

Magnitude redshift relation in the Brans-Dicke model constrained from Big-Bang Nucleosynthesis

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Abstract

The magnitude redshift relation ($m-z$) in the Brans-Dicke theory with both a variable and constant cosmological term is investigated. Observations of Type Ia Supernovae (SNIa) are used, in the redshift range of $0.01 < z < 2$. The contribution of the matter and a variable cosmological term (Λ) is examined. As the next approach BDA model with a constant cosmological term has been investigated. Later Big Bang Nucleosynthesis has been used to constrain the parameters in BDA model for coupling constant $\omega = 10^4$.

Keywords : nucleosynthesis, accelerating universe

1 Introduction

To explain some puzzles in cosmology, new modified theories beyond the standard model are needed. To solve the cosmological constant problem, it can be imagined that the cosmological term decreases from large value at the early epoch to the present value. Therefore various functional forms have been suggested Refs. [1], [2]. Among them the Brans-Dicke(BD) theory is considered with a variable cosmological term Λ as a function of the scalar field Φ . This model has been investigated for the early universe of the Big Bang Nucleosynthesis Refs. [1],[2],[3]. However, an answer is needed to the question "How this model work at the present epoch?" . Therefore to investigate this model, the magnitude-redshift relation of SNIa observation is adopted. Cosmological models with a cosmological term are tightly constrained by the $m-z$ relation derived from SNIa observations. This is because, the cosmological term affects to the cosmic expansion rate of the universe significantly at the low redshifts. SNIa observations imply that the universe is accelerating around the present redshift times Refs. [4],[5], [6]. The magnitude-redshift relation in the Brans-Dicke theory with both a variable and constant cosmological terms for the flat universe are studied in this paper.

2 Brans-Dicke model with a variable cosmological term

The equations of motion in the $B\Lambda$ model are written as follows Refs. [1]:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3\phi}(\rho_m + \rho_\gamma) - \frac{k}{a^2} + \frac{\Lambda}{3} + \frac{\omega}{6}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi}, \quad (1)$$

$$\dot{\phi} = \frac{1}{a^3} \left[\frac{8\pi\mu}{2\omega + 3} \left(\rho_{m_0} t + \int (\rho_e - 3p_e) dt \right) + B \right], \quad (2)$$

where $a(t)$, k , ρ , p and ω are the scale factor, the curvature constant, the energy density, the pressure and the coupling constant respectively.

where $\rho_\gamma = \rho_{rad} + \rho_\nu + \rho_{e^\pm}$ at $t \leq 1s$. Subscript m , rad , ν and e^\pm are for matter, photon, neutrino

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and electron-positron respectively. Here energy density of matter varies as $\rho_m = \rho_{m0}a^{-3}$. The energy density of radiation is written as $\rho_\gamma = \rho_{\gamma0}a^{-4}$ except e^\pm epoch. Subscript "0" means the values at the present epoch.

Evolution of the cosmological term (Λ) and the Gravitational term in the BDA model describe as,

$$\Lambda = \frac{2\pi(\mu-1)}{\phi}\rho_{m0}a^{-3}, G = \frac{1}{2}\left(3 - \frac{2\omega+1}{2\omega+3}\mu\right)\frac{1}{\phi}. \quad (3)$$

where μ is a constant. Original Brans-Dicke theory is deduced for $\mu = 1$. To solve above equations numerically, physical parameters are used as $\omega = 10^4$ Refs. [8], $G_0 = 6.6726 \times 10^{-8} \text{ dyn cm}^2 \text{ g}^{-2}$, $H_0 = 71 \text{ km s}^{-1} \text{ Mpc}^{-1}$ Refs. [9], $-1 \leq \mu \leq 2$ Refs. [3], [2] and $-10 \leq B^* \leq 10$ Refs. [3].

3 $m - z$ relation in the BDA model

The apparent magnitude m of the source at the redshift z is,

$$m = 5\log_{10}\left(\frac{1+z}{10\text{pc}}r_l\right) + M, \quad (4)$$

where M is absolute magnitude and r_l is the radial distance in the units of parsecs (pc).

