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An interacting fermion-antifermion pair in the spacetime background generated by static cosmic string

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ABSTRACT

We consider a general relativistic fermion antifermion pair that they interact via an attractive Coulomb type interparticle interaction potential in the $2+1$ dimensional spacetime background spanned by cosmic string. By performing an exact solution of the corresponding fully-covariant two body Dirac Coulomb type equation we obtain an energy spectrum that depends on angular deficit parameter of the static cosmic string spacetime background for such a composite system. We arrive that the influence of cosmic string spacetime topology on the binding energy of Positronium-like atoms can be seen in all order of the coupling strength constant, even in the well-known non-relativistic binding energy term ($\propto \alpha_c^2$). We obtain that the angular deficit of the static cosmic string spacetime background causes a screening effect. For a predicted value of angular deficit parameter, $\alpha \sim 1-10^{-6}$, we apply the obtained result to an ortho-positronium, which is an unstable atom formed by an electron and its antimatter counterpart a positron, and then we determine the shift in the ground state binding energy level as 27, 2 μeV. We also arrive that, in principle, the shift in ground state binding energy of ortho-positronium can be measured even for $\alpha \sim 1-10^{-11}$ value with current techniques in use today. Moreover, this also gives us an opportunity to determine the altered total annihilation energy transmitted by the annihilation photons. The yields also impose that the total lifetime of an ortho-positronium can be changed by the topological feature of static cosmic string spacetime background. In principle, we show that an ortho-positronium system has a potential to prove the existence of such a spacetime background.

1. Introduction

The idea of a theory that merges the quantum physics with the general relativity leaded to the generalization of the relativistic wave equations to curved spacetimes. Therefore, the dynamics of quantum systems in curved spacetimes has been attracted great attention for many years [1–6]. The influence of a curved spacetime background on the single-particle energy states are also considerable interest for the construction of a theory that stands for the combination of the quantum physics ($\hbar$) and gravity (G). The energy levels of an atom placed in a curved spacetime background are shifted as a result of the interaction of the atom with the spacetime curvature [1–5]. Thus, the affected spectral lines of an atom placed in a curved spacetime background give information about the feature of the spacetime background [6,7]. Moreover, these shifts in the energy levels can be different for each quantum state of the atom. This feature can be used to differentiate the other shifts in the energy levels that may originate from the Doppler effect, gravitational or cosmological redshifts, since their effects are same for each spectral lines of the atom [7].

Topological defects such as the monopoles, domain walls and the cosmic strings may have been formed during expansion of the universe. The cosmic strings are one dimensional stable topological defects probably arising during the early stages of the universe and have survived to the present day [8,9]. Initially, these hypotetical objects were described as general relativistic solutions of Kerr space-time in 3-dimensions [10], and then these solutions were naturally extended to the 4-dimensional spacetime background [11], in which the dynamics of physical systems remains invariant under Lorentz boost in the added spatial coordinate [12] provided that the spacetime background is static. The existence of such structures was also supported by an extension of the standard model of particle physics [13] and by superstring theories with extra dimensions [14]. The spacetime background spanned by such structures includes conical singularity [9]. Out of this singularity, static cosmic strings, as well as their rotating stationary counterparts, present a flat spacetime background.

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However, these structures carry remarkable properties, since they are not flat globally. Thus, they may be responsible for a number of interesting effects, such as gravitational lensing [15,16], particle production [17,18], electrostatic self-force on the electric charges at rest [19–21], bremsstrahlung process [22,23] and the so-called gravitational Aharonov-Bohm effect (non-quantum) [24]. The cosmic strings are characterized by a wedge parameter α, ($\alpha = 1 - 4G\mu_s/c^2$, in which G is Newtonian gravitational constant and c is light speed) that depends on their linear mass density μ_s and by the linear density of angular momentum j_s [25–27]. The gravitational or electrostatic effects of these structures on physical systems depend on the $G\mu_s/c^2$ [19,28]. The α parameter is not a free parameter, in principle. However, there is currently no consensus regarding the value of string tension ($\propto \mu_s$) [28]. In general, the existing constraints on cosmic strings refer to string tension [28,29]. Early models of cosmic strings predict that $G\mu_s/c^2 \sim 10^{-6}$, which is might expect for symmetry breaking at the ground unification scale [28]. The constraints on the string tension have been considered by a number of authors [30–32] (more details can be found in [28]). They have found a bound on $G\mu_s/c^2$ and this bound is of the order $\sim 10^{-6}$. Also, of course, various cosmological searches have been carried out to detect the cosmic strings. Cosmic strings can induce the temperature anisotropies of the cosmic microwave background radiation and hence cosmic microwave background observations can provide info about the large scale structure perturbations in the universe [28,33,34]. Current limits obtained from cosmic microwave observations lead $G\mu_s/c^2 \lesssim 10^{-7}$ [35] (see [36]). A loop can be formed by reconnection of long string with itself [28,29,36]. This object can oscillate relativistically and lose their energy eventually [28,36]. Therefore, it is considered that one of the best hope for detecting cosmic string "network", which consists of infinite strings and a distribution of loops of various sizes, is the observation of gravitational waves produced by oscillating string loops [28,36–39]. Gravitational waves produced by the cosmic strings can modulate the pulsar timing and accordingly this effect has been used to look for cosmic strings by the pulsar timing observations [28,36,40]. Recent simulations favor such networks and clearly indicate that large initial loop sizes ($\alpha \lesssim 1$) are preferred [29,36]. In this case gravitational decay of loops (large scale) can takes many Hubble times [29,36]. The influence of cosmic strings on physical systems have been studied for a long time by both observational or theoretical methods. Effects induced by the passage of the cosmic string through the earth was considered and the results showed that the cosmic string can effect global oscillation amplitude of our planet, since it was obtained that the oscillation amplitude of the earth linearly depends on the string tension [41] besides to the string velocity. Therefore, it is considered that the cosmic strings may also cause distinctive eartquakes, which may be detected, in principle, by instruments such as global positioning system and seismometer [41] (also see [42,43]). Moreover, it seems also possible to detect the gravitational waves with the help of quantum mechanical systems, in principle [44]. Such problems have been considered many times. Also, the effect of gravitational fields produced by cosmic strings on the quantum systems have been attracted great attention for a long time [45–60]. In this present paper, we want to determine the alterations in the energy of a fermion-antifermion pair that they interact via an attractive Coulomb type interparticle interaction potential in the 3-dimensional spacetime background induced by cosmic string. Here, it is useful to mentioned that the absence of the third spatial coordinate permits a number of interesting physical and mathematical phenomena [61,62] such as Fractional statistics [63] and Chern-Simmons gauge fields among others [61]. One of such studies has shown that the total energy of a two-dimensional Dirac oscillator depends on spin of the fermion [64].

