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# He-star donor AM CVn stars and their progenitors as LISA sources

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## ABSTRACT

*Context.* Ultracompact cataclysmic variables (CVs) of the AM CVn type are deemed to be important verification sources for the future space gravitational wave detectors such as the Laser Interferometer Space Antenna (LISA).
*Aims.* We model the present-day Galactic population of AM CVn stars with He-star donors. Such a population has long expected to exist, though only a couple of candidates are known.
*Methods.* We applied the hybrid method of binary population synthesis (BPS) which combines a simulation of the population of immediate precursors of AM CVn stars by a fast BPS code with subsequent tracking of their evolution by a full evolutionary code.
*Results.* The model predicts that the present birthrate of He-donor AM CVn stars in the Galaxy is $4.6 \times 10^{-4}$ yr$^{-1}$ and the Galaxy may harbour $\simeq 112\,000$ objects of this class which have orbital periods $P \lesssim 42-43$ min. The foreground confusion limit and instrumental noise of LISA prevent the discovery of longer periods systems in gravitational waves. We find that about 500 He-star AM CVns may be detected by LISA with signal-to-noise ratio $(S/N) > 5$ during a 4 yr mission. Within 1 Kpc from the Sun, there may exist up to 130 He-star AM CVns with the periods in the same range, which may serve as verification binaries, if detected in the electromagnetic spectrum. In the Milky Way, there are also $\simeq 14\,800$ immediate precursors of AM CVn stars. They are detached systems with a stripped low-mass He-star and a white dwarf companion, out of which about 75 may potentially be observed by LISA during its mission.

**Key words.** gravitational waves – binaries: close – novae, cataclysmic variables – stars: evolution – subdwarfs – white dwarfs

## 1. Introduction

AM CVn stars are a small group of ultracompact cataclysmic binaries (CVs) with He-rich (He white dwarf or stripped He star) donors and carbon-oxygen (CO) white dwarf (WD) accretors. Currently, about 70 definite and candidate AM CVn stars are known (see Ramsay et al. 2018, Table 1; Wevers et al. 2016; Burdge et al. 2020; Kato & Kojiguchi 2021; and van Roestel et al. 2021). Estimated orbital periods of AM CVns range from 5.4 min to 67.8 min. Their evolution is driven by gravitational waves radiation (GWR, Paczyński 1967). Detailed reviews of them were published by Solheim (2010) and Ramsay et al. (2018).

Lipunov & Postnov (1987) estimated that binary WDs may be the strongest GWR sources detected using lasers in space. Hellings (1996) recognised that AM CVn stars are also expected to be detectable by space GWR antennas. Along with massive black hole binaries and extreme or intermediate mass ratio inspirals, compact binaries are among the main objects expected to be observed by the planned space GWR observatories such as the Laser Interferometer Space Antenna (LISA, see Amaro-Seoane et al. 2022, and references therein), Taiji, and TianQin (Gong et al. 2021).

The special importance of AM CVn stars for GWR astrophysics stems from the fact that some of them belong to the so-called verification binaries (or guaranteed sources) for space GWR detectors (S. Phinney 2001, unpublished; Stroeer & Vecchio 2006; Nelemans 2009; Kupfer et al. 2018; and Huang et al. 2020). It is expected that they will be discovered rather soon after the beginning of the missions because of

a strong signal due to the proximity to the Sun; additionally, thanks to the knowledge of mass ratios of components, orbital periods and distances from observations in the electromagnetic spectrum, they will serve for testing and calibrating detectors.

Theoretical modelling and observations suggest three formation channels for AM CVn stars. A detailed analysis of them may be found, for instance, in Nelemans (2005) and Postnov & Yungelson (2014). Here, we recall only basic details of these channels.

The double-degenerate (DD, Paczyński 1967) channel envisions formation of a binary harbouring a CO WD and a less massive He WD companion. If the system is sufficiently tight, the He WD may overflow its Roche lobe in a time shorter than the Hubble time due to angular momentum loss (AML) via GWR, and under certain conditions stable mass exchange is expected to commence (see also Tutukov & Yungelson 1979; Nelemans et al. 2001a; Marsh et al. 2004). In the DD channel, the binary may evolve from $P_{orb} \approx (2-3)$ min at the RLOF to $P_{orb} \sim 1$ h.

