Non-radial f-modes in neutron stars with hyperons and delta baryons

O. P. Jyothilakshmi[1],* P. E. Sravan Krishnan[2], Sreekanth V. [1], Harsh Chandrakar[3], and T. K. Jha[3]
[1]*Department of Physics, Amrita School of Physical Sciences, Coimbatore - 641112, Amrita Vishwa Vidyapeetham, India*
[2]*Argelander-Institut für Astronomie, Universität Bonn, Auf dem Hügel 71, 53121 Bonn, Germany and*
[3]*Department of Physics, BITS PILANI K K Birla Goa Campus, Goa 403726, India*

The gravitational waves generated by the oscillation of compact objects give a way to explore the internal structure of such stars. Neutron stars oscillate in various quasi-normal modes such as f-mode, p-mode, r-mode, etc., essentially categorized based on the restoring force that returns the system back to its equilibrium position. Studying these oscillation modes involves solving the perturbed fluid equations within the framework of general relativity. The relativistic Cowling approximation [1], which neglects metric perturbations, is commonly employed to study f-mode oscillations in compact stars [2].

High-mass pulsars place significant constraints on the equations of state (EoS) and particle interactions, especially when we consider exotic particles like hyperons or quarks in their cores. While these particles tend to soften the EoS, a stiffer EoS is needed to support massive neutron stars. Several approaches are used to achieve high-mass stars, such as looking at repulsive hyperon-hyperon interactions, hyperonic three-body forces, or phase transitions to quark matter. In this work, we study the f-mode oscillations using the Cowling approximation for two different models of neutron stars, which includes the octet of baryons $(n, p, \Lambda^0, \Sigma^{-,0,+}, \Xi^{-,0})$ and with delta's (n, p, $\Delta^{++,+,0,-}$) within the effective chiral model.

We employ the effective Lagrangian used in Refs. [3, 4] given by:

$$\begin{aligned}\mathcal{L} = \bar{\psi}_B\Big[\Big(i\gamma\mu\partial^\mu - g_\omega\gamma_\mu\omega^\mu - \frac{1}{2}g_\rho\vec{\rho_\mu}.\vec{\tau}\gamma^\mu\Big) \\ - g_\sigma(\sigma + i\gamma_5\vec{\tau}.\vec{\pi})\Big]\psi_B + \frac{1}{2}(\partial_\mu\vec{\pi}.\partial^\mu\vec{\pi} + \partial_\mu\sigma\partial^\mu\sigma) \\ - \frac{\lambda}{4}\left(x^2 - x_0^2\right)^2 - \frac{\lambda b}{6m^2}(x^2 - x_0^2)^3 \\ - \frac{\lambda c}{8m^4}(x^2 - x_0^2)^4 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}g_\omega^2x^2\left(\omega_\mu\omega^\mu\right) \\ - \frac{1}{4}\vec{R_{\mu\nu}}.\vec{R^{\mu\nu}} + \frac{1}{2}m_\rho'^{\,2}\vec{\rho_\mu}.\vec{\rho^\mu} + \eta_1\left(\frac{1}{2}g_\rho^2x^2\vec{\rho_\mu}.\vec{\rho^\mu}\right)\end{aligned} \tag{1}$$

Here, the nucleon isospin doublet ψ_B interacts through the exchange of the pseudo-scalar meson π, the scalar meson σ, the iso-vector meson ρ and the vector meson ω.

One of the important parameters that helps in describing the stiffness of EoS at a given energy density ϵ and pressure P is the adiabatic index, defined as follows:

$$\gamma = \frac{\epsilon + P}{P}\frac{d\epsilon}{dP}, \tag{2}$$

and it is plotted as a function of ϵ in Fig. 1. We find that both Δ and H EoSs follow a similar trend qualitatively and it shows a slight difference quantitatively especially in the high density region. We also note that the Δ EoS has a higher value of γ, which suggests a stiffer EoS. The stability criterion requires the value of γ to be greater than 4/3, which is well satisfied for both Δ and H EoSs.

