in Ref.[59]). For each one of these choices, the diagrams of ω_R and ω_I for some suitable multipole number ℓ and overtone number n were illustrated and compared qualitatively.

2.3. *Constraining modified gravity theories with physical black holes*

Presented by: Sebastian Murk
In collaboration with: Daniel R. Terno
Based on the Ref.[60].

In this work, the authors emphasize that a physical black hole is a celestial object which has a smooth apparent horizon and trapped surface.[61] Requesting the smoothness of these surfaces constrains the gravitational models which govern the dynamics of such solutions. To be more accurate, the existence of semiclassical physical black holes in modified theories of gravity induces some necessary conditions.

In order to find such necessary conditions, Sebastian Murk as a PhD student and his colleague have focused on the spherically symmetric black hole solutions which are presented as expansions in the coordinate distance from the apparent horizon and do not require a General Relativity solution as the zeroth-order perturbative solution of the modified theory. The only condition which they impose is the regularity of apparent horizon for black hole solutions. They pick finiteness of T^{μ}_{μ} and $T^{\mu\nu}T_{\mu\nu}$ as the regularity condition on the trapped horizon. By perturbing the Einstein equation with a new term $G_{\mu\nu} + \lambda\epsilon_{\mu\nu} = 8\pi T_{\mu\nu}$ and a generic ansatz for a spherically symmetric black hole metric, the necessary conditions on the component of the $\epsilon_{\mu\nu}$ are investigated. In the end, the results are studied more for a special modified gravity, the Starobinsky model.[62]

2.4. *Black holes, stationary clouds and magnetic fields*

Presented by: Nuno Santos
In collaboration with: Carlos A.R. Herdeiro
Based on the Ref.[63].

Stationary bosonic clouds are stable configuration of scalar fields around a black hole resulting in a bound state. Suggested by many examples, in order to have such a bound state, two conditions are necessary: (1) the possibility for superradiance, (2) a confinement mechanism. The former is necessary to synchronize the bosonic cloud with the black hole rotation, and the latter is needed to make the configuration stable. Interestingly, stationary clouds are characterized by a discrete number of nodes in the radial direction, n, the orbital angular momentum, ℓ , and the azimuthal harmonic index m. In this regard, they resemble orbital configurations in a Hydrogen atom.[11, 12, 64, 65]

In this presentation, the bosonic clouds are studied around a Reissner-Nordström black hole immersed in a magnetic field. These family of black hole solutions are called Reissner-Nordström-Melvin black holes.[66] They are solutions to Einstein-Maxwell theory, and their asymptotics resemble a magnetic Melvin universe. These black holes are stationary and axially symmetric. The presence of an external magnetic field provides both of the conditions for the stability of a bosonic cloud, i.e., the ergoregion and the confinement mechanism. The scalar field in this analysis has been considered to be complex, massless and minimally coupled to gravity. So, the theory is described by the action:

$$\mathcal{I} = \frac{1}{4\pi}\int d^4x\sqrt{-g}\big[\frac{R}{4} - \frac{F^2}{4} - (\nabla^\mu\Psi^*)(\nabla_\mu\Psi)\big]. \tag{2}$$

The bosonic clouds in this model, and on the Reissner-Nordström-Melvin solution are studied, and it is shown that for specific mass to charge ratios, there exist stable bosonic clouds.

2.5. *Bardeen black hole from a self-dual radius in spacetime*

Presented by: Michael Florian Wondrak
In collaboration with: Marcus Bleicher, Piero Nicolini, and Euro Spallucci
Based on the Ref.[67, 68].