On the other hand, similar to the non-relativistic quantum theory, in the relativistic framework, the dynamics of the interacting particles are described by one-time equations that are established as a sum of free Hamiltonians for each particle plus a phenomenologically written interparticle interaction potentials [65,66], in general. One of the main problems in such equations is the selection of the interparticle interaction potentials. Moreover, there is also two-time problem in the determination of relativistic dynamics of two interacting particles. After suggestions formulated by Heisenberg and Pauli, the excepted first relativistic two-body equation was introduced by Breit [67,68]. By assuming the electron velocity as the Dirac matrices and then by substituting them into the Lienard-Wiechert potentials, Breit obtained first order correction to the Coulomb potential of the Darwin Lagrangian. Then, he concluded that there must be a strict relationship between the electron motion and the spin in the relativistic framework [68,69]. However, this equation does not hold when velocities of the particles are high or radial distance between the particles is large, due to the retardation effects. Another formalism was introduced by Bethe and Salpeter [70]. Even though this formalism provides an approach to the bound-state problems such as hydrogen-like atoms, it has given energy solutions that have negative magnitude. The relativistic two body equation introduced by Kemmer, Fermi and Yang has quite importance, even though this equation was written as phenomenologically [65,66]. Similar to this equation, a complete fully-covariant two-body Dirac-Coulomb type equation (one-time) [71] that takes into account the behavior of the second fermion was derived from Quantum Electrodynamics with the help of Action principle. This equation includes the most general electric and magnetic potentials. In 4-dimensions, this equation gives a 16×16 dimensional matrix equation. By using group theoretical methods the relative motion coordinates and the center of mass motion coordinates can be separated covariantly [72]. Nevertheless, the well-known energy spectrum for Hydrogen-like atoms (in all orders of the coupling strength constant) can be obtained only a perturbative solution of this matrix equation [72–74]. For the systems that have dynamical symmetries [48,75,76], the fully-covariant two body Dirac-Coulomb equation may be exactly solvable for a fermion-antifermion pair [75]. Positronium (Ps) atom (or antiatom) as a low-energy bound-state system containing a fermion-antifermion pair is one of the fundamental problems in Quantum Electrodynamics, due to the fact its purely leptonic structure [77–79]. This atom is a unique system, as it is the purely leptonic and lightest atom known, that unaffected by color and finite size effects [78]. The Ps is formed by an electron and a positron (anti-electron) pair interacting through attractive Coulomb type interparticle interaction potential. When the angular momentum equals zero, if there is no external effect, a Ps atom can be formed in the spin symmetric ($s = 1$) quantum state known as ortho-positronium (o-Ps, $^3S_1$) with probability 3/4 and spin anti-symmetric ($s = 0$) quantum state known as para-positronium (p-Ps, $^1S_0$) with probability 1/4 [79]. These systems consist of electromagnetically interacting particles and it is known that the laws of electrodynamics remain invariant under the interchange of electron and positron and accordingly overall C-symmetry is conserved. Due to the purely leptonic structure and attractive force between the electron and positron the both mentioned quantum states collapse and then the annihilation event occurs when the annihilation condition is satisfied (which requires $l = 0$). As a result, a Ps atom can decay into odd or even number of gamma photons (γ) via self annihilation process [78]. Number of the resulting photons is determined according to the overall charge conjugation symmetry $(-(-1)^{l+s})$ [77]. The charge conjugation parity of a system of N photons is $(-1)^N$, and hence the o-Ps system ($l = 0$, $s = 1$) must decay into an odd number of gamma photons [77–79]. We should note that this system can not decay into a single photon due to the conservation of momentum [77–79]. Therefore, an o-Ps atom decays into three photons most probably [79], since the higher order annihilations, such as 5γ, 7γ, are strongly suppressed [77–79]. The history of Ps studies goes long way back [78]. The announced results show that the o-Ps decays, most often, into 3γ with lifetime ∼142 ns and the p-Ps decays most probably into 2γ with lifetime ∼125 ps. Due to the fact that there is a huge difference in the mentioned lifetimes, majority of the experimental studies on Ps physics are based on o-Ps [77–79]. On this subject, recent theoretical and experimental studies on Ps system can be found in [80] and [78], respectively. It is obvious that theoretical precision for hyperfine interval of o-Ps is much higher than that of any experimental measurements [78]. Today, there is fact that the most precise measurements about the Ps hyperfine interval can be performed at a few MHz (∼ nano-electron-volt) level sensitivity. Many research group have studied on such measurements [81–85] and efforts to increase experimental measurement sensitivity are still continue today [78,79,86–89]. But, the theoretical calculations are at the kHz (∼ pico-electron-volt) level sensitivity. Thanks to the current developments in experimental Ps physics [78,79,86] and the fascinating properties of an o-Ps system, we can think that an o-Ps can also be regarded as a probe to detect gravitational fields originating from some cosmological objects, in principle at least. This is because of the o-Ps system can enable us, at least, two crucial information. The first of them is based on the fact that it has relatively longer lifetime than the other fermion-antifermion systems. Therefore, it may be possible to measure the shifts in its real oscillation modes with the current techniques in use provided that the shifts are greater than a few MHz level [78]. Here, of course we should ignore the argues about the possibility that matter and antimatter may have different gravitational interactions, since there is no a direct measurement [78]. The second is based on the fact that total annihilation energy transmitted by annihilation photons carry information about the physical properties (morphological, electrical and topological) of spacetime background where the annihilation event occurs. However, it is also important to say that the interaction of a gamma photon with a scintillator occurs mostly through the Compton scattering [79,86,89]. Even though it is possible to measure very sensitively the energy of a photon with semi-conductor detectors (but not cost-effective), a precise measurement of the energy transmitted by an annihilation photon is not seem to be possible today [79,86,89]. However, we can learn some crucial informations about the physical properties of spacetime backgrounds via self annihilation properties of the o-Ps atoms in nearly event-by-event basis. This results are obtained by not only measuring the total lifetime of the o-Ps systems but also measuring the energy transmitted by annihilation photons [79,86]. Therefore, to discuss whether such an effect can be observed or not, we can consider an o-Ps system interacting with the gravitational field produced by a cosmic string. In principle, we think that it is possible to determine the influence of a spacetime background on an o-Ps system. So, we can use the generalized form of fully-covariant two-body Dirac equation [90] (well-established) in order to arrive at a non-perturbative spectrum (in closed-form) in energy domain for the mentioned quantum system.

