Semiclassical Analysis of ICF, CF and TF Excitation Functions for $^{7}Li + ^{124}Sn$ System at around Barrier Energies

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The study of nuclear fusion reactions involving the weakly bound projectiles at around barrier energies is a topic of contemporary interest in nuclear physics [1, 2]. In these reactions, in addition to the Direct Complete Fusion (DCF), where the entire projectile merges with the target nucleus, the phenomenon of projectile breakup before fusion leads to some unusual fusion mechanisms such as Incomplete Fusion (ICF), where only one of the fragments resulting from the breakup undergoes fusion with the target, and Sequential Complete Fusion (SCF), where all resultant fragments are sequentially absorbed by the target nucleus. The sum of DCF and SCF is referred to as Complete Fusion (CF), while that of DCF, SCF and ICF is called as Total Fusion (TF). From the Experimental point of view, it is not possible to separate out SCF events from DCF while CF and ICF can be measured separately.

Theoretically classical, semiclassical and fully quantum mechanical models are developed to calculate separately the CF and ICF cross sections [3-5]. In the present work, we have employed a semiclassical model developed in Ref. [4] to analyse the excitation functions of ICF, CF and TF processes for $^{7}Li + ^{124}Sn$ system at near barrier energies measured by V.V. Parkar et al. [6].

In this semiclassical treatment, the projectile-target relative motion is described classically while the internal dynamics of the projectile is treated quantum mechanically. The Hamiltonian of the projectile-target system is given by $h = h_0(\xi) + V(\mathbf{r}, \xi)$ with $h_0(\xi)$ as the intrinsic Hamiltonian of the projectile and $V(\mathbf{r}, \xi)$ as the projectile–target interaction. The wave function in intrinsic space expanded in terms of eigenvectors of $h_0(\xi)$, obtained by solving the equation $h_0|\Phi_\alpha\rangle = \varepsilon_\alpha |\Phi_0\rangle$, is written as

$$\psi(\xi, t) = \sum_\alpha a_\alpha(\ell, t)\, \Phi_\alpha(\xi)\, e^{-i\varepsilon_\alpha t/\hbar}$$

Substitution of this wave function $\psi(\xi, t)$ into the time dependent Schrödinger equation leads to the following Alder-Winther (AW) [7] equations

$$i\hbar \dot{a}_\alpha(\ell, t) = \sum_\beta \langle \Phi_\alpha | V_\ell(\xi, t) | \Phi_\beta \rangle e^{i(\varepsilon_\alpha - \varepsilon_\beta)t/\hbar}\, a_\beta(\ell, t)$$

which are solved numerically to obtain a_α by assuming that initially the projectile was in ground state. The probability to populate channel α in a collision with angular momentum ℓ is given by $P_\ell^{(\alpha)} = |a_\alpha(\ell, t \to +\infty)|^2$.
The fusion probability is approximated as the product of $\bar{P}_\ell^{(\alpha)}$, the probability that the system is in channel α at the point of closest approach on the classical trajectory, and of the tunnelling probability $T_\ell^{(\alpha)}(E_\alpha)$ through the potential barrier in channel α that is $P_\ell^F(\alpha) \simeq \bar{P}_\ell^{(\alpha)}\, T_\ell^{(\alpha)}(E_\alpha)$. The tunneling probability $T_\ell^{(\alpha)}(E_\alpha)$ is described very well under the parabolic approximation for the barrier due to effective potential by the following Hill-Wheeler formula

$$T_\ell^{(\alpha)}(E_\alpha) = \left[1 + exp\left\{\frac{2\pi}{\hbar\omega_\ell}(B_\ell - E_\alpha)\right\}\right]^{-1}$$

where B_ℓ and ω_ℓ are the height and curvature of the fitted parabola respectively.
The fusion cross section is given by

$$\sigma_F = \sum_\alpha \left[\frac{\pi}{k^2} \sum_\ell (2\ell + 1) P_\ell^F(\alpha)\right]$$

The label α = 0 corresponds to the ground state, the only bound state of the projectile, and contribute only to CF events. The label α ≠ 0 corresponds to the breakup states represented by a single effective bound state and contributes only to ICF events.

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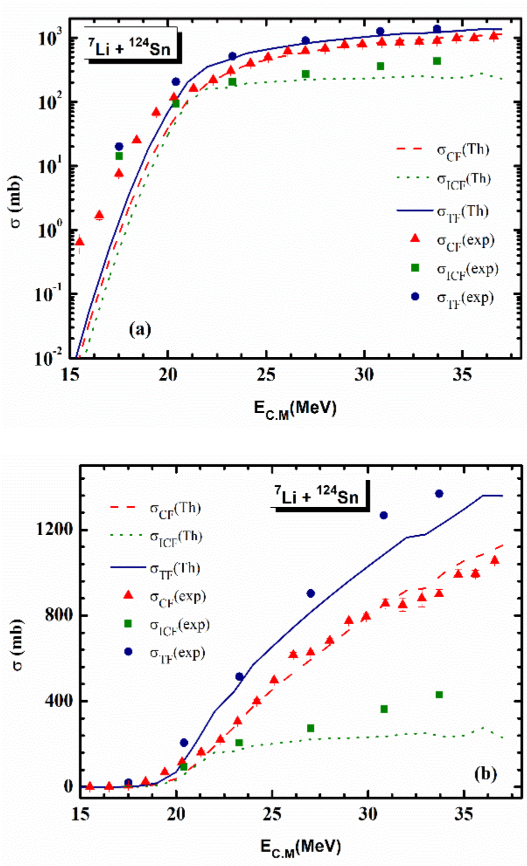

Fig. 1 Fusion excitation functions for CF, ICF and TF processes for ^{7}Li + ^{124}Sn reaction are compared with the corresponding experimental data taken from Ref. [6].

In Fig. 1(a), fusion excitation functions for CF, ICF and TF processes induced by ^{7}Li projectile on ^{124}Sn target at around barrier ($V_B \approx$ 20 MeV) energies are compared with the corresponding experimental data taken from Ref. [6]. It can be clearly seen in this figure that the matching between the data and predictions is reasonably well for the qualitative description in the energy region ranging from slightly below to above barrier energies. But the measured CF cross sections are substantially underestimated by the calculations in the deep sub barrier region where the ICF are not available. It may be ascribed to the fact that the coupling with the target excited states is neglected in the present analysis.

In order to see the fine details of the comparison of data and prediction of various fusion cross sections at above barrier energies, the results of Fig. 1(a) are replotted on a linear scale in Fig. 1(b). It is noticed in this figure that at energies much above the barrier energy the predicted ICF cross section is smaller than the measured one and consequently the theoretical CF cross section is more than the observed values. This discrepancy between data and predictions may be attributed to Coulomb dipole approximation for coupling which is not a reasonable approximation at higher energies. In this energy regime the distance of closest approach between the projectile and target is small and there is also a strong nuclear coupling between the interacting nuclei.

References

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