String T-duality is an equivalence between two string theories on spacetimes with at least one compactified extra dimension – provided that the compactification radii are inversely related to each other, i.e. $R_1 = R$ and $R_2 = \frac{(R^\star)^2}{R}$. The special case of $R = R^\star$ is called self-dual radius. Beginning from T-duality in bosonic string theory, Michael Florian Wondrak first gave an overview on how quantum field propagators are deformed.[69] The scalar propagator obtained via Schwinger's proper time formalism is read to be

$$G(k) = -\frac{\ell_0 K_1(\ell_0\sqrt{k^2+m^2})}{\sqrt{k^2+m^2}}. \tag{3}$$

In this relation, $K_\nu(x)$ is the Modified Bessel function of second kind, and ℓ_0 is the zero point length.[70] Studying the potential induced by this propagator, a spherically symmetric black hole solution was derived,

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \qquad f(r) = 1 - \frac{2Mr^2}{(r^2+\ell_0^2)^{3/2}}. \tag{4}$$

This metric resembles the Bardeen black hole with a new interpretation of the UV cut-off in terms of the zero point length ℓ_0 instead of a magnetic monopole charge. Then, the thermodynamics of this black hole was studied and it was shown that at the end of evaporation process, the evaporation stops with a cold remnant instead of a final explosion. The remnants of these black holes were investigated regarding a possible fraction of dark matter. With a mass below $5\times10^{-8}\,M_\odot$, they comply with recent constraints on primordial black holes.[71] In the rest of the talk, observability of ℓ_0 was discussed: From the hydrogen energy spectrum, an upper bound of 4×10^{-19} meter was deduced.

2.6. *Asymptotically flat black hole solution in modified gravity*

Presented by: Surajit Kalita
In collaboration with: Banibrata Mukhopadhyay
Based on the Ref.[72].

$f(R)$ gravities are among alternative theories of gravity which have observational motivations to be studied. In these theories, one replaces the Ricci scalar R in the Lagrangian of General Relativity by a function of R, which is called $f(R)$.

$$\mathcal{I} = \frac{1}{16\pi}\int d^4x\sqrt{-g}f(R). \tag{5}$$

Although the equations of motion are higher-order in derivatives, they do not suffer from Ostrogradsky instability. In this presentation, Surajit Kalita presented the construction of spherically symmetric asymptotic flat black hole solutions to a subset of $f(R)$ theories. This subset is parametrized by a constant B in the relation $\frac{df}{dR} = 1 + \frac{B}{r}$. The metric ansatz is chosen to be $g_{\mu\nu} = \mathrm{diag}(g_{tt}, g_{rr}, r^2, r^2\sin^2\theta)$. By inserting this ansatz in the equations of motion of $f(R)$ gravities, the unknown components (g_{tt}, g_{rr}) are found as an expansion if powers of B and $\frac{1}{r}$. Then, some properties of these solutions are analyzed, including marginally stable and bound orbits, and spherical accretion flows. Moreover, from these solutions it is deduced that the Birkhoff theorem can be violated in $f(R)$ gravities.

2.7. *Infinitely degenerate exact Ricci-flat solutions in $f(R)$ gravity*

Presented by: Semin Xavier
In collaboration with: Jose Mathew and S. Shankaranarayanan
Based on the Ref.[73].

In this talk, Semin Xavier presented their results for an infinite number of solutions to a subset of $f(R)$ gravities. The theory is described by the action

$$\mathcal{I} = \frac{1}{16\pi}\int d^4x\sqrt{-g}f(R), \qquad f(R) = (\alpha_0 + \alpha_1 R)^p, \tag{6}$$

for a real number $p > 1$, and constants α_0 and α_1. The metric ansatz is chosen to be spherically symmetric:

$$ds^2 = -A(r)e^{\delta(r)}dt^2 + \frac{dr^2}{A(r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2). \tag{7}$$

Solving the equations of motion for the unknown functions $A(r)$ and $\delta(r)$ it is shown that there are an infinite number of possibility for these functions. As a result, it is discussed that in $f(r)$ gravities the Birkhoff theorem may be violated. In order to show this explicitly, two black hole solutions are constructed and studied.

2.8. *Does the Penrose suggestion as to black holes from a prior universe showing up in today's universe have credibility? Examined from a singular, and nonsingular beginning of cosmological expansion*

Presented by: Andrew Walcott Beckwith
Based on the Ref.[74].