In this present paper, we deal with a general fermion-antifermion pair that they interact via an attractive Coulomb type interparticle interaction potential in the 2+1 dimensional spacetime background induced by cosmic string [25–27]. In order to arrive at a non-perturbative spectrum of such a composite structure we solve the corresponding fully-covariant two body Dirac equation under the following assumption. This assumption is that the center of mass of the composite structure is static and locates at the origin of the static cosmic string spacetime background. At that rate, we arrive at a wave equation that includes a general interparticle interaction potential for a general static fermion-antifermion pair. By an exact solution of the wave equation for such a fermion-antifermion pair interacting through an attractive Coulomb type interparticle interaction potential, we obtain an energy spectrum that depends on angular deficit parameter of the static cosmic string spacetime background. To determine the influence of the gravitational field produced by static cosmic string on the energy of the system that under scrutiny this non-perturbative energy spectrum can be reduced to the "usual" case where the angular deficit of the spacetime background vanishes ($\alpha = 1$). This, leads a binding energy value that agrees well with the current literature when we apply the usual energy spectrum to the triplet quantum states of an electron-positron pair [77,78]. We analyze the effect stemming from the topological feature of the spacetime background on the energy of the system by comparing the obtained both non-perturbative energy results. For an o-Ps, we arrive at an explicit expression that gives the shift in binding energy level. This also provide to determine the effected total annihilation energy [91], which transmitted by the annihilation photons (most probably 3γ), of the system. We discuss the obtained results according to the effective string tension values that can vary as $\propto 1/r$, in principle [19]. The obtained results imply that static cosmic strings may also cause some differences in the formation time and total lifetime of unstable composite systems such as o-Ps and accordingly they can also be responsible for the production of different high energetic photons, since the existence of a static cosmic string change the coupling strength between the fermion and its own antiparticle an antifermion. We arrive at explicit expressions that can allow for practical calculations to determine the altered binding energy and accordingly the altered total annihilation energy values in terms of arbitrary α values. We calculate the mentioned alterations from $G\mu_s/c^2 \sim 10^{-6}$ to $G\mu_s/c^2 \sim 10^{-13}$ values and we discuss the findings whether the alterations can be measurable with current techniques or not (see Table 1).

2. The two body Dirac equation in a general 2+1 dimensional spacetime background

To describe the dynamics of an interacting fermion-antifermion pair in a general $2+1$ dimensional curved spacetime, the fully-covariant two-body Dirac equation can be used in the following form [75],

$$\left\{H^{(1)} \otimes \gamma^{0^{(2)}}(\mathbf{x}_2) + \gamma^{0^{(1)}}(\mathbf{x}_1) \otimes H^{(2)}\right\} \Psi(\mathbf{x}_1, \mathbf{x}_2) = 0, \quad (1)$$

where the superscripts (1) and (2) refer to the first and second fermions, respectively. The bi-local massive field $\Psi(\mathbf{x}_1, \mathbf{x}_2)$ is constructed by a direct production ($\otimes$) of two arbitrary massive Dirac fields,

$$\Psi(\mathbf{x}_1, \mathbf{x}_2) = \xi(\mathbf{x}_1) \otimes \chi(\mathbf{x}_2), \quad (2)$$

also the written Dirac Hamiltonians for this interacting fermion-fermion system are given as follows;

$$H^{(1)} = \left[\gamma^{\lambda^{(1)}} P^{(1)}_{\lambda} - m_1 c \mathbf{I}_2\right], \quad H^{(2)} = \left[\gamma^{\lambda^{(2)}} P^{(2)}_{\lambda} - m_2 c \mathbf{I}_2\right], \quad (\lambda = 0, 1, 2.), \quad (3)$$

in which, m_1, m_2 and c stand for the masses of fermions and speed of light in the vacuum. These Hamiltonians include 2×2 dimensional unit matrices $\mathbf{I}_2$, the generalized Dirac matrices $\gamma^{\lambda}(\mathbf{x})$ and momentum operators that read,

$$P^{(1)}_{\lambda} = i\hbar\left\{\partial^{(1)}_{\lambda} + \frac{ie_1}{\hbar c} A^{(2)}_{\lambda} - \Gamma^{(1)}_{\lambda}\right\},$$
$$P^{(2)}_{\lambda} = i\hbar\left\{\partial^{(2)}_{\lambda} + \frac{ie_2}{\hbar c} A^{(1)}_{\lambda} - \Gamma^{(2)}_{\lambda}\right\}, \quad (\lambda = 0, 1, 2.), \quad (4)$$

here, e_1, e_2 stand for the charges of the interacting two Dirac particles, $\hbar$ is the reduced Planck constant, $A_\lambda(\mathbf{x})$ and $\Gamma_\lambda(\mathbf{x})$ correspond to the electromagnetic vector potentials and spinor connections, respectively. Even though Eq. (1) do not look at first to be manifestly covariant, the γ^0 in everywhere means actually $\gamma^\lambda\eta_\lambda$ in which η_λ is a timelike vector in 3-dimensions. For $\eta_\lambda=(100)$ one can recover the given form in this equation, which includes the spin algebra spanned by the Kronocker (direct) production of the generalized Dirac matrices (for each particle $\gamma^{(1)}\otimes\gamma^{(2)}$). More details about the Eq. (1) can be found in [71].

The 2+1 dimensional spacetime background induced by a spinning cosmic string can be represented by the following line element [26],

$$\begin{aligned} ds^2 &= (cdt+\varpi d\vartheta)^2 - dr^2 - \alpha^2 r^2 d\vartheta^2 \\ &= c^2dt^2 + 2\varpi\, cdtd\vartheta - dr^2 - \left(\alpha^2 r^2 - \varpi^2\right) d\vartheta^2, \\ &\quad \alpha \in (0,1], \end{aligned} \tag{5}$$

where, the angular deficit parameter $\alpha = 1 - 4\mu_s G/c^2$ depends on the linear mass density (μ_s) of the cosmic string and the letter G stands for gravitational Newton constant ($0 \leq 4\mu_s G/c^2 < 1$). The $\varpi = 4 j_s G/c^3$, which has unit of distance, is rotational parameter of the string that carries linear angular momentum density j_s. For $\alpha = 1 - 4\mu_s G/c^2 < 1$, the (r,ϑ)-surface gets a conical topology. This background is locally flat except for the apex of the cone. For $\alpha = 1$, $j_s = 0$ the spacetime given in Eq. (5) describes Minkowski spacetime in polar coordinates. Hence, the topology and accordingly the symmetry of the Minkowski spacetime is changed by μ_s and j_s parameters. For the line element given in Eq. (5), covariant metric tensor is written as;

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & \varpi \\ 0 & -1 & 0 \\ \varpi & 0 & -(\alpha^2 r^2 - \varpi^2) \end{pmatrix}, \tag{6}$$

and its contravariant form becomes,

$$g^{\mu\nu} = \begin{pmatrix} 1 - \frac{\varpi^2}{\alpha^2 r^2} & 0 & \frac{\varpi}{\alpha^2 r^2} \\ 0 & -1 & 0 \\ \frac{\varpi}{\alpha^2 r^2} & 0 & -\frac{1}{\alpha^2 r^2} \end{pmatrix}. \tag{7}$$