In the single degenerate (SD) channel (Faulkner et al. 1972 and Savonije et al. 1986), a stripped 'semidegenerate' He-star may accompany CO WD. In this case, RLOF by the progenitor of the donor should occur before He exhaustion in its core (Iben 1987). At the contact, $P_{orb}$ is several dozen minutes, depending on the masses of components and the extent of exhaustion of He in the core of the future donor. The binary first evolves to $P_{min} \sim$ 10 min, which is attributable to the change in sign of the $M - R$ relation when the thermal timescale of the donor becomes substantially longer than the timescale of angular momentum loss by GWR (see Yungelson 2008, for more details). The scenario of

formation of He-donor AM CVn stars was extensively discussed, for example, by Tutukov & Yungelson (1996), Nelemans et al. (2001a), Postnov & Yungelson (2006, 2014), Solheim (2010), Götberg et al. (2020), and Bauer & Kupfer (2021).

Following Nelemans et al. (2001a), the objects that formed via an SD channel are classified as AM CVns after passing $P_{min}$. However, as noted by Bauer & Kupfer (2021), already before reaching $P_{min}$, relatively high-mass subdwarf donors may transfer matter from $(0.2$–$0.3) M_\odot$ outer He shells that contain the ashes of CNO burning. In this case, accretion disks spectra resemble the spectra typical for AM CVns.

In the SD channel, when the donor mass decreases to $(0.02$–$0.03) M_\odot$, it begins to cool, its thermal timescale becomes shorter than the mass-loss timescale, it becomes more degenerate, and the $M - R$ relation gradually merges with that for the cool WD (see Wong & Bildsten 2021, for a detailed discussion). The formation of a particular AM CVn system via a DD or SD channel may be inferred from the abundances of elements and their ratios, especially N/He, N/C, N/O, O/He, and O/C (Nelemans et al. 2010); however, the derivation of abundances is a formidable task.

The evolved donor channel (Tutukov et al. 1985) is, in fact, the standard scenario of formation of CVs, in which the donor overflows the Roche lobe when the hydrogen abundance in the core becomes $\lesssim 0.1$. The donor becomes a star with an almost H-depleted core and a thin H envelope, which later may be lost. Hypothetical AM CVn stars, which formed via this channel, typically have $P_{orb} \gtrsim 30$ min and very rarely may evolve to $P_{orb} \sim 10$ min (Podsiadlowski et al. 2003; Goliasch & Nelson 2015; Kalomeni et al. 2016; Yungelson 2018; Liu et al. 2021). Strictly speaking, this channel does not produce 'classic' AM CVn stars, since an abundance of H at the surface of the donors hardly decreases below $10^{-4}$–$10^{-3}$, while spectral lines of H should be observed for an abundance exceeding $10^{-5}$ (Nagel et al. 2009). The parameters of the well-studied system Gaia14aae are apparently matched best by the evolved donors channel, but no H is observed in its spectrum (Green et al. 2018).

For completeness, we note that about a dozen so-called HeCVs with an enhanced He/H abundance ratio have $P_{orb}$ below conventional $P_{orb,min}$ (Breedt et al. 2012; Breedt 2015; Kennedy et al. 2015; Lee et al. 2022). Some of them may finish their evolution in the sub-minimum periods' range or bounce (change the sign of $\dot{P}$) and return to $P_{orb} \gtrsim 70$–$80$ min, retaining H in the envelopes. Envelopes of other HeCVs may contract after the mass of them drops below a certain minimum. Then they experience a detached stage of evolution and join the family of DD AM CVns, due to continuing angular momentum losses. The rest of the H envelopes may be expected to be lost very soon after RLOF, when $\dot{M}$ is high.

Observational data suggest that the population of AM CVn stars is totally dominated by the objects that formed via the DD channel. Currently, only several candidate AM CVn stars with a He-star donor are known – SDSS J0926+3624 (Copperwheat et al. 2011), ZTFJ1637+49, and ZTFJ0220+21 (van Roestel et al. 2021).

