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Inflation with Einstein-Gauss-Bonnet, non-minimal, and non-minimal derivative couplings: the constant-roll condition

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Abstract. In our research, we consider and study the inflation theories of Einstein-Gauss-Bonnet, minimal, non-minimal, and non-minimal derivative couplings with the constant-roll condition. Given the gravitational wave event, GW170817, which produces the speed of gravitational waves that was almost equal to the speed of light in vacuum, we constrain that the speed of the tensor perturbation was nearly equal to unity, $c_T^2 \simeq 1$. We involve a scalar field potential whose function can be obtained from an equation of motion by choosing the Gauss-Bonnet coupling functions. We use the linear and quadratic coupling functions, which are their simplest functions. The one result obtained are able to produce observed quantities such as the spectral index for scalar perturbation, $n_s = 0.9642$, the spectral index for tensor perturbation, $n_T = -5.2471 \times 10^{-4}$, and tensor-scalar ratio, $r = 0.0041$, which are compatible with the newest Planck data using the slow-roll parameters obtained analytically.

1. Introduction

The assumption of the inflation theory is dominated by vacuum energy, so it is liable for the exponential expansion of the early universe [1]. A phase transition occurs where the vacuum energy density turns into matter and radiation *(reheating)* which ended its exponential expansion and then the Friedmann equation began its evolution [2]. Inflation also predicts the production of gravitational waves that theoretically resulted from the tensor perturbation of the metric [3]. The one is the gravitational waves generated by the merging of two neutron stars in the GW170817 event which after its merger occurred a kilonova. It gives the fact that the gravitational waves nearly arrive at the same time as the electromagnetic radiation emitted by a kilonova, thus producing the speed of gravitational waves was almost equal to the speed of light in vacuum. We constrain that the speed of the tensor perturbation is almost equal to unity, $c_T^2 \simeq 1$. This fact causes some generalizations of Einstein's theory of relativity must be modified because the gravity theory predicts difference between the speed of gravitational waves and the speed of light, which refers to the speed of the tensor perturbation [4].

The formulation of the speed of the tensor perturbation can be obtained by reviewing the effective field theory (EFT) of cosmological perturbations that has been studied in relation with inflation characterize the low-energy degree of freedom of a most general gravitational theory [5,6]. This approach makes it possible to cope with all possible high energy corrections to standard slow-roll inflation driven by a single scalar field [7]. In particular, the most general single field modified gravity scenario in terms of a Lagrangian depends on the lapse function and some geometrical scalar quantities naturally appearing in the Arnowitt-Deser-Misner (ADM) formalism on flat Friedmann-Lemaître-Robertson-Walker (FLRW) cosmological background [8]. Horndeski theory [9] has received much concern [10–12] as the most general scalar-tensor theory with second-order differential equations of motion. It is caused by the covariant Galileons generalization [13–15] allowing for the realizations of cosmic acceleration [16]. The explanation of Ref. [8] shows that the Horndeski theory is supported in the EFT framework of inflation as a special condition. In fact, the Horndeski theory fulfills conditions for the absence of spatial derivatives higher than second order in the equations of linear cosmological perturbations. Gleyzes et al. [8] gave explanations connecting the variables between the Horndeski theory and the EFT of inflation.

The theory of Einstein-Gauss-Bonnet is one of theories that describe the inflation era and provides a solution of GW170817 since it is based on the string theory by reviewing the canonical scalar fields with minimal coupling [17]. It has been calculated [18] that assuming the constant-roll of a scalar field from the theory of Einstein-Gauss-Bonnet yields observational quantities that are compatible with the newest Planck data [19]. Another method that has been calculated is to add the non-minimal [20] and non-minimal derivative coupling terms separately [21], giving results that are also compatible with the latest Planck data. Therefore, with high motivation, we combine all the previously worked correction terms to show the tensor and perturbation and the scalar perturbation will produce the solutions of the gravitational wave with the constant-roll condition and the spectral index to compare with the observational data based on the GW170817 event.

2. Theoretical framework of minimal and non-minimal derivative couplings with string correction from the Horndeski theory

By using the EFT framework, we consider the most general scalar-tensor theory only up to second-order differential equations of motion is named the Horndeski theory [9]. This theory is described by the action $S = \int d^4x\sqrt{-g}L$ with the Lagrangian [22],

$$L = \sum_{i=2}^{5} L_i, \tag{1}$$

where,

$$\begin{aligned}
L_2 &= G_2(\phi, X), && (2)\\
L_3 &= G_3(\phi, X)\Box\phi, && (3)\\
L_4 &= G_4(\phi, X)R - 2G_{4X}(\phi, X)\left[(\Box\phi)^2 - \phi^{;\mu\nu}\phi_{;\mu\nu}\right], && (4)\\
L_5 &= G_5(\phi, X)G_{\mu\nu}\phi^{;\mu\nu} \\
&\quad + \frac{1}{3}G_{5X}(\phi, X)\left[(\Box\phi)^3 - 3(\Box\phi)\phi_{;\mu\nu}\phi^{;\mu\nu} + 2\phi_{;\mu\nu}\phi^{;\mu\sigma}\phi^{;\nu}_{;\sigma}\right]. && (5)
\end{aligned}$$