Conformal Cyclic Cosmology (CCC) is a cosmological model in the framework of General Relativity, which is proposed by Roger Penrose.[75] In this model of cosmology, the universe iterates through infinite cycles, with the future timelike infinity of each previous iteration being identified with the past timelike infinity of the next universe. From the observational point of view, it has been suggested that the black holes in the previous universe can have implications in our universe, imprinted in the cosmic microwave background (CMB).[76,77]

In this talk, Andrew Walcott Beckwith discussed on the feasibility and credibility of the proposed methods for checking CCC using CMB data. He discussed the Penrose singularity theorem, and investigated the two cases of the CCC: cycling through singular or non-singular starting/ending points.

2.9. *Analytical computation of quasi-normal modes of slowly-rotating black-holes in dCS gravity*

Presented by: Manu Srivastava
In collaboration with: Yanbei Chen and S. Shankaranarayanan
Based on the Ref.[78].

Gravitational waves have provided interesting data about the inspiral and ring-down phases of black hole mergers. The waves in the ring-down phase are quasi-normal modes of the black hole merger. Therefore, by studying the data in the ring-down phase we can investigate the quasi-normal modes of the system.[36,37,40] However, quasi-normal mode frequencies depend on the gravitational theory. So, they provide a new tool to distinguish and examine alternative theories of gravity.

In this line of research, Manu Srivastava and his collaborators have focused on dynamical Chern-Simons gravity (dCS), which is described by the following action:[79]

$$\begin{aligned}\mathcal{I} =& \frac{1}{16\pi}\int d^4x\sqrt{-g}R - \frac{\beta}{2}\int d^4x\sqrt{-g}(\nabla_\mu\vartheta\nabla^\mu\vartheta + V(\vartheta))\\ &+ \frac{\alpha}{4}\int d^4x\sqrt{-g}\vartheta * RR + \mathcal{I}_{\text{matter}},\end{aligned} \tag{8}$$

in which ϑ is a pseudo-scalar field, and $*RR$ is the Pontryagin density

$$*RR \equiv \frac{1}{2}\epsilon^{cdef}R^a{}_{bef}R^b{}_{acd}. \tag{9}$$

where ϵ^{cdef} is the Levi-Civita tensor. The background is chosen to be a slowly rotating black hole introduced in Ref.[80]. The parameter of slow rotation is denoted by a. In the analysis, quasi-normal modes in the axial and polar sectors are studied up to linear order in a and quadratic order in α. The results of this study, along with the data from gravitational wave observations, can be used as a test for dCS gravity and to constrain coupling parameters.

2.10. *Scalar perturbations of Kerr black-holes in hybrid metric-Palatini gravity*

Presented by: João Luís Rosa
In collaboration with: José P. S. Lemos and Francisco S. N. Lobo
Based on the Ref.[81].

In General Relativity, the connection $\Gamma^\lambda_{\mu\nu}$ is assumed to be the metric connection, which is related to the metric by the relation

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2}g^{\lambda\sigma}\left(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu}\right). \tag{10}$$

However, in the Palatini formulation of gravity (see a review in Ref.[82]), the connection $\hat{\Gamma}^\lambda_{\mu\nu}$ is considered independent of the metric. If we denote the Ricci scalar which is built upon $\hat{\Gamma}$ by $\mathcal{R}$, then the generalized hybrid metric-Palatini (GHMP) gravity[83–85] (see a review in Ref.[86]) is described by the following action:

$$\mathcal{I} = \frac{1}{16\pi}\int d^4x\sqrt{-g}f(R,\mathcal{R}) + \mathcal{I}_{\text{matter}}. \tag{11}$$

By the equations of motion, it turns out that the $\mathcal{R}$ is the Ricci scalar calculated for a metric which is conformal to $g_{\mu\nu}$. Motivated by this, João Luís Rosa and his collaborators have shown that for any vacuum solution to GR, there is a non-trivial function $f(R,\mathcal{R})$ such that it is a solution to the GHMP too.