For the homogeneous isotropic universe, the relation between the radial distance and the redshift is derived from the Robertson-Walker metric as the followings,

$$\int_0^z \frac{dz}{H} = \begin{cases} k^{-1/2} \sin^{-1}(\sqrt{k}r_l) & k > 0, \text{close universe} \\ r_l & k = 0, \text{Flat universe} \\ |k|^{-1/2} \sinh^{-1}(\sqrt{|k|}r_l) & k < 0, \text{Open Universe} \end{cases}$$

where $H = \dot{a}/a$ is the equation to describe the expansion of the universe for the modified BD theory is written from the equation(1) as,

$$H = \pm \left[\frac{1}{4} \left(\frac{\dot{\phi}}{\phi} \right)^2 - (1+z)^2 k - \frac{\Lambda}{3} - \frac{\omega}{6} \left(\frac{\dot{\phi}}{\phi} \right)^{\frac{1}{2}} - \frac{8\pi}{3} \frac{\rho}{\phi} \right]^{\frac{1}{2}} - \frac{1}{2} \frac{\dot{\phi}}{\phi}. \quad (5)$$

If the universe is flat ($k = 0$) at the present, BDA model is written as,

$$H_0^2 = \frac{1}{3} \left(\frac{8\pi\rho_{m0}}{\phi_0} + \Lambda_0 \right) + \frac{\omega}{6} \left(\frac{\dot{\phi}}{\phi} \right)_0^2 - \left(\frac{\dot{\phi}}{\phi} H \right)_0. \quad (6)$$

Since Last two terms are small compared with the other two terms in the equation (6), energy densities are defined as, $\rho_{m0} = 4\rho_c^{BDA}/\mu + 3$ and $\rho_c^{BDA} = 3\phi_0 H_0^2/8\pi$:

where ρ_c^{BDA} is the critical density in the BDA model.

Then the two energy density parameters are written as, $\Omega_{m0} = \rho_{m0}/\rho_c^{BDA}$ and $\Omega_{\Lambda0} = (\mu-1)\rho_{m0})/4\rho_c^{BDA}$.

Fig. 1 shows the $m - z$ relation in the BDA model with SNIa observations. Matter is dominant in this model. Specially in the parameter region $0.7 < \mu < 2$, energy density of the cosmological term is always less than 30%. When we compare with the Friedmann model with the energy density parameters of $(\Omega_m, \Omega_\Lambda) = (1.0, 0.0)$, is merged with this BDA model with $\chi^2 = 416$. This is inconsistent with present accelerating universe. Therefore as the next approach, BDA model is modified by adding constant cosmological term Λ_{c0} .

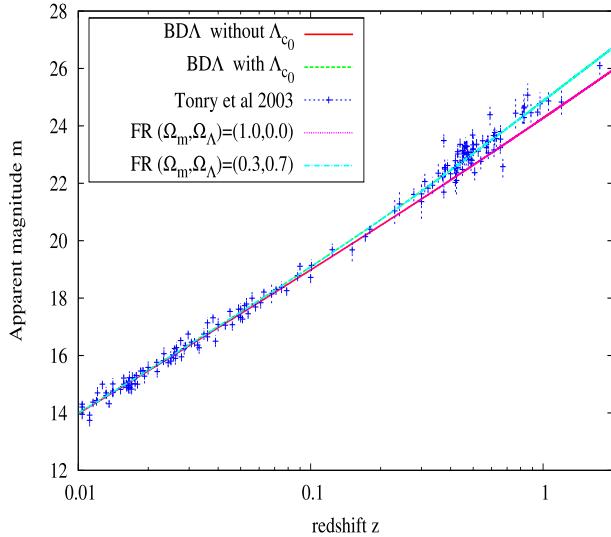


Figure 1: $m - z$ relation for the flat universe in the Friedmann model and BDA model with and without constant cosmological term for $\mu = 0.7$ and $B^* = -10$, constrain from SNIa observations from Supernova Cosmology Project and High-z Supernovae search team Refs. [10]