Here $G_i(i = 2, 3, 4, 5)$ are functions in terms of a single scalar field ϕ and its kinetic energy $X \equiv \partial^\mu\phi\partial_\mu\phi = (\nabla\phi)^2$ with the partial derivative $G_{iX} \equiv \frac{\partial G_i}{\partial X}$ and $G_{i\phi} \equiv \frac{\partial G_i}{\partial\phi}$, $\Box \equiv \partial^\mu\partial_\mu = \frac{\partial^2}{\partial t^2} - \nabla^2$ is d'Alembert operator, R is the scalar curvature, and $G_{\mu\nu}$ is the Einstein tensor. Based on our model, the corresponding G_i functions are,

$$G_2 = -\frac{X}{2} - V(\phi) + \frac{1}{4} f^{(4)} X^2 \left[3 - \ln\left(-\frac{X}{2}\right)\right], \tag{6}$$

$$G_3 = -\frac{1}{4} f^{(3)} X \left[7 - 3\ln\left(-\frac{X}{2}\right)\right], \tag{7}$$

$$G_4 = \frac{1}{2}\left(1 - \zeta\phi^2\right) - \frac{1}{4} f^{(2)} X \left[2 - \ln\left(-\frac{X}{2}\right)\right], \tag{8}$$

$$G_5 = \xi\phi - \frac{1}{2} f^{(1)} \ln\left(-\frac{X}{2}\right). \tag{9}$$

where $f^{(n)} = \partial^n f(\phi)/\partial\phi^n$. Substitute Eqs. (6)–(9) to the Horndeski Lagrangian in Eqs. (2)–(5) are obtained the action [23],

$$S = \int d^4x\sqrt{-g}\left\{\frac{R}{2} - \frac{1}{2}\left(\nabla\phi\right)^2 + \frac{1}{8} f(\phi) R_{GB}^2 - \frac{1}{2}\zeta\phi^2 R + \xi\phi G_{\mu\nu}\phi^{;\mu\nu} - V(\phi)\right\}, \tag{10}$$

where g is the metric determinant, $f(\phi)$ is the Gauss-Bonnet coupling function, while R_{GB}^2 expressed the Gauss-Bonnet invariant, $R_{GB}^2 = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2$, with $R_{\mu\nu}$ and $R_{\mu\nu\alpha\beta}$ are the Ricci tensor and the Riemann tensor respectively, ζ and ξ are non-minimal and non-minimal derivative coupling constants respectively, and $V(\phi)$ is the scalar potential function. Furthermore, we shall assume that the cosmological geometric background is flat Friedmann-Lemaître-Robertson-Walker (FLRW), and consequently the line element reads,

$$ds^2 = -dt^2 + a^2(t)\sum_{i=1}^{3}\left(dx^i\right)^2, \tag{11}$$

where $a(t)$ is the cosmic scale factor. Therefore, the Ricci scalar and the Gauss–Bonnet invariant are $R = 6(2H^2 + \dot{H})$ and $R_{GB}^2 = 24H^2(H^2 + \dot{H})$, where $H \equiv \frac{\dot{a}}{a}$ is Hubble's parameter and the "dot" signifies differentiation with respect to cosmic time t.

By varying the action in Eq. (10) with respect to the metric tensor $g^{\mu\nu}$ and the scalar field ϕ are obtained the equations of motion as follows,

$$3H^2 = \frac{1}{2}\dot{\phi}^2 + V + 3\zeta\left(H^2\phi^2 + 2H\phi\dot{\phi}\right) - 9\xi H^2\dot{\phi}^2 - 3H^3\dot{f}, \tag{12}$$

$$\begin{aligned} -2\dot{H} &= \dot{\phi}^2 + 2\zeta\left(H\phi\dot{\phi} - \dot{H}\phi^2 - \dot{\phi}^2 - \phi\ddot{\phi}\right) \\ &\quad -2\xi\left(3H^2\dot{\phi}^2 - \dot{H}\dot{\phi}^2 - 2H\dot{\phi}\ddot{\phi}\right) + 2H\dot{H}\dot{f} + H^2\left(\ddot{f} - H\dot{f}\right), \end{aligned} \tag{13}$$

$$\begin{aligned} &\left(1 - 6\xi H^2\right)\left(\ddot{\phi} + 3H\dot{\phi}\right) + 6\zeta\left(2H^2 + \dot{H}\right)\phi \\ &-12\xi H\dot{H}\dot{\phi} - 3f'H^2\left(H^2 + \dot{H}\right) + V' = 0, \end{aligned} \tag{14}$$

where "prime" signifies differentiation with respect to the scalar field ϕ. Fortunately, in this case we consider the universe during inflation, then we shall implement the inflation and the constant-roll condition as follows,

$$H^2 \gg \dot{H}, \qquad V \gg \frac{1}{2}\dot{\phi}^2, \qquad \ddot{\phi} = \beta H\dot{\phi}, \tag{15}$$

where β is the constant-roll parameter. Furthermore, we shall assume that the string terms are also negligible from the equations of motion, leaving only the canonical scalar field terms. As an example, we show the approximation for Eq. (12) which are,

$$V \gg -3H^3\dot{f}, \qquad V \gg -9\xi H^2\dot{\phi}^2. \tag{16}$$

In principle, we assume that $f \ll 1$ and $\xi \ll 1$ with all the possible combinations appearing Eqs. (12)–(14), while ζ is still used in the background because it is directly coupled to the Ricci scalar [20]. For the above assumptions, the equations of motion in Eqs. (12)–(14) become very simple namely,

$$\left(1-\zeta\phi^2\right)H^2 \simeq \frac{1}{3}V + 2\zeta H\phi\dot{\phi}, \tag{17}$$

$$-2\left(1-\zeta\phi^2\right)\dot{H} \simeq (1-2\zeta)\,\dot{\phi}^2 + 2\zeta\,(1-\beta)\,H\phi\dot{\phi}, \tag{18}$$

$$(\beta+3)\,H\dot{\phi} + 12\zeta H^2\phi + V' \simeq 0. \tag{19}$$

However, before we go any further calculation, we shall apply certain additional constraints to get compatibility with the newest observations.