The generalized Dirac matrices can be written in terms of spacetime independent Dirac matrices ($\overline{\gamma}^k$) via triads ($e^\lambda_{(k)}(\mathbf{x})$), as in the following;

$$\gamma^\lambda(\mathbf{x}) = e^\lambda_{(k)}(\mathbf{x})\,\overline{\gamma}^k, \quad (k,\lambda = 0,1,2.), \tag{8}$$

and the spacetime independent Dirac matrices are reduced to the Pauli spin matrices for a general 3-dimensional spacetime background. For the signature $+,-,-$, the spacetime independent Dirac matrices can be chosen as $\overline{\gamma}^0 = \sigma^z, \overline{\gamma}^1 = -i\sigma^x, \overline{\gamma}^2 = i\sigma^y$ [75,92]. Now, one can obtain the triads via the following expressions,

$$\begin{aligned} & g_{\mu\tau} = e^{(i)}_\mu e^{(j)}_\tau \eta_{(i)(j)}, \quad e^\mu_{(i)} = g^{\mu\tau} e^{(j)}_\tau \eta_{(i)(j)}, \\ & \eta_{(i)(j)} = diag\,(1,-1,-1), \quad (\mu,\tau,i,j = 0,1,2.), \end{aligned}$$

as;

$$e^\mu_{\ (i)} = \begin{pmatrix} 1 & 0 & -\frac{\varpi}{\alpha r} \\ 0 & -1 & 0 \\ 0 & 0 & \frac{1}{\alpha r} \end{pmatrix}. \tag{9}$$

With the help of Eq. (8) and Eq. (9), the generalized Dirac matrices are obtained,

$$\begin{aligned} & \gamma^t = \sigma^z - \tfrac{\varpi}{\alpha r} i\sigma^y, \quad \gamma^r = i\sigma^x, \\ & \gamma^\vartheta = \tfrac{1}{\alpha r} i\sigma^y, \quad \sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \\ & \sigma^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \end{aligned} \tag{10}$$

and then the spinor connections $\Gamma_\lambda(\mathbf{x})$ are found via the following expression,

$$\begin{aligned} \Gamma_\lambda(\mathbf{x}) &= \tfrac{1}{4} g_{\mu\tau}(\mathbf{x})\,(e^i_{\nu,\lambda}(\mathbf{x})\, e^\tau_i(\mathbf{x}) - \Gamma^\tau_{\nu\lambda}(\mathbf{x})) s^{\mu\nu}(\mathbf{x}), \\ & \qquad (i,\mu,\nu,\lambda,\tau = 0,1,2.), \end{aligned} \tag{11}$$

which includes spin operators $s^{\mu\nu}(\mathbf{x})$,

$$s^{\mu\nu}(\mathbf{x}) = \frac{1}{2}[\gamma^\mu(\mathbf{x}), \gamma^\nu(\mathbf{x})], \tag{12}$$

and Christoffel symbols $\Gamma^\tau_{\nu\lambda}(\mathbf{x})$,

$$\begin{aligned} \Gamma^\tau_{\nu\lambda}(\mathbf{x}) &= \tfrac{g^{\tau\epsilon}(\mathbf{x})}{2} \left\{ \partial_\nu g_{\lambda\tau}(\mathbf{x}) + \partial_\lambda g_{\epsilon\nu}(\mathbf{x}) - \partial_\epsilon g_{\nu\lambda}(\mathbf{x}) \right\}, \\ & \qquad (\nu,\lambda,\tau,\epsilon = 0,1,2.), \end{aligned} \tag{13}$$

where $g^{\tau\epsilon}(\mathbf{x})$ is the contravariant metric tensor.

By using the line element given in Eq. (5) that represents to the $2+1$-dimensional spacetime background generated by spinning cosmic string, one can obtain the non-vanishing components of Christoffel symbols as in the following,

$$\Gamma^t_{r\vartheta} = -\frac{\varpi}{r}, \quad \Gamma^r_{\vartheta\vartheta} = -\alpha^2 r, \quad \Gamma^\vartheta_{r\vartheta} = \frac{1}{r}. \tag{14}$$

Afterwards, the spinor connections are found via Eq. (10), Eq. (11), Eq. (12), Eq. (14) as follows,

$$\Gamma_t = 0, \quad \Gamma_r = 0, \quad \Gamma_\vartheta = \frac{\alpha}{2} i\sigma^z. \tag{15}$$

For interacting two fermions that located at different positions in the spacetime background induced by static cosmic string ($j_s = 0$), the generalized Dirac matrices and the spinor connections are determined,

$$\begin{aligned} & \gamma^{t^{(1)}} = \gamma^{t^{(2)}} = \sigma^z, \quad \gamma^{r^{(1)}} = \gamma^{r^{(2)}} = i\sigma^x, \\ & \gamma^{\vartheta^{(1)}} = \tfrac{i}{\alpha r_1}\sigma^y, \quad \gamma^{\vartheta^{(2)}} = \tfrac{i}{\alpha r_2}\sigma^y, \\ & \Gamma^{(1)}_t = \Gamma^{(2)}_t = 0, \quad \Gamma^{(1)}_r = \Gamma^{(2)}_r = 0, \\ & \Gamma^{(1)}_\vartheta = \Gamma^{(2)}_\vartheta = \tfrac{\alpha}{2} i\sigma^z. \end{aligned} \tag{16}$$

As is usual with two-body problems, the center of mass motion coordinates and relative motion coordinates can be separated with help of the following expressions that obtained for equal massive two particles [75],

$$\begin{aligned} & \mathbf{R} = \frac{\mathbf{x}_1 + \mathbf{x}_2}{2}, \qquad \mathbf{r} = \mathbf{x}_1 - \mathbf{x}_2, \\ & \mathbf{x}_1 = \frac{1}{2}\mathbf{r} + \mathbf{R}, \qquad \mathbf{x}_2 = -\frac{1}{2}\mathbf{r} + \mathbf{R}, \\ & \mathbf{x}_1 = \left\{ t^{(1)}, r^{(1)}, \vartheta^{(1)} \right\}, \quad \mathbf{x}_2 = \left\{ t^{(2)}, r^{(2)}, \vartheta^{(2)} \right\}, \\ & \partial^{(1)}_{x_\lambda} = \partial_{x_\lambda} + \frac{\partial_{R_\lambda}}{2}, \qquad \partial^{(2)}_{x_\lambda} = -\partial_{x_\lambda} + \frac{\partial_{R_\lambda}}{2}, \\ & \partial_\lambda = \left(\frac{\partial_t}{c}, \partial_r, \partial_\vartheta \right), \quad (\lambda = 0,1,2.). \end{aligned} \tag{17}$$