In this context, the Kerr black hole can also be considered as a solution to the GHMP. The stability of the Kerr constrains the $f(R,\mathcal{R})$, and this is analyzed in this presentation by studying massive scalar perturbations around the Kerr black hole. In particular, it is shown that the Kerr black hole is stable for some specific $f(R,\mathcal{R})$ theories and masses for the scalar perturbation.

2.11. *Emergent magnetic monopoles in degenerate theory*

Presented by: Suvikranth Gera
In collaboration with: Sandipan Sengupta
Based on the Ref.[87].

Magnetic monopoles are theoretical counterparts for electric charges in electric-magnetic duality.[88,89] Although they have been studied extensively, there is not yet observational evidence for their existence in nature.

In this talk, Suvikranth Gera presented a non-invertible metric which resembles a magnetic monopole in the first-order formulation of gravity.[90–92] Unlike the usual metric formulations, the non-invertible metric is well-defined in the tetrad formulations. In the presentation, he first presented the metric explicitly, and then followed by the calculation the spin-connection and its field strength in the first-order formulation. The magnetic charge is calculated, and the topological origin and its observability is discussed at the end, and it is mentioned that this solution has not any curvature singularity. Moreover, for the observers moving on timelike geodesics, this emergent magnetic charge is not accessible observationally, although it affects the curvature of the space-time.

2.12. *Black holes in metric-affine gravity: properties and observational discriminators*

Presented by: Diego Rubiera-Garcia
Based on the Ref.[93].

A generic connection $\Gamma^{\lambda}_{\mu\nu}$ can be independent of the metric (see the review in Ref.[82]). Such a generic connection is called affine connection. Accordingly, it can be regarded as an independent field in the Lagrangian. These models of gravity are called metric-affine gravities. In a subset of such models, which is called "Ricci based gravities," the connection $\Gamma^{\lambda}_{\mu\nu}$ appears in the Lagrangian only through a symmetric Ricci tensor $R_{\mu\nu}$,

$$\mathcal{I} = \frac{1}{16\pi}\int d^4x\sqrt{-g}\mathcal{L}_G[g_{\mu\nu}, R_{\mu\nu}(\Gamma)] + \mathcal{I}_{\rm matter}[g_{\mu\nu}, \psi_m]. \tag{12}$$

In this talk, Diego Rubiera-Garcia presented an overview on recent results for the black holes in these models of gravity. It is shown that for spherically symmetric black holes, the curvature singularity can be smoothed out, and the geodesic completeness is restored. Moreover, for such solutions there exist "double critical curves." This feature makes the shadow of the black hole to appear as two or more bright rings.

2.13. *Holography for rotating black holes in $f(T)$ gravity*

Presented by: Masoud Ghezelbash
In collaboration with: Canisius Bernard
Based on the Ref.[94].

General relativity is based on the Riemann curvature, while the torsion and non-metricity are considered to vanish. However, it is equivalent to make gravitational theories solely based on torsion, or non-metricity. These equivalent formulations in the literature are referred as Teleparallel and Symmetric Teleparallel (or coincident) gravities respectively. Similar to the Ricci scalar R which is made from the Riemann tensor, one can appropriately define the torsion scalar T, or the non-metricity scalar Q. According to the equivalence alluded to above, one can have analogous theories for $f(R)$, namely $f(T)$ or $f(Q)$.