Since string corrections of the Gauss-Bonnet term are applied, the speed of gravitational waves which is propagated through spacetime does not necessarily coincide with the speed of light in vacuum. Particularly, the tensor propagation speed square is given by [22],

$$c_T^2 \equiv \frac{\mathcal{E}}{L_{\mathcal{S}}}, \tag{20}$$

in natural units, and $\mathcal{E}$ and $L_{\mathcal{S}}$ are defined as,

$$\mathcal{E} = G_4 + \frac{1}{2}XG_{5\phi} - XG_{5X}\ddot{\phi}, \tag{21}$$

$$L_{\mathcal{S}} = G_4 - 2XG_{4X} + H(-X)^{3/2}G_{5X} - \frac{1}{2}XG_{5\phi}. \tag{22}$$

Then, use the functions of G_4 and G_5 in Eqs. (8) and (9) to calculate Eqs. (21) and (22), we get,

$$\mathcal{E} = \frac{1}{2}(1-\zeta\phi^2 - \xi\dot{\phi}^2 + \ddot{f}), \tag{23}$$

$$L_{\mathcal{S}} = \frac{1}{2}(1-\zeta\phi^2 + \xi\dot{\phi}^2 + H\dot{f}). \tag{24}$$

As a result, compatibility with the GW170817 event was recoverable by reviewing that the speed of the primordial tensor perturbation was nearly equal to unity, $c_T^2 \simeq 1$. By using Eqs. (23) and (24), so that Eq. (20) becomes,

$$\dot{\phi} = \frac{(1-\beta)\,Hf'}{f''-2\xi}, \tag{25}$$

where the differential symbol $\frac{d}{dt}$ is equivalent to $\dot{\phi}\frac{d}{d\phi}$. By applying the limit $\beta = 0$, Eq. (25) in equivalent to the slow-roll case. Thus, the equations of motion in Eqs. (17)–(19) are rewritten in this case becomes,

$$H^2 \simeq \frac{1}{3}V\left[\frac{f''-2\xi}{(1-\zeta\phi^2)\,(f''-2\xi) - 2\,(1-\beta)\,\zeta\phi f'}\right], \tag{26}$$

$$-2\left(1-\zeta\phi^2\right)\dot{H} \quad \simeq \quad \frac{(1-\beta)^2 H^2 f'}{f''-2\xi}\left[2\zeta\phi+\frac{(1-2\zeta)\,f'}{f''-2\xi}\right], \tag{27}$$

$$V'+\left[\frac{(1+\beta/3)\,(1-\beta)\,f'+4\zeta\phi\,(f''-2\xi)}{-2\,(1-\beta)\,\zeta\phi f'+\left(1-\zeta\phi^2\right)(f''-2\xi)}\right]V\simeq 0. \tag{28}$$

The three above equations are much more simple to calculate analytically. In Eq. (27) produces the slow-roll parameter expression, $\varepsilon \equiv -\dot{H}/H^2$, which is useful because it is directly related to the constant-roll parameter β and the Gauss-Bonnet coupling function $f(\phi)$. While for Eq. (28), the inflaton potential function can be known explicitly if the coupling function $f(\phi)$ is specifically set and the corresponding parameters are appropriately given.

Consider the six slow-roll parameters on cosmic inflation which can be expressed as,

$$\varepsilon \equiv -\frac{\dot{H}}{H^2}, \quad \beta \equiv \frac{\ddot{\phi}}{H\dot{\phi}}, \quad \delta_{Q_t} \equiv \frac{\dot{Q}_t}{HQ_t}, \quad \delta_{c_T} \equiv \frac{\dot{c}_T}{Hc_T}, \quad \delta_{Q_s} \equiv \frac{\dot{Q}_s}{HQ_s}, \quad \delta_{c_s} \equiv \frac{\dot{c}_s}{Hc_s}, \tag{29}$$

where,

$$Q_t \equiv \frac{L_{\mathcal{S}}}{2}, \qquad Q_s \equiv \frac{2L_{\mathcal{S}}}{3\mathcal{W}^2}\left(9\mathcal{W}^2+8L_{\mathcal{S}}w\right), \tag{30}$$

and,

$$c_s^2 \equiv \frac{2}{Q_s}\left(\dot{\mathcal{M}}+H\mathcal{M}-\mathcal{E}\right), \tag{31}$$

which is the speed of the primordial scalar perturbation. The quantities of $\mathcal{W}$, w, and $\mathcal{M}$ are defined as,

$$\begin{aligned}
\mathcal{W} &= 4HG_4+2\dot{\phi}XG_{3X}-16H(XG_{4X}+X^2G_{4XX})+2\dot{\phi}(G_{4\phi}+2XG_{4\phi X}) \\
&\quad -2H^2\dot{\phi}(5XG_{5X}+2X^2G_{5XX})-2HX(3G_{5\phi}+2XG_{5\phi X}), && (32)\\
w &= -18H^2G_4+3\left(XG_{2X}+2X^2G_{2XX}\right)-18H\dot{\phi}\left(2XG_{3X}+X^2G_{3XX}\right) \\
&\quad -3X\left(G_{3\phi}+XG_{3\phi X}\right)+18H^2\left(7XG_{4X}+16X^2G_{4XX}+4X^3G_{4XXX}\right) \\
&\quad -18H\dot{\phi}\left(G_{4\phi}+5XG_{4\phi X}+2X^2G_{4\phi XX}\right)+6H^3\dot{\phi}\left(15XG_{5X}+13X^2G_{5XX}\right. \\
&\quad \left.+2X^3G_{5XXX}\right)+9H^2X\left(6G_{5\phi}+9XG_{5\phi X}+2X^2G_{5\phi XX}\right), && (33)\\
\mathcal{M} &= \frac{4L_{\mathcal{S}}^2}{\mathcal{W}}. && (34)
\end{aligned}$$