We assume that the two Dirac particles interact via Coulomb type interparticle interaction potential which is described by the following components of the electromagnetic vector potential A_λ,

$$A_t = V(\mathbf{x}_1 - \mathbf{x}_2), \quad A_r = 0, \quad A_\vartheta = 0. \tag{18}$$

3. Radial equations

In a general $2+1$ dimensional spacetime background, the fully-covariant two body Dirac equation gives a 4×4 dimensional matrix equation, since the spacetime independent Dirac matrices are reduced to the Pauli spin matrices. For a fermion-antifermion pair, each of mass m_f, that they interact via an attractive Coulomb type interparticle interaction potential in the static cosmic string spacetime background, the fully covariant two body Dirac equation can be explicitly written as in the following form,

$$\gamma^{t^{(1)}}\otimes\gamma^{t^{(2)}}\left[\frac{\partial_t^{(1)}}{c}+\frac{\partial_t^{(2)}}{c}+iV\left(\mathbf{x}_1-\mathbf{x}_2\right)\right]\Psi+\gamma^{r^{(1)}}\otimes\gamma^{t^{(2)}}\partial_r^{(1)}\Psi$$
$$+\gamma^{t^{(1)}}\otimes\gamma^{r^{(2)}}\partial_r^{(2)}\Psi+\gamma^{\vartheta^{(1)}}\otimes\gamma^{t^{(2)}}\partial_\vartheta^{(1)}\Psi+\gamma^{t^{(1)}}\otimes\gamma^{\vartheta^{(2)}}\partial_\vartheta^{(2)}\Psi$$
$$-\left[\gamma^{\vartheta^{(1)}}\Gamma_\vartheta^{(1)}\right]\otimes\gamma^{t^{(2)}}\Psi-\gamma^{t^{(1)}}\otimes\left[\gamma^{\vartheta^{(2)}}\Gamma_\vartheta^{(2)}\right]\Psi$$
$$+i\frac{m_fc}{\hbar}\left[\mathbf{I}_2\otimes\gamma^{t^{(2)}}+\gamma^{t^{(1)}}\otimes\mathbf{I}_2\right]\Psi=0, \tag{19}$$

where $V\left(\mathbf{x}_1-\mathbf{x}_2\right)$ stands for the interparticle interaction potential. Provided that center of mass momentum is a constant of motion, the symmetry of the background and the type of the interaction between the fermions allow the factorization of the solution of the equation given above, since there is no relative time differences between these fermions (see Eq. (17) and Eq. (19) for ∂_{x_0}). Therefore, we can define the bi-spinor Ψ as in the following,

$$\Psi\left(t,\vec{r},\vec{R}\right)=e^{-i\omega t}e^{is\vartheta}e^{i\mathbf{K}.\mathbf{R}}\begin{pmatrix}\phi_1(r)\\ \phi_2(r)\\ \phi_3(r)\\ \phi_4(r)\end{pmatrix}, \tag{20}$$

if and only if the interaction is time-independent. In Eq. (20), $\mathbf{K}$ relates with the spatial momentum of the center of mass motion ($\hbar\mathbf{K}$), the components of $\mathbf{R}$ correspond to the spatial coordinates of center of mass motion, r is radial distance between the fermions, ω is total frequency of the system and s stands for total spin quantum number of the system that under scrutiny. One of the most interesting case is that of zero total center of mass momenta, $\hbar\mathbf{K}=0$, where the two particles must be have opposite momenta. This is relatively simple case, but any pairing effect becomes important in this case. For a static fermion-antifermion pair ($K=0$), via Eq. (16), Eq. (17), Eq. (18), Eq. (19) and Eq. (20), one can arrive at a system of coupled equations that reads,

$$\Phi(r)\,\psi_1(r)-2b\psi_2(r)-2\left(\partial_r-\tfrac{1}{r}\right)\psi_3(r)+\tfrac{4s}{\alpha r}\psi_4(r)=0,$$
$$\Phi(r)\,\psi_2(r)-2b\psi_1(r)=0,$$
$$\Phi(r)\,\psi_3(r)+2\left(\partial_r-\tfrac{1}{r}\right)\psi_1(r)=0,$$
$$\Phi(r)\,\psi_4(r)+\tfrac{4s}{\alpha r}\psi_1(r)=0, \tag{21}$$

where,

$$\psi_1(r)=\phi_1(r)+\phi_4(r),\quad \psi_2(r)=\phi_1(r)-\phi_4(r),$$
$$\psi_3(r)=\phi_2(r)-\phi_3(r),\quad \psi_4(r)=\phi_2(r)+\phi_3(r),$$
$$\Phi(r)=\tfrac{\omega}{c}-V(r),\quad b=\tfrac{m_fc}{\hbar}. \tag{22}$$

4. Energy spectrum

The solution of the equation system for $\psi_1(r)$ gives the following wave equation,

$$\partial_r^2\psi_1(r)-\left[\tfrac{2}{r}+\kappa(r)\right]\partial_r\psi_1(r)$$
$$+\left[\tfrac{\Phi(r)^2-B^2}{4}-\tfrac{4s^2}{\alpha^2r^2}+\tfrac{\kappa(r)}{r}+\tfrac{2}{r^2}\right]\psi_1(r)=0,$$
$$B=2b,\quad \kappa(r)=\tfrac{\partial_r\Phi(r)}{\Phi(r)}. \tag{23}$$

For an attractive Coulomb type interparticle interaction potential, $V(r)=-\frac{\alpha_c}{r}$, the solution function of the wave equation can be defined as follows,[1]

$$\psi_1(r)=\overline{\psi}(r)\,e^{-\frac{\sqrt{B^2c^2-\omega^2}}{2c}}r^{1+\frac{\zeta}{2}},$$

and a dimensionless independent variable that reads $y=-\frac{\omega r}{\alpha c}$ can be introduced. Afterwards, one can obtain the wave equation as in the following,

$$\partial_y^2\psi(y)+\left[\Omega+\frac{1+\zeta}{y}+\frac{1+\varepsilon}{y-1}\right]\partial_y\psi(y)$$
$$+\left[\frac{\Delta}{y}+\frac{\Lambda}{y-1}\right]\psi(y)=0, \tag{24}$$

which is the simplest uniform shape of Heun differential equation. This equation has two regular singular points at $y=0$ and $y=1$ and one irregular singular point at spatial infinity. Around the regular singular point $y=0$, the solution functions of Eq. (24) form linearly independent solutions if and only if ζ is not an integer. In Eq. (24), the used parameters and the relations between among them are given as follows,