The first binary population synthesis (BPS) studies of AM CVn stars, including both DD and SD channels, were performed by Tutukov & Yungelson (1996), and Nelemans et al. (2001a, 2004). Later studies, as a rule, were aimed at DD. This is mainly related to the apparent scarcity of observed He-donor AM CVns and the badly understood consequences of accretion of He onto WDs at the expected accretion rates, especially for rotating WDs. Even so, the studies of He-star AM CVns may be important; this is because if they exist, but they are simply not recog-

nised due to selection effects, they may increase the sample of LISA verification binaries.

For the present paper, we modelled the Galactic population of the stripped He stars with WD companions which later form AM CVn stars with He-star donors. We modelled the formation of them and followed their evolution to RLOF and through the mass-transfer stage to the instant when the mass of accretors reached $M_{Ch}$ or the mass of the donor decreased to $(0.02 - 0.03) M_\odot$, the limit imposed by the evolutionary code used. We estimate the number of AM CVn stars and their direct precursors and evaluate the number of AM CVn stars that may be detected in GW with a signal-to-noise ratio $(S/N) > 5$ by space-detector LISA during a 4 yr long mission. We present our model in Sect. 2. In Sect. 3 the results of the modelling are presented. We discuss the results and summarise our conclusions in Sect. 4.

## 2. The model

### 2.1. Population synthesis

For the modelling, we applied the hybrid BPS method (Nelson 2012; Chen et al. 2014; Goliasch & Nelson 2015): the evolution of binaries up to the formation of precursors of AM CVn systems, WDs accompanied by He stars, was computed by means of an updated fast analytic BPS code BSE (Hurley et al. 2002)[1], while their further evolution was simulated using a grid of precomputed full evolutionary tracks. The advantage of hybrid population synthesis over other algorithms of BPS, implemented, in particular, in BSE, is that the latter are usually based on analytic approximations to the evolutionary tracks for relatively well-explored single stars. Description of the evolution of close binaries, for which systematic studies are much scarcer, since they are more complicated, is often based on 'educated guesses'. This is particularly true for the second RLOF in the systems with compact accretors. Furthermore, as test runs show, BSE does not reproduce the evolution of semi-detached systems with He donors, which gradually become more degenerate.

The crucial assumptions were as follows. The initial mass function (IMF) of primary components followed the Salpeter law ($dN/dM \sim M^{-2.35}$) in the mass range $1 \leq M_1/M_\odot \leq 100$. A flat distribution of mass ratios of components $q = M_2/M_1 \leq 1$ in the [0.1,1] range (Kraicheva et al. 1989) was assumed[2]. The stellar binarity rate was set to 50%, that is to say two-thirds of all stars are binary components. The initial distribution of close binaries over orbital periods was accepted after Sana et al. (2012): $f(\log P_{orb}) \propto \log P_{orb}^{-0.55}$. For common envelopes (CE) treatment, we applied the energy-balance formalism of Webbink (1984) and de Kool (1990) with 'CE efficiency' parameter $\alpha_{ce}=1$ and binding energy parameter $\lambda$ values from Loveridge et al. (2011).

Evolutionary tracks for tracing further evolution were computed with the updated P. P. Eggleton evolutionary code STARS (Eggleton 1971, year 2006 version, priv. comm.); for more details, readers can refer to Yungelson (2008). All computations were carried out for metallicity $Z = 0.02$. The STARS code lacks opacity tables that would allow one to compute models with

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[1] After the update, our version of BSE did in fact become rather similar to the latest published version of this code (Banerjee et al. 2020).
[2] Korol et al. (2017) found that the flat distribution provides better agreement of the model with the number of WDs observed in the immediate vicinity of the Sun than often used $f(q) \propto q^{-1}$.

masses $\lesssim 0.02\,M_\odot$ corresponding to $P_{\rm orb} \gtrsim$ (42–43 min)[3]. By coincidence, at $P_{\rm orb}$ which slightly exceeds 40 min, it is impossible to detect a GWR signal of AM CVn stars by LISA, due to the foreground and antenna noise.