Then, use the functions of G_i in Eqs. (6)–(9) to calculate Eqs. (32)–(34), we get,

$$\mathcal{W} = 2H-2\zeta\left(H\phi^2+\phi\dot{\phi}\right)+6\xi H\dot{\phi}^2+3H^2\dot{f}, \tag{35}$$

$$w = 3\left[-3H^2+\frac{1}{2}\dot{\phi}^2+3\zeta\left(H^2\phi^2+2H\phi\dot{\phi}\right)-18\xi H^2\dot{\phi}^2-6H^3\dot{f}\right], \tag{36}$$

$$\mathcal{M} = \frac{\left(1-\zeta\phi^2+\xi\dot{\phi}^2+H\dot{f}\right)^2}{2H-2\zeta\left(H\phi^2+\phi\dot{\phi}\right)+6\xi H\dot{\phi}^2+3H^2\dot{f}}. \tag{37}$$

Hence, according to Eqs. (24), (27) and (30), the first three slow-roll parameters, ε, β, and δ_{Q_t} can be analytically rewritten as,

$$\varepsilon = \frac{(1-\beta)^2}{2}\left[\left(\frac{2\zeta\phi}{1-\zeta\phi^2}\right)\left(\frac{f'}{f''-2\xi}\right)+\left(\frac{1-2\zeta}{1-\zeta\phi^2}\right)\left(\frac{f'}{f''-2\xi}\right)^2\right], \tag{38}$$

$$\delta_{Q_t} = \frac{2\beta\left(1-\beta\right)^2 \xi H^2 f'^2 + \left(1-\beta\right)\left(f''-2\xi\right)\left[-2\zeta\phi + \left(\beta-\varepsilon\right)H^2 f' + H f''\right] f'}{\left(1-\zeta\phi^2\right)\left(f''-2\xi\right)^2 + \left(1-\beta\right)^2 \xi H^2 f'^2 + \left(1-\beta\right)\left(f''-2\xi\right)H^2 f'^2}, \tag{39}$$

and β will be chosen in the section of the coupling function models. For the last three slow-roll parameters, δ_{c_T}, δ_{Q_s}, and δ_{c_s}, we do not calculate in detail, because the equations are too long, then its alternative solutions can be calculated using computations. In Eq. (38) can produce eternal inflation when $\varepsilon = 0$, so $\beta = 1$ does not fulfill these conditions. Lastly, we discuss the observational quantities in the case of models we will choose. The scalar spectral index n_s, the tensor spectral index n_T, and the tensor-to-scalar ratio r in terms of the slow-roll parameters are defined as,

$$n_s = 1 - 2\varepsilon - \delta_{Q_s} - 3\delta_{c_s}, \tag{40}$$
$$n_T = -2\varepsilon - \delta_{Q_t} - 3\delta_{c_T}, \tag{41}$$
$$r = 4\left(\frac{Q_s}{Q_t}\right)\left(\frac{c_s}{c_T}\right)^3. \tag{42}$$

We can do this calculation by first evaluating the final value of the scalar field ϕ_f. This value can be calculated by equating the slow-roll parameter ε in Eq. (38) equal to one. Consequently, the initial value of the scalar field ϕ_i can be calculated from the e-foldings number, expressed as $N = \int_{t_i}^{t_f} H dt = \int_{\phi_i}^{\phi_f} \frac{H}{\dot\phi} d\phi$, where the difference $t_f - t_i$ denotes the duration of inflation. Recalling expression of $\dot\phi$ in Eq. (25), we will find that ϕ_i can be derived from,

$$N = \frac{1}{1-\beta}\int_{\phi_i}^{\phi_f} \frac{f''-2\xi}{f'} d\phi. \tag{43}$$

Therefore, by using expression of ϕ_i as an calculation input in Eqs. (40)–(42), we will calculate and check if there exist values for the free parameters which gives compatible results with the newest Planck data which specifically constrains n_s and r as follows,

$$n_s = 0.9649 \pm 0.0042, \qquad r < 0.064, \tag{44}$$

with 68% C.L and 95% C.L respectively. Referring to the tensor spectral index n_T, until now no specific value is known because the B-mode has not been observed [19]. It can be seen that from Eqs. (43) and (38), it is obvious that choosing the right coupling function, is the key to simplify the results. In the next section, we will consider certain functional forms of this coupling function and derive the scalar potential function from Eq. (28), then returns the results for the observational quantities introduced earlier.

3. Specific models of Gauss-Bonnet coupling function and its compatibility with observational data

In this section, we will analyze two models that can provide the most feasible phenomenology, by appropriately choosing and the simple scalar coupling function $f(\phi)$. Therefore, we will define specifically the Gauss-Bonnet coupling scalar function $f(\phi)$ i.e. the linear and quadratic coupling functions. After that, we will find the potential function $V(\phi)$ obtained from Eq. (28) based on the selection of the coupling scalar function. Next, we will calculate the slow-roll parameters ε in Eq. (38) equal to unity to discover the final value of the scalar field ϕ_f, and from the expression of the e-foldings number from Eq. (43), the initial value of the scalar field ϕ_i can be determined as an input, and fundamentally, we must choose best values for these parameters. Accordingly, by comparing the numerical value of the observational indices obtained by the model, namely the scalar spectral index n_s, the tensor spectral index n_T, and the tensor-to-scalar ratio r, which are coming from the newest Planck 2018 collaboration, the validity of the models can be confirmed.