$$\delta=\Delta+\Lambda-\Omega\tfrac{(\zeta+\varepsilon+2)}{2},\quad \delta=-\tfrac{\alpha_c^2}{2},$$
$$\Omega=\tfrac{\alpha_c}{\omega}\sqrt{B^2c^2-\omega^2},\quad \zeta=-\tfrac{\sqrt{16s^2-\alpha_c^2\alpha^2}}{\alpha},\quad \varepsilon=-2,$$
$$\Pi=(1+\zeta)\tfrac{\Omega}{2}-\tfrac{(\zeta+\varepsilon+\zeta\varepsilon)}{2}-\Delta,\quad \Pi=1+\tfrac{\alpha_c^2}{2},$$

and there is no a physical reason that impose the ζ must be an integer. Therefore, the linearly independent solutions of Eq. (24) are found in terms of Heun functions (H_C) as follows,

$$\psi(y)=Q_1H_C(\Omega,\zeta,\varepsilon,\delta,\Pi,y)+\frac{Q_2}{y^\zeta}H_C(\Omega,-\zeta,\varepsilon,\delta,\Pi,y), \tag{25}$$

in which Q_1 and Q_2 are arbitrary constants and asymptotically acceptable physical solution is the latest. Hence, we can get that $Q_1=0$. However, the asymptotically acceptable physical solution function can only be reduced to polynomial of degree $n\geq 1$ with respect to the variable y, provided that the following necessary condition is satisfied [93],

$$(n+1+\frac{\zeta+\varepsilon}{2})\Omega=-\delta.$$

At last, we arrive to a non-perturbative energy spectrum that reads,

$$E=\pm 2m_fc^2\sqrt{1-\frac{\alpha_c^2}{4\left(n^2-\frac{n}{\alpha}\sqrt{16s^2-\alpha_c^2\alpha^2}+\frac{4s^2}{\alpha^2}\right)}},$$
$$\alpha=\left(1-\tfrac{4\mu_sG}{c^2}\right)\in(0,1],\quad n\geq 1. \tag{26}$$

In this non-perturbative energy spectrum, n, E, m_f, α_c, s and α represent to the principal quantum number, total energy of the system, mass of the fermions, coupling strength constant, total spin quantum number of this composite system and the angular deficit parameter of the spacetime background that relates with the linear mass density μ_s of static cosmic string, respectively.

[1] To avoid any confusion the coupling strength constant was taken as α_c.

In the usual case ($\alpha = 1$) that corresponds to the flat Minkowski case, the non-perturbative energy spectrum given in Eq. (26) becomes,

$$E = 2m_f c^2 \sqrt{1 - \frac{\alpha_c^2}{4\left(n^2 - n\sqrt{16s^2 - \alpha_c^2 + 4s^2}\right)}}. \tag{27}$$

Via power series expansion method applied to Eq. (27), the "usual" energy spectrum can be acquired as follows,

$$E \approx \pm 2m_f c^2 \left\{1 - \frac{\alpha_c^2}{8} + \frac{\alpha_c^4}{128} - \frac{3\alpha_c^6}{4096}\right\}, \tag{28}$$

for $n = 1, s = 1$ quantum state of the composite system that under scrutiny. In Eq. (28), the first term corresponds to the total rest mass energy of the system formed by equal massive two Dirac particles ($2m_f c^2$) and the second term ($\propto \alpha_c^2$) gives the well-known non-relativistic ground state binding energy ($-m_f c^2 \frac{\alpha_c^2}{4}$) of such a system [77,78]. The obtained both energy spectrums (Eq. (26) and Eq. (27)) can be used in order to determine the shifts in the binding energy level of a general system formed by a fermiyon-antifermiyon pair that they interact via an attractive Coulomb type interparticle interaction potential. An experimental measurement of the binding energy of an o-Ps is relatively easy, since its lifetime is remarkable longer than the other fermion-antifermion systems. Even though we do not know the initial quantum state at which an electron and positron couples, it is also trivial that at some stage this composite system will reach to "ground state" where the wave functions of these fermions overlap and the fermions annihilate each other [77,78]. Hence, to check the findings, we can apply the result in Eq. (28) to an o-Ps ($s = 1$) in ground state [77]. The Eq. (28), which includes the relativistic correction, gives ground state binding energy ($E_{n,s}$) of an o-Ps as follows,

$$E_{1,1}\left(\alpha_c, \alpha = 1\right) = -6,802822706 \text{ eV}. \tag{29}$$

This binding energy value is in a good agreement with the current literature (for details see [78]).

For $n = 1, s = 1$, expanding Eq. (26) in a series of powers of α_c we have the following leading terms (for + signature),

$$E_{1,1}\left(\alpha_c, \alpha < 1\right) \approx 2m_f c^2 - m_f c^2 \frac{\alpha_c^2\alpha^2}{4(2-\alpha)^2}$$
$$+ m_f c^2 \frac{\alpha_c^4\alpha^4(2\alpha-1)}{64(2-\alpha)^4} - m_f c^2 \frac{\alpha_c^6\alpha^6\left(4-12\alpha+12\alpha^2-\alpha^3\right)}{2048(2-\alpha)^6},$$
$$\alpha_c = \frac{e^2}{4\pi\epsilon_0\hbar c}, \tag{30}$$