It is useful to study gravitational features in GR, in the Teleparallel gravity or Symmetric Teleparallel gravity. In this talk, Masoud Ghezelbash has focused on the Kerr/CFT correspondence in $f(T)$ gravities. Kerr/CFT is a correspondence between the near horizon region of extremal black holes (originally Kerr black hole[95]) with a (chiral) two dimensional CFT. In this correspondence, the entropy of the black hole is calculated via the Cardy-formula for the entropy in a CFT:[96,97]

$$S = \frac{\pi^2}{3}(c_L T_L + c_R T_R), \tag{13}$$

in which the c's are the central charge of the left and right sectors, and the T's are Frolov-Thorne temperatures.[98] The result of such analyses shows that the entropy is proportional to the angular momentum associated with the axial symmetry which eventually enhances to the Virasoro sectors in the CFT (a review on Kerr/CFT and its extensions can be found in Ref.[99]). Besides, there are many works attempting to realize black hole microstates from the CFT using the Kerr/CFT correspondence (see e.g.[100–108]).

In this study, the model of gravity has been chosen to be

$$\mathcal{I} = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left(f(T) - 2\Lambda - F^2 \right), \tag{14}$$

in which Λ is the cosmological constant, and F^2-term is the Maxwell theory. A rotating charged black hole as a solution to this theory (introduced in Ref.[109]) is chosen to be studied, and the wave equation for a massless scalar field is approximated in the near horizon geometry of this black hole. Instead of a chiral Virasoro, a full Virasoro algebra is reported in this near horizon, and the Cardy formula (13) is used to reproduce the black hole entropy.

2.14. *Universe in a black hole with spin and torsion*

Presented by: Nikodem Poplawski
Based on the Ref.[110].

The Einstein-Cartan theory (EC) of gravity[111–115] is the simplest theory with torsion and curvature. In this theory, the Lagrangian is the same as the Einstein gravity

$$\mathcal{I} = \frac{1}{2\kappa}\int d^4x\sqrt{-g}(R + \mathcal{L}_{\text{matter}}), \tag{15}$$

but in addition to the metric, the torsion is considered as a dynamical field. In this talk, Nikodem Poplawski studies the collapse of spherically symmetric fermionic matter to form a black hole in EC. He explains that if a fermionic field is considered as a perfect fluid with energy density ϵ and pressure p as the matter in this theory, the effect of the dynamics of torsion can be absorbed in the energy-momentum tensor[116] by

$$T_{\mu\nu} = \epsilon u_\mu u_\nu - p(g_{\mu\nu} - u_\mu u_\nu) \quad \rightarrow \quad \tilde{T}_{\mu\nu} = \tilde{\epsilon} u_\mu u_\nu - \tilde{p}(g_{\mu\nu} - u_\mu u_\nu), \tag{16}$$

in which

$$\tilde{\epsilon} = \epsilon - \alpha n_f^2, \qquad \tilde{p} = p - \alpha n_f^2. \tag{17}$$

In this relation, u_μ is the four-velocity, n_f is the number density of fermions, and $\alpha = \frac{\kappa}{32}$. Using $\tilde{T}_{\mu\nu}$, the equation of motion can be written as the usual Einstein equation:

$$G_{\mu\nu} = \kappa\tilde{T}_{\mu\nu}. \tag{18}$$

In this setup, the collapse is studied by the Tolman ansatz[117] for the metric:

$$ds^2 = e^{\nu(\tau,R)}d\tau^2 - e^{\lambda(\tau,R)}dR^2 - e^{\mu(\tau,R)}(d\theta^2 + \sin^2\theta d\varphi^2), \tag{19}$$

and solved for the unknown functions (ν, μ, λ). The results show that in this collapse, the singularity is prevented by the effects of the torsion. The geometry of the universe on the other side of the horizon is calculated, and it is shown that the geometry is an oscillating FLRW metric and it describes a closed universe. It is also calculated that due to the pair particle production, the frequency of the oscillation is reduced in time.

2.15. *Absorption by deformed black holes*

Presented by: Renan B. Magalhães
In collaboration with: Luiz C.S. Leite and Luís C.B. Crispino
Based on the Ref.[118].