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3.1. A Linear Coupling Function

We choose a linear coupling function which is expressed as,

$$f(\phi) = \lambda\phi, \tag{45}$$

where λ is a dimensionless constant. Whereas, the first and second derivatives of $f(\phi)$ to the scalar field respectively are,

$$f'(\phi) = \lambda, \qquad f''(\phi) = 0. \tag{46}$$

Then, plug the above conditions into Eq. (28) to discover the explicit function of the scalar potential function $V(\phi)$, and we briefly get,

$$V(\phi) = V_0 e^{(\tau_1+\tau_2)}, \tag{47}$$

where V_0 is an initial potential, as well as τ_1 and τ_2 are the function of the scalar field which is expressed as,

$$\tau_1 = 2\ln\left[\frac{|\left(1-\zeta\phi^2\right)\xi + (1-\beta)\lambda\zeta\phi|}{|\xi|}\right], \tag{48}$$

and,

$$\begin{aligned}\tau_2 \;=\; & -\frac{(1-\beta)(4\zeta-\beta/3-1)\lambda}{\sqrt{-4\zeta\xi^2-(1-\beta)^2\zeta^2\lambda^2}}\left[\arctan\left(\frac{-2\zeta\xi\phi+(1-\beta)\zeta\lambda}{\sqrt{-4\zeta\xi^2-(1-\beta)^2\zeta^2\lambda^2}}\right)\right.\\ & \left.-\arctan\left(\frac{(1-\beta)\zeta\lambda}{\sqrt{-4\zeta\xi^2-(1-\beta)^2\zeta^2\lambda^2}}\right)\right].\end{aligned} \tag{49}$$

By applying the conditions (45) and (46) to the slow-roll parameters in Eqs. (38) and (39), we have,

$$\varepsilon = (1-\beta)^2\left[\frac{-4\zeta\xi\lambda\phi+(1-2\zeta)\lambda^2}{8\left(1-\zeta\phi^2\right)\xi^2}\right], \tag{50}$$

and,

$$\delta_{Q_t} = \frac{4(1-\beta)\zeta\lambda\phi + 2(1-\beta)\left(\varepsilon-\beta^2\right)\lambda^2H^2}{4\left(1-\zeta\phi^2\right)\xi - \left(1-\beta^2\right)\lambda^2H^2}, \tag{51}$$

where the expression of H^2 in Eq. (26) becomes,

$$H^2 \simeq \frac{1}{3}V\left[\frac{\xi}{\left(1-\zeta\phi^2\right)\xi+(1-\beta)\zeta\lambda\phi}\right]. \tag{52}$$

You can see only three the slow-roll parameters, namely ε, β, and δ_{Q_t} which have the simplest form compared to δ_{c_T}, δ_{Q_s}, and δ_{c_s} which will be calculated numerically. Since expression of ε is the simplest slow-roll expression, the initial and final value of the scalar field are easily obtained as follows,

$$\phi_i = \phi_f + \frac{(1-\beta)\lambda N}{2\xi}, \tag{53}$$

$$\phi_f = \frac{(1-\beta)^2\zeta\lambda \pm \left[(1-\beta)^4\zeta^2\lambda^2 - 2(1-\beta)^2(1-2\zeta)\zeta\lambda^2 + 16\zeta\xi^2\right]^{\frac{1}{2}}}{4\zeta\xi}. \tag{54}$$

The resulting theory from the above equations can be compatible with the observational data for several free parameters. For instance by selecting $(\lambda, V_0, N, \beta, \zeta, \xi) = \left(-3\times10^{-7}, 10^3, 60, -0.0345, 0.279, 10^{-5}\right)$ in Planck units, subsequently the observational quantities reads,

$$n_s = 0.964154, \quad n_T = -5.247070\times10^{-4}, \quad r = 0.004100, \tag{55}$$

which are compatible with the newest Planck data [19]. Furthermore, the scalar field increases with time i.e $\phi_i = 0.954096$ whereas $\phi_f = 1.885146$, the sound wave velocity is equal to unity as expected and moreover, the slow-roll parameters are valid since these numerical values are of order $\mathcal{O}\left(10^{-2}\right)$ and lesser. In particular, $\varepsilon = 0.005799$, $\beta = -0.0345$, $\delta_{Q_t} = -0.011074$, $\delta_{c_T} = 4.4\times10^{-10}$, $\delta_{Q_s} = -0.041322$, and $\delta_{c_s} = 0.021856$. In this model, the scalar potential function has about $V = 5.454349\times10^2$ and the square of the Hubble parameter about $H^2 = 2.464357\times10^2$. The best results are shown in $|\lambda| \sim 10^{-7}$ since it has a wide interval,