Now, we calculate the f-mode frequencies using the Cowling approximation and it is plotted as a function of stellar mass M in Fig. 2 for the $l = 2$, 3 and 4 modes for both H and Δ EoSs. We also show the mass limits obtained from pulsar observations. The maximum masses obtained for H

*Electronic address: op_jyothilakshmi@cb.students.amrita.edu

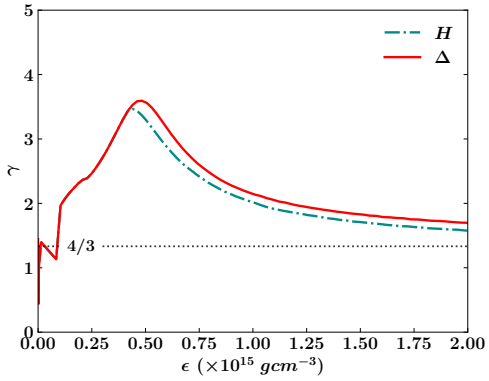

FIG. 1: The dimensionless adiabatic index γ as a function of the energy density ϵ for Δ and H EoSs.

and Δ EoSs are 2.03 and 2.20 $M_\odot$ respectively and they are found to be consistent with the mass limit given by PSR J0348+0432 and PSR J0740+6620. We find that the f-mode frequencies of H and Δ EoSs show a considerable difference in the higher mass region ($M > 1.2M_\odot$), while the mode frequencies remain almost equal in the lower mass region. The $l = 2$ mode frequency (f_{max}) corresponding to the maximum mass for H and Δ baryon EoSs are 2.42 and 2.39 kHz. We also note that f_{max} for Δ EoS is slightly lower. The f_{max} values for $l = 3$ (4) for H and Δ EoSs are 2.94 (3.42) kHz and 2.98 (3.43) kHz respectively.

Next, in Fig. 3 we show the f-mode frequencies as a function of tidal deformability Λ for the models considered. The tidal deformability offers important insights into the properties of stellar matter and depends on the EoS through both the neutron star radius R and the dimensionless Love number k_2 as $\Lambda = 2k_2R^5/3$. The f-mode frequency as a function of Λ for H and Δ EoSs are easily distinguishable and the value of Λ corresponding to the maximum mass for H and Δ EoS are 30.5 and 17.8 respectively. The dimensionless tidal deformability of a canonical NS ($\Lambda_{1.4}$) for H and Δ EoS is $\sim$ 800, which is in considerable agreement with the constraints given by the GW170817 event. We also find that the frequency corresponding to a mass of $1.4M_\odot$ of $l = 2$ f-mode for H and Δ EoSs is approximately $\sim$ 2 kHz, which satisfies the limit provided for f-mode frequency (*i.e.*, 1.67 − 2.18 kHz) [5]. We find that various stellar properties studied in this work show a close agreement with various observational constraints from pulsars and GW events.

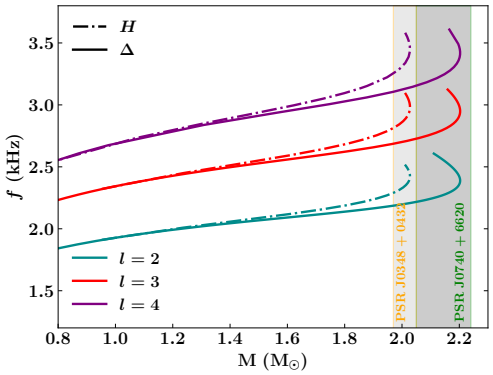

FIG. 2: The frequencies of $l = 2$, 3, and 4 f-modes as a function of stellar mass M for the EoSs considered.

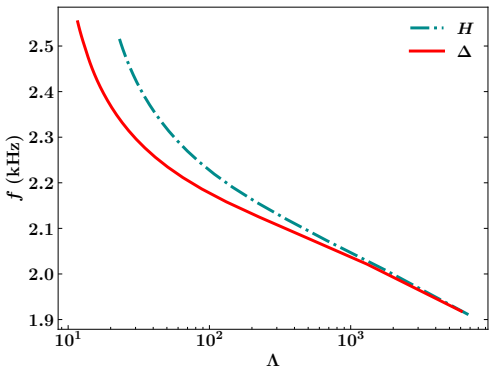

FIG. 3: The frequencies of f-mode f as a function of tidal deformability Λ for the EoSs considered.

References

[1] T. G. Cowling, Mon. Not. Roy. Astron. Soc. 101, 367 (1941).
[2] O. P. Jyothilakshmi, L. J. Naik, and V. Sreekanth, Eur. Phys. J. C 84, 427 (2024).
[3] T. Malik, K. Banerjee, T. K. Jha, and B. K. Agrawal, Phys. Rev. C 96, 035803 (2017).
[4] O. P. Jyothilakshmi, P. E. S. Krishnan, P. Thakur, V. Sreekanth, and T. K. Jha, Mon. Not. Roy. Astron. Soc. 516, 3381 (2022).
[5] D.-H. Wen, B.-A. Li, H.-Y. Chen, and N.-B. Zhang, Phys. Rev. C 99, 045806 (2019).