which tell us what the dependence of each term with the parameter α is. It can be seen that the angular deficit parameter ($\alpha < 1$) of the spacetime background decreases the magnitude of ground state binding energy of such a system. Actually, this result is not very surprising. It has been shown that in Grand Unified Theories the cosmic strings have radius (r_s) $r_s \sim 10^{-29}$ cm which is, of course, much smaller than Bohr radius of the Ps, $\sim 10^{-8}$ cm [94]. Moreover, it has been obtained that in the physically interesting limit ($\alpha \to 1$), there is repulsive electrostatic long-range and finite interaction between a charged particle (massive) and cosmic string [19]. This interaction, of course, associates with the spacetime topology of cosmic string [19,94]. We can consider this repulsive interaction, which obtained in [19], as $\nu(r) \approx 5G\mu_s/c^2 r$ for each charged particle whether $e < 0$ or $e > 0$, since we assume that the center of mass of the o-Ps locates at the origin of the static cosmic string spacetime background. This effect [19], of course, naturally appears in the obtained energy spectrum (see Eq. (30)). It would be quite appropriate to say that the effects of the gravitational field produced by the cosmic string on an o-Ps system will decrease as the cosmic string moves away from the particles and increase as it gets closer, since $\nu(r) \propto 1/r$ [19]. Here, we can think that the strength of repulsive interaction between the static cosmic string and each particle increases as the particles close to cosmic string. Therefore, the alterations in binding energy can change according to the distance between the cosmic string and particles. We can consider this effect in terms of effective string tension values. Our results show that the alterations in binding energy and total annihilation energy becomes more noticeable for relatively big μ_s values. If we consider now the collapsing process of an o-Ps system (as far as we know), total collapsing duration will be longer than $\alpha = 1$ case, since the strength of the effective attractive force between the fermion-antifermion pair becomes weaker than $\alpha = 1$ case. It is obvious that the repulsive interaction between the particles and cosmic string leads to relatively longer lifetime of any composite structure formed by a fermion and its own antimatter counterpart an antifermion pair interacting through attractive Coulomb type interparticle interaction potential. It is well-known that the decay rates of such unstable composite structures are determined according to the coupling strength between the particles (for more details see the review in [78]). Therefore, according to the main results in here, one can also infer that the existence of gravitational field produced by static cosmic string change formation times and total lifetimes of o-Ps like unstable composite structures. Even though there is no a direct evidence about the existence of such structures today, some indirect evidences indicate that cosmic strings could really be exist [95]. We should note that Eq. (30) not only gives the energy shift but also provides to see how the cosmic string effect the total annihilation energy transmitted by the resulting photons (see [91]). Eq. (30) shows that the total annihilation energy (E_{ann}) that transmitted by the resulting photons carry information about the cosmic string spacetime topology, even if the alteration can be very small. It is important to notice that the effect of the spacetime background on the binding energy level of o-Ps can be seen in all order of the coupling strength constant (see Eq. (30)), even in the non-relativistic binding energy term. This result supports the findings in [7]. Additionally, the terms including α_c with the angular deficit parameter α of the spacetime background in the Eq. (30) can be considered as an effective coupling strength constant that determined by the topological feature of the spacetime background, since the angular deficit of the cosmic string spacetime background causes a screening effect. Therefore, our results imply that the decay rates [96] of such unstable systems are changed due to the interactions of particles with the cosmic string. Also, in principle, our findings impose that an o-Ps or a triplet-exciton [75] system can also be used as a probe in order to determine some topological properties of any $2D$ samples (see also [97]).

Eq. (28) and Eq. (30) lead to following expression that allows to calculate the shift in the ground state binding energy of an o-Ps,

$$\left|\triangle E_{1,1}\left(\alpha_c, \alpha < 1\right)\right| \approx$$
$$m_f c^2 \alpha_c^2 \left\{\frac{1-\alpha}{(2-\alpha)^2} - \frac{\alpha_c^2}{64}\left(1 - \frac{2\alpha-1}{(2-\alpha)^4}\right)\right\}$$
$$+ m_f c^2 \frac{\alpha_c^6}{2048}\left\{3 - \frac{\alpha^6\left(4-12\alpha+12\alpha^2-\alpha^3\right)}{(2-\alpha)^6}\right\}, \tag{31}$$

which, of course, vanishes when $\alpha = 1$. This result clearly shows that the shift in ground state binding energy of o-Ps becomes more noticeable for big μ_s values (see Fig. 1). For a predicted value for GUT string, $\alpha = 1 - 10^{-6}$, we obtain the shift as 27,2 μeV if $\alpha_c = 1/137,06$ and m_f is mass of an electron (m_e). This means that the normal oscillation mode of the system in ground state is shifted as $\sim 6,577$ GHz (see also [6]). Therefore, in principle, it is seem to be possible to measure this shift with the current techniques in use in experimental Ps physics (for more details see the review in [78]).

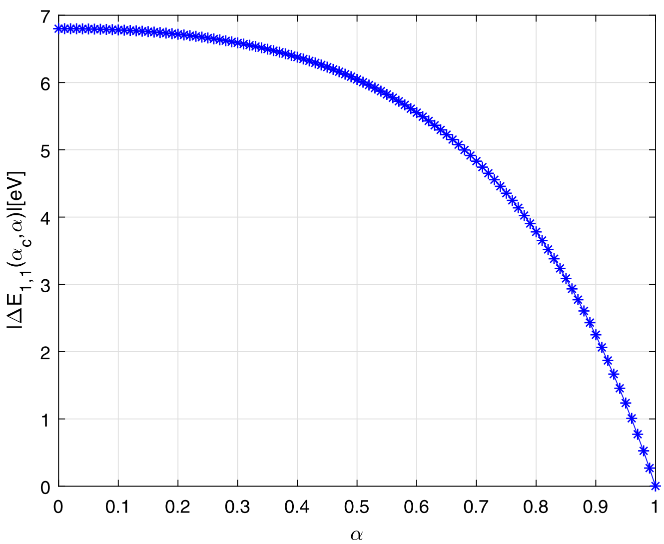

Fig. 1. Dependence of the energy shift on angular deficit parameter of the static cosmic string spacetime background for $\alpha_c = 1/137,06$.

Furthermore, this means that the altered total annihilation energy transmitted by the resulting photons will be bigger by 27, 2 μeV than $\alpha = 1$ case if $\alpha = 1 - 10^{-6}$ and $N = 3$. Now, for practical calculations, with the help of Eq. (28), Eq. (30) and Eq. (31), we can write an explicit expression that gives the effected energy,

$$E_{ann}(\alpha_c, \alpha < 1)/N = 2m_e c^2/N - |E_{1,1}(\alpha_c, \alpha = 1)|/N + |\triangle E_{1,1}(\alpha_c, \alpha < 1)|/N, \tag{32}$$

transmitted by each resulting photon, if there exist no a symmetry violation [78,79,86]. Therefore, if there exist an interaction between a cosmic string and an electron-positron pair the energy transmitted by each annihilation photon [91] will be bigger than the energy corresponding to the usual case ($\alpha = 1$). This result also shows that gamma photons hitting sensitive detectors outside the earth's atmosphere can be distinguished, in principle.