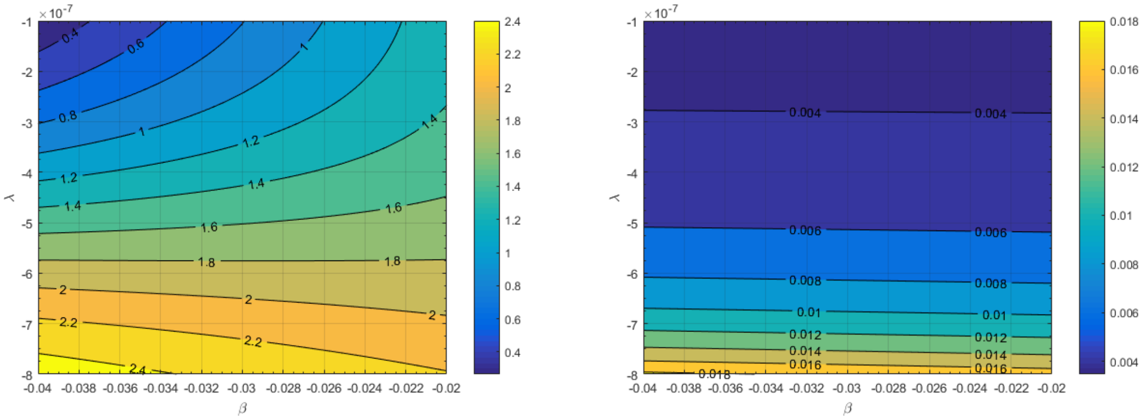

Figure 1. A linear coupling function model: the contour plot of the scalar spectral index n_s (left) and tensor-to-scalar ratio r (right) depending on parameters β and λ on intervals $[-0.04, -0.02]$ and $\left[-8\times10^{-7}, -1\times10^{-7}\right]$ respectively. It can be seen that the value of the scalar spectral index of are very much determined by the constant-roll parameter β and has a narrow area to be compatible with observational data.

hence has a significant difference between ϕ_i and ϕ_f which indicates the scalar field growth during inflation, as well as V and H^2 are almost close to V_0. The largest order of λ in this model is about $|-1.2\times10^{-6}|$ and if it is more than that, the other quantities are imaginary. The smaller the order of λ causes ϕ_i and ϕ_f to be almost the same value, the orders of V and H^2 extremely drops, and β the nearly same to produce n_s that fits the data. Now, we consider variations of $\zeta = (10, 50, 100)$ at $|\lambda| \sim 10^{-8}$ resulting ϕ_i and ϕ_f are almost the same, V, H^2, and ε decreases, as well as $\beta \to 1$ and $\delta_{Q_t} > 1$ so that it does not match the expected values. Therefore, the order of $\zeta \sim 10^{-1}$ works well in this model. On the other hand, if we consider $\xi > 10^{-5}$, the values of ϕ_i and ϕ_f are almost the same, V and H^2 decreases, as well as ε and β become relatively constant. Therefore, the order of ξ works well in this model. This proves that the assumption of the coupling ξ and $f \sim \lambda$ works on the relatively small order of the proposed models. The most important thing from the results (55) obtained analytically is how the values changes when varied against independent parameters, such as β which affects the scalar spectral index value, and λ which affects the Gauss-Bonnet coupling function value. Shown at **Figure 1** above, the contour plot of the scalar spectral index perturbation n_s and the scalar-to-tensor ratio r at corresponding intervals.

Finally, we validate whether the approximations assumed in Eqs. (15) and (16) holds true. Firstly, by choosing $(\lambda, V_0, N, \beta, \zeta, \xi) = \left(-3\times 10^{-7}, 10^3, 60, -0.0345, 0.279, 10^{-5}\right)$, we have $\dot{H} \sim \mathcal{O}(1)$ compared to $H^2 \sim \mathcal{O}\left(10^2\right)$ holds true. Similarly, $\frac{1}{2}\dot{\phi}^2 \sim \mathcal{O}\left(10^{-2}\right)$ while $V \sim \mathcal{O}\left(10^2\right)$ also holds true. In addition, we have $-9\xi H^2\dot{\phi}^2 \sim \mathcal{O}\left(10^{-3}\right)$ and $-3H^3\dot{f} \sim \mathcal{O}\left(10^{-4}\right)$ compared to $3\zeta H^2\phi^2 \sim \mathcal{O}\left(10^2\right)$, $6\zeta H\phi\dot{\phi} \sim \mathcal{O}(1)$, and potential term holds true. Also $2H\dot{H}\dot{f} \sim \mathcal{O}\left(10^{-6}\right)$, $H^2\left(\ddot{f} - H\dot{f}\right) \sim \mathcal{O}\left(10^{-4}\right)$, $-2\xi\left(3H^2\dot{\phi}^2 - \dot{H}\dot{\phi}^2 - 2H\dot{\phi}\ddot{\phi}\right) \sim \mathcal{O}\left(10^{-4}\right)$, compared to $2\zeta\left(H\phi\dot{\phi} - \dot{H}\phi^2 - \dot{\phi}^2 - \phi\ddot{\phi}\right) \sim \mathcal{O}(1)$ and kinetic term holds true. As well $-6\xi H^2 \sim \mathcal{O}\left(10^{-2}\right)$, $-12\xi H\dot{H}\dot{\phi} \sim \mathcal{O}\left(10^{-4}\right)$, $-3f'H^2\left(H^2 + \dot{H}\right) \sim \mathcal{O}\left(10^{-2}\right)$ compared to $12\zeta H^2\phi \sim \mathcal{O}\left(10^2\right)$ and $\ddot{\phi} + H\dot{\phi} \sim \mathcal{O}\left(10^1\right)$ also holds true.