4 $m - z$ relation in the BDA model with a constant cosmological term Λ_{c_0}

Hubble parameter for BDA model with constant cosmological term (Λ_{c_0}) is written as,

$$H = \left[\frac{1}{4} \left(\frac{\dot{\phi}}{\phi} \right)^2 - (1+z)^2 k - \frac{\Lambda}{3} + \frac{\Lambda_{c_0}}{3} - \frac{\omega}{6} \left(\frac{\dot{\phi}}{\phi} \right)^2 - \frac{8\pi\rho}{3\phi} \right]^{\frac{1}{2}} - \frac{1}{2} \frac{\dot{\phi}}{\phi}. \quad (7)$$

Here energy density parameter of the constant cosmological term is fixed as 0.7. Fig. 1 shows that this model is consistent with SNIa observations. Total cosmological term is dominant in this model and consistent with present accelerating universe with $\chi^2 = 196$. For $B^* = 10$ and $\mu = 2$, BDA model with Λ_{c_0} predicts $\Omega_\Lambda = 6.0 \times 10^{-2}$ and $\Omega_m = 0.24$. It is concluded that the BDA model with Λ_{c_0} has the nearly same energy density parameters as the Friedmann model with $(\Omega_m, \Omega_\Lambda) = (0.3, 0.7)$.

All the parameters which is inherent in the BDA model become independent as far as $m - z$ relation is concerned at the present epoch. Therefore as the next approach we investigate these parameters for the present values of ω using Big Bang Nucleosynthesis.

5 Parameter constrained from Big Bang Nucleosynthesis

The parameters inherent in the BDA model have been investigated for $\omega = 500$ Refs. [3]. But these parameters become independent, as far as the $m - z$ relation at the present epoch is concerned. Therefore here we use the Big Bang Nucleosynthesis to investigate these parameter for $\omega = 10000$. We adopt the observed abundances of 4He , D/H and $7\text{Li}/\text{H}$ as follows: $\text{Yp} = 0.242 \pm 0.002$ Refs. [11], $\text{D}/\text{H} = (2.82 \pm 0.21) \times 10^{-6}$ Refs. [12], $7\text{Li}/\text{H} = (2.19 \pm 0.28) \times 10^{-10}$ Refs. [13]. The abundance of 4He , D and 7Li are calculated by considering the value of $\eta = (6.225 \pm 0.170) \times 10^{-10}$ Refs. [14].

(Fig. 2) shows that the 4He and D are consistent with the value of η for $\omega = 10^4$ in the range of $-0.9 \leq \mu \leq 0.9$ and $-3 \leq B^* \leq 4$.

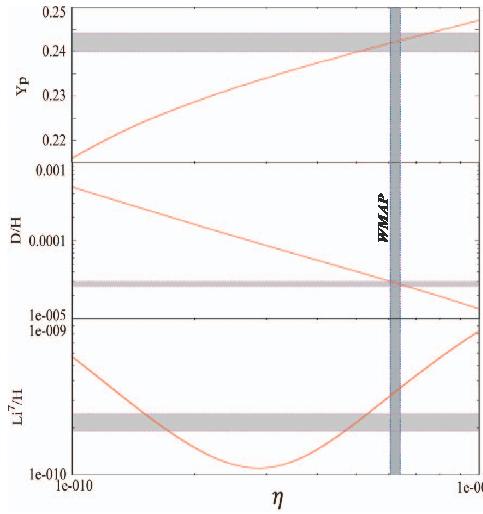


Figure 2: Light elementary abundances of 4He , D and 7Li vs η for $B^* = 1$, $\mu = 0.8$, $\omega = 10^4$

6 Concluding Remarks

The models whose parameters are inherent in the BDA model become independent as far as the $m - z$ relation at the present epoch is concerned. Therefore we can not constrain these parameters around the present epoch using $m - z$ relation. Therefore we limit these parameters for $\omega = 10^4$ by BBN. BBN calculations with the observational abundances and the obtained value of the η from the WMAP restricted the parameters range as $-0.5 \leq \mu \leq 0.8$ and $-10 \leq B^* \leq 10$ Refs. [3] for $\omega = 500$. Comparing with our result, large value of ω is affected to decrease the parameter range of B^* . It is oppositely affected to the parameter μ . Since BDA model is inconsistent with present accelerating universe, we have done a modification by adding a constant cosmological term. Therefore it is worthwhile to introduce more general functional form to the cosmological term.

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