5. Conclusion and discussion

Here, in order to determine the influence of static cosmic string on the energy of Positronium-like atoms, we use the generalized form of a fully-covariant two-body Dirac-Coulomb type equation that derived from Quantum Electrodynamics with the help of Action principle. Also, in our model, we consider that the both fermions interact with the static cosmic string spacetime background and they interact with each other (also see [7]). As is usual with two-body problems, we define relative motion coordinates and center of mass motion coordinates and then we arrive at a system of coupled equations provided that the center of mass is rest at the origin of the static cosmic string spacetime background. By an exact solution of the equation system we obtain a spectrum in energy domain. This energy spectrum depends on the angular deficit parameter of the static cosmic string spacetime background. We check the obtained results by reducing the spectrum into the "usual" case where the angular deficit of the cosmic string spacetime background vanishes. The arrived "usual" spectrum, which includes relativistic correction, is applied to an o-Ps in ground state. Here, we deal with an o-Ps system, since it has remarkable longer lifetime than the other unstable fermion-antifermion systems. Furthermore, the o-Ps system can be regarded as a useful probe to detect any physical properties of spacetime backgrounds. This is because of binding energy and total lifetime of an o-Ps besides to the annihilation photons that appear when the annihilation event occurs can give enlightening information about the electrical, morphological and topological properties of the spacetime backgrounds. This may provide an experimental advantage, in principle. The energy spectrum in Eq. (28) leads a ground state binding energy value (see Eq. (29)) that agrees well with the current literature [78]. This provide us to determine the energy shift stemming from the cosmic string spacetime topology on the binding energy of an o-Ps. In order to determine the static cosmic string spacetime topology on the o-Ps system we compare the both energy spectrums in Eq. (28) and in Eq. (30). This leads to an expression, given in Eq. (31), that can be used to determine the shift in the ground state binding energy of such a fermion-antifermion pair and it is useful for practical calculations for different angular deficit parameter values (see Eq. (31)). The dependence of the shift in energy on the string tension can be seen in Fig. 1, in which, of course, the energy shift closes to zero as the spacetime topology closes to flat topology. In principle, the decrease in the magnitude of ground state binding energy of the system that under scrutiny becomes more noticeable for a static cosmic string that has very big linear mass density values (in the mathematically defined interval) (see Fig. 2). For a predicted value of α for GUT string [7,9,28], $\alpha = 1 - 10^{-6}$, we calculate the shift in the ground state binding energy of the system and we arrive at an energy shift value. We obtain this shift as 27, 2 μeV for an o-Ps system in ground state. This means that the normal oscillation mode of an o-Ps in ground state is shifted approximately as $\sim 6,577$ GHz and it seems to be possible to measure this shift with current techniques in experimental Ps physics, since the similar most precise measurements can be performed at MHz (∼neV) level sensitivity (see review in [78]). Moreover, we obtain an expression for practical calculations of the altered total annihilation energy transmitted by the annihilation photons. This expression is given in Eq. (32) in terms of the both α and number of annihilation photons N. With the help of Eq. (32), we can see that the alteration in the energy transmitted by each annihilation photon increases as approximately 9, 066 μeV if $\alpha \sim 1 - 10^{-6}$ and $N = 3$. Furthermore, we calculate the alterations in ground state binding energy of o-Ps and accordingly the effected total annihilation energy values for several α values that vary from $1 - 10^{-6}$ to $1 - 10^{-13}$ [28,29,36,41]. The results can be found in Table 1. These results indicate that the shift in ground state binding energy of o-Ps may be measurable with current techniques even for $\alpha \sim 1 - 10^{-11}$ value, in principle (see [78]). However, it seems not possible today to measure this small alterations in the energy transmitted by each annihilation photon, since the interaction of a gamma photon with a scintillator occurs mostly through the Compton scattering [79,86,89]. In principle, it may be possible to measure very sensitively the energy of an arbitrary photon with semi-conductor detectors, but a precise measurement of the energy transmitted by an annihilation photon is not seems to be possible today [79,86]. Furthermore, our results also imply that the formation time and total lifetime of o-Ps can be changed by the cosmic string spacetime topology. This is because of the fact these durations depend on both the interaction between the cosmic string and particles and the coupling strength between the fermion-antifermion pair. In Eq. (30), we should note that the angular deficit of static cosmic string spacetime background causes a screening effect. Due to this repulsive interaction between the cosmic string and particles, we may infer that the static cosmic string can alter not only total lifetime of an o-Ps but also it may effect the formation time somehow. We may associate the effect of cosmic string on an o-Ps atom with the distance between the cosmic string and particles whether $e > 0$ or $e < 0$. As it we discussed before, we can think that the "effective" string tension value will increase when the particles close to the static cosmic string [19]. We should notice that the effect of static

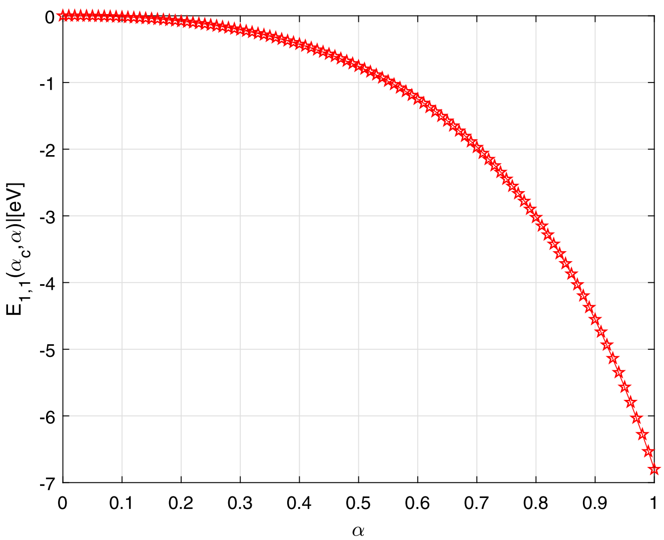

Fig. 2. Dependence of the binding energy on angular deficit parameter of the static cosmic string spacetime background for $\alpha_c = 1/137, 06$.

Table 1
The altered ground state binding energy and accordingly alterations in the total annihilation energy values for several α values. Here, $\alpha_c = 1/(137, 06)$.

$1-\alpha$	$\mid \triangle E_{1,1}(\alpha_c, \alpha < 1) \mid$	$\triangle E_{ann}(\alpha_c, \alpha < 1)$
10^{-6}	27, 20161 μeV	27, 20161 μeV
10^{-7}	2, 72016 μeV	2, 72016 μeV
10^{-8}	0, 27201 μeV	0, 27201 μeV
10^{-9}	27, 20161 neV	27, 20161 neV
10^{-10}	2, 72016 neV	2, 72016 neV
10^{-11}	0, 27201 neV	0, 27201 neV
10^{-12}	27, 20161 peV	27, 20161 peV
10^{-13}	2, 72016 peV	2, 72016 peV

cosmic string on an o-Ps system will be relatively more noticeable when the string tension value increases (see Fig. 1, Fig. 2 and Table 1). Of course, the o-Ps system can be considered as only one of the various physical probing mechanism, in principle, in order to detect the effect of cosmic strings (see also [28,29,36,41]). As a result, in principle, our findings show that the o-Ps has a potential to prove the existence of a static cosmic string, since these objects can alter not only binding energy and total lifetime of an o-Ps but also total annihilation energy transmitted by the resulting photons. The results in Table 1 show that an o-Ps system may provide enlightening information about the existence of static cosmic strings if we concentrate mainly on observational consequences. In addition, the other conclusion is that not only the binding energy and total lifetime of o-Ps-like unstable systems, but also the production of gamma photons carrying different energy may differ for different points in the universe if cosmic strings are really exist.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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