3.2. A Quadratic Coupling Function

We choose again a quadratic coupling function which is expressed as,

$$f(\phi) = \lambda\phi^2, \tag{56}$$

where λ is a dimensionless constant. Whereas, the first and second derivatives of $f(\phi)$ to the scalar field respectively are,

$$f'(\phi) = 2\lambda\phi, \qquad f''(\phi) = 2\lambda. \tag{57}$$

Then, plug the above conditions into Eq. (28) to discover the explicit function of the scalar potential function $V(\phi)$, and we briefly get,

$$V(\phi) = V_0\left[\frac{|k\phi^2 + c|}{|c|}\right]^m, \tag{58}$$

where V_0 is an initial potential, as well as k, c, and m are constants which is expressed as,

$$k = 2(1-\beta)\zeta\lambda + \zeta(\lambda - \xi), \quad c = -(\lambda - \xi), \quad m = \frac{(1+\beta/3)(1-\beta)\lambda + 4\zeta(\lambda - \xi)}{4(1-\beta)\zeta\lambda + 2\zeta(\lambda - \xi)}. \tag{59}$$

By applying the conditions (56) and (57) to the slow-roll parameters in Eqs. (38) and (39), we have,

$$\varepsilon = \frac{(1-\beta)^2}{2}\left[\frac{\left(\lambda^2 - 2\zeta\xi\lambda\right)\phi^2}{(1-\zeta\phi^2)(\lambda-\xi)^2}\right], \tag{60}$$

and,

$$\delta_{Q_t} = \frac{2\beta(1-\beta)^2\xi\lambda^2H^2\phi^2 + 2(1-\beta)(\lambda-\xi)\left[-\zeta\lambda\phi^2 + (\beta-\varepsilon)\lambda^2H^2\phi^2 + \lambda^2H\phi\right]}{(1-\zeta\phi^2)(\lambda-\xi)^2 + (1-\beta)^2\xi\lambda^2H^2\phi^2 + 2(1-\beta)(\lambda-\xi)\lambda^2H^2\phi^2}, \tag{61}$$

where the expression of H^2 in Eq. (26) becomes,

$$H^2 \simeq \frac{1}{3}V\left[\frac{\lambda-\xi}{(1-\zeta\phi^2)(\lambda-\xi) - 2(1-\beta)\zeta\lambda\phi^2}\right]. \tag{62}$$

You can see only three the slow-roll parameters, namely ε, β, and δ_{Q_t} which have the simplest form compared to δ_{c_T}, δ_{Q_s}, and δ_{c_s} which will be calculated numerically. Since expression of ε is the simplest slow-roll expression, the initial and final value of the scalar field are easily obtained as follows,

$$\phi_i = \phi_f \exp\left[\frac{(1-\beta)\lambda N}{\xi - \lambda}\right], \tag{63}$$

$$\phi_f = \left[\frac{2\left(\lambda - \xi\right)^2}{\left(1-\beta\right)^2\left(\lambda^2 - 2\zeta\xi\lambda\right) + 2\zeta\left(\lambda-\xi\right)^2} \right]^{\frac{1}{2}}. \tag{64}$$

The resulting theory from the above equations can be compatible with the observational data for several free parameters. For instance by selecting $(\lambda, V_0, N, \beta, \zeta, \xi) = \left(-3\times 10^{-7}, 10^3, 60, -0.0349, 0.1, 10^{-5}\right)$ in Planck units, subsequently the observational quantities reads,

$$n_s = 0.964266, \quad n_T = -2.497178\times 10^{-4}, \quad r = 0.001973, \tag{65}$$

which are compatible with the newest Planck data [19]. Furthermore, the scalar field increases with time i.e $\phi_i = 0.509468$ whereas $\phi_f = 3.108606$, the sound wave velocity is equal to unity as expected and moreover, the slow-roll parameters are valid since these numerical values are of order $\mathcal{O}\left(10^{-2}\right)$ and lesser. In particular, $\varepsilon = 9.281029\times 10^{-4}$, $\beta = -0.0349$, $\delta_{Q_t} = -0.001607$, $\delta_{c_T} = 2.9\times 10^{-9}$, $\delta_{Q_s} = -0.066916$, and $\delta_{c_s} = 0.033598$. In this model, the scalar potential function has about $V = 9.450100\times 10^2$ and the square of the Hubble parameter about $H^2 = 3.239177\times 10^2$. The best results are shown in $|\lambda| \sim 10^{-7}$ since it has a wide interval,

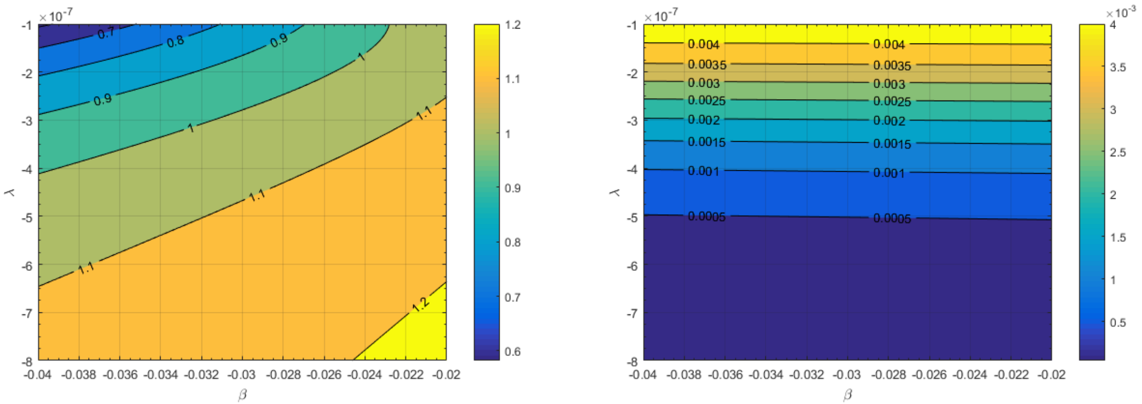

Figure 2. A quadratic coupling function model: the contour plot of the scalar spectral index n_s (left) and tensor-to-scalar ratio r (right) depending on parameters β and λ on intervals $[-0.04, -0.02]$ and $\left[-8\times 10^{-7}, -1\times 10^{-7}\right]$ respectively. It can be seen that the value of the scalar spectral index are also very much determined by the constant-roll parameter β and also has a narrow area to be compatible with observational data.