The usual way to study alternative theories of gravity is to change the Lagrangian or the dynamical fields. Then, the equations of motion follow, and one can study the solutions to the new equations. However, in Ref.[119], Johannsen and

Psaltis introduced a parametric deviation approach, which is the reverse of the procedure mentioned above. They deform black holes by some parameters such that the black holes remain smooth, free of pathology, and have some suitable properties. Then, the equations of motion which could have such solutions are investigated. Following this method, Konoplya and Zhidenko in Ref.[120] have deformed the Kerr black hole by some parameters, while keeping its suitable features intact.

In this talk, Renan B. Magalhães presented the analysis of the absorption cross section of a massless scalar field for the static Konoplya-Zhidenko black hole. The metric of this black hole is similar to the Schwarzschild black hole with a deformed mass:

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2 d\Omega^2, \qquad f = 1 - \frac{2M}{r}, \qquad M \to M + \frac{1}{2}\sum_i \frac{\eta_i}{r^i}, \tag{20}$$

with the special choice of $\eta_i = \delta_{i2}$. So, the f is chosen to be $f = 1 - \frac{2M}{r} + \frac{\eta}{r^3}$ for some parameter η. In order to have a horizon, $\eta_{\rm min} < \eta < 0$ in which $\eta_{\rm min} = \frac{-32}{27M^3}$. On this specific background, Renan B. Magalhães and his collaborators have studied the wave equation for a scalar field $\Box\psi = 0$. He presented the result of the radial potential for different spherical harmonic modes, illustrating them from numerical calculations. Moreover, the absorption cross sections are derived numerically.

2.16. *Shadow of a charged black hole surrounded by an anisotropic matter field*

Presented by: Javier Badía
In collaboration with: Ernesto F. Eiroa
Based on the Ref.[121].

Observation of the shadow of a supermassive black hole at the center of galaxy M87 has been one of the main progresses in black hole physics in recent years.[122] In parallel with this observation, theoretical studies of black hole shadows have been one of the active lines of research (see Ref.[123] for a review). Especially, the presence of matter fields surrounding black holes, and its effect on their shadows has been investigated (see e.g.[124–127]).

In this talk, Javier Badía presents the results of the calculation of shadows for rotating charged black holes surrounded by an anisotropic matter field.[128, 129] In the spherical coordinates (t, r, θ, φ) the anisotropic matter is considered to be a perfect fluid which is described by the energy-momentum tensor

$$T_\mu^{\ \nu} = \mathrm{diag}(-\rho, p_1, p_2, p_2), \qquad p_1 = -\rho, \quad p_2 = \mathrm{w}\rho. \tag{21}$$

The rotating charged black hole surrounded by this matter is derived by the Newman-Janis algorithm to the following spherically symmetric spacetime:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \qquad f(r) = 1 - \frac{2m(r)}{r}, \qquad m(r) = M - \frac{Q^2}{2r} + \frac{K}{2r^{2\mathrm{w}-1}}. \tag{22}$$

The M, Q, and K are integration constants, and w is restricted to be $\frac{1}{2} < \mathrm{w} < 1$ by physical energy conditions. The resulting metric by the Newman-Janis algorithm is the Kerr metric in which $M \to m(r)$, i.e.

$$\begin{aligned} ds^2 =& -(1-f)dt^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 - 2fa\sin^2\theta\, dtd\varphi \\ &+ \left(r^2 + a^2 + fa^2\sin^2\theta\right)\sin^2\theta\, d\varphi^2, \\ &\rho^2 = r^2 + a^2\cos^2\theta\,, \qquad \Delta = r^2 - 2m(r)r + a^2\,, \qquad f = \frac{2m(r)r}{\rho^2}, \end{aligned} \tag{23}$$

and $m(r) = M - \frac{Q^2}{2r} + \frac{K}{2r^{2\mathrm{w}-1}}$.

On this background, the Hamilton-Jacobi equation for the null geodesics is separable, and reduces to ordinary differential equations with radial derivatives. The result of the shadow calculations for an asymptotic observer on the equator is presented by illustrations. Moreover, three observables which characterize the shadows, named as the area, oblateness, and centroid of the shadows are discussed.[121,130]

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