hence has a significant difference between ϕ_i and ϕ_f which indicates the scalar field growth during inflation, as well as V and H^2 are almost close to V_0. The largest order of λ in this model is about $|-2.4\times 10^{-6}|$ and if it is more than that, the other quantities are imaginary. The smaller the order of λ causes ϕ_i and ϕ_f to be almost the same value, the orders of V and H^2 extremely drops, and β the nearly same to produce n_s that fits the data. Now, we consider variations of $\zeta = (10, 50, 100)$ at $|\lambda| \sim 10^{-8}$ resulting ϕ_i and ϕ_f are almost the same, V and H^2 decreases, as well as $\beta \to 1$ and $\delta_{Q_t} > 1$ so that it does not match the expected values. Therefore, the order of $\zeta \sim 10^{-1}$ works well in this model. On the other hand, if we consider $\xi > 10^{-5}$, the values of ϕ_i and ϕ_f are almost the same, V and H^2 decreases, as well as ε and β become relatively constant. Therefore, the order of ξ works well in this model. This proves that the assumption of the coupling ξ and $f \sim \lambda$ works on the relatively small order of the proposed models. The most important thing from the results (65) obtained analytically is how the values changes when varied against independent parameters, such as β which affects the value of the scalar spectral index, and λ which affects the value of the Gauss-Bonnet coupling function. Shown at **Figure 2** above, the contour plot of the scalar spectral index n_s and the scalar-to-tensor ratio r at corresponding intervals.

Lastly, we validate whether the approximations assumed in Eqs. (15) and (16) holds true. Firstly, by choosing $(\lambda, V_0, N, \beta, \zeta, \xi) = \left(-3\times10^{-7}, 10^3, 60, -0.0349, 0.1, 10^{-5}\right)$, we have $\dot{H} \sim \mathcal{O}\left(10^{-1}\right)$ compared to $H^2 \sim \mathcal{O}\left(10^2\right)$ holds true. Similarly, $\frac{1}{2}\dot{\phi}^2 \sim \mathcal{O}\left(10^{-2}\right)$ while $V \sim \mathcal{O}\left(10^2\right)$ also holds true. In addition, we have $-9\xi H^2\dot{\phi}^2 \sim \mathcal{O}\left(10^{-3}\right)$ and $-3H^3\dot{f} \sim \mathcal{O}\left(10^{-3}\right)$ compared to $3\zeta H^2\phi^2 \sim \mathcal{O}\left(10^1\right)$, $6\zeta H\phi\dot{\phi} \sim \mathcal{O}\left(1\right)$, and potential term holds true. Also $2H\dot{H}\dot{f} \sim \mathcal{O}\left(10^{-7}\right)$, $H^2\left(\ddot{f} - H\dot{f}\right) \sim \mathcal{O}\left(10^{-4}\right)$, $-2\xi\left(3H^2\dot{\phi}^2 - \dot{H}\dot{\phi}^2 - 2H\dot{\phi}\ddot{\phi}\right) \sim \mathcal{O}\left(10^{-3}\right)$, compared to $2\zeta\left(H\phi\dot{\phi} - \dot{H}\phi^2 - \dot{\phi}^2 - \phi\ddot{\phi}\right) \sim \mathcal{O}\left(10^{-1}\right)$ and kinetic term holds true. As well $-6\xi H^2 \sim \mathcal{O}\left(10^{-2}\right)$, $-12\xi H\dot{H}\dot{\phi} \sim \mathcal{O}\left(10^{-4}\right)$, $-3f'H^2\left(H^2 + \dot{H}\right) \sim \mathcal{O}\left(10^{-2}\right)$ compared to $12\zeta H^2\phi \sim \mathcal{O}\left(10^2\right)$ and $\ddot{\phi} + H\dot{\phi} \sim \mathcal{O}\left(10^1\right)$ also holds true.

4. Conclusion

In this work, we studied the gravitation models with string correction term which is the Einstein-Gauss-Bonnet term, minimal, non-minimal, and non-minimal derivative couplings during cosmic inflation in which we combined the action function in previous work [18,20,21]. Theories of gravity in Eq. (10) belong to a wider class of Horndeski's theory. The GW170817 event provides evidence that gravitational waves travel at almost the same to the speed of light in vacuum, so we impose the constraint $c_T^2 \simeq 1$ [4]. This constraint makes the expression of the rate of the scalar field $\dot{\phi}$ depends on the Gauss-Bonnet coupling function $f(\phi)$, non-minimal derivative coupling constant ξ, and the constant-roll β, which greatly affects calculations so that compatible with data. The constraint $c_T^2 \simeq 1$ actually refers to non-minimal derivative coupling which is phenomenologically viable in correcting the Einstein-Gauss-Bonnet inflationary theory.

By using the constant-roll assumption in our theoretical work, we can demonstrate its feasibility by phenomenology. As prove, if we saw from the contour plots presented in the previous section, the correct selection of β produces observational quantities (analytically obtained) which is compatible with the latest Planck data [19]. The Gauss-Bonnet coupling function models we chose are also very simple functions. This proves that even the simple models also give viable results.

Thus, one can use Eq. (25) as an additional constraint and provide numerical solutions for cosmological and astrophysical interest. Our task in planning future theoretical research also relies on future observational research data relating to the speed of gravitational waves, or events in the early universe.

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