### 2.2. Computation of gravitational waves' strain

Via BPS, we found the masses of WD ($M_{\rm WD}$), stripped He stars ($M_{\rm He}$), and orbital periods just after the end of RLOF or common-envelope phases $P_i$ for binaries, in which a He star may later initiate stable mass transfer. For the mission lifetime of LISA $T = 4$ yr, the characteristic strain of an inspiraling binary can be calculated as (Thorne 1987)

$$h_{\rm c} \approx 3.75 \times 10^{-19} \left(\frac{f_{\rm gw}}{1\,{\rm mHz}}\right)^{7/6} \left(\frac{\mathcal{M}}{1\,M_\odot}\right)^{5/3} \left(\frac{1\,{\rm Kpc}}{d}\right), \tag{1}$$

where $f_{\rm gw}[{\rm Hz}] = 2/P_{\rm orb}[{\rm s}]$ is the GW frequency, $d$ is the distance to the object, and

$$\mathcal{M} = \frac{(M_{\rm WD} M_{\rm He})^{3/5}}{(M_{\rm WD} + M_{\rm He})^{1/5}} \tag{2}$$

is the so-called chirp mass.

Helium-donor AM CVn stars are young objects ($t \lesssim 2$ Gyr) and belong predominantly to the thin disk population (Ramsay et al. 2018). Therefore, to determine $d$, we assumed that the space distribution of progenitors of AM CVn stars in the Galaxy may be described as

$$\rho \propto \exp\left(-\frac{R}{R_d}\right) {\rm sech}^2\left(\frac{z}{z_d}\right), \tag{3}$$

where $1 \leq R \leq 16$ Kpc is the galactocentric radial distance, $R_d = 2.5$ Kpc is the characteristic radial scale, $z$ is the distance to the Galactic plane, and $z_d = 0.3$ Kpc is the characteristic scale height of the disk (Jurić et al. 2008). We did not consider the inner region of the Galaxy with $R \leq 1$ Kpc, hosting a 'bulge/bar', where young stars are absent (Rich et al. 2020; Johnson et al. 2020; Joyce et al. 2022). The volume of the 'excluded' region is quite conservative, taking the complicated structure of the inner region of the Milky Way into account (e.g. Valenti et al. 2016). The Galactic thin disk age was set to 10 Gyr.

The current number of proto- and AM CVn systems was obtained by convolving the birthrates of model systems with their lifetimes and star formation rate (SFR). considered to be constant over the past 2-3 Gyr and equal to 2 $M_\odot$ yr$^{-1}$ (Chomiuk & Povich 2011; Licquia & Newman 2015). The birthrate of AM CVns precursors for SFR 1 $M_\odot$ yr$^{-1}$ may be found as $\nu = C \times N_{\rm AM}/N_{\rm BSE}$, where $N_{\rm AM}$ is the number of precursors obtained by evolving $N_{\rm BSE}$ initial systems by BSE and $C = 0.045$ is the percentage of systems with $M_1 \geq 1\,M_\odot$ in the $[0.1, 100] M_\odot$ range (for the Salpeter IMF). The number of stars that spend time $\Delta t_k$ in the rectangular cell of $(f, h_c)$ space limited by $[f_i, f_i + \Delta f, (h_c)_j, (h_c)_j + \Delta h_c]$ (per initiated star), where $\Delta f$ and $\Delta h_c$ are the steps of the regular grid, respectively, was computed as follows:

$$\Delta N(t_k, t_k + \Delta t_k) = \frac{\sum_{l=1}^{N_{{\rm STARS}_{i,j}}} \Delta t^{k,l}}{\Delta t_k} \times \frac{1}{N_{\rm BSE}}, \tag{4}$$

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[3] This is the problem common to many evolutionary codes (see, e.g. discussion in Wong & Bildsten 2021).

where $0 \leq \Delta t^{k,l} \leq \Delta t_k$ is the duration of the AM CVn stage for the system $l$, $N_{{\rm STARS}_{i,j}}$ is the number of systems in the given cell according to the STARS grid, and $N_{\rm BSE}$ is the number of systems initialised in BSE. We used $N_{\rm BSE} = 10^5$.

## 3. Results of computations

### 3.1. Formation of He-donor AM CVn stars

Modelling the evolution of $10^5$ stars in BSE resulted in the production of 524 progenitors of He-star AM CVns. Corner plots in Fig. 1 show the relations between the masses of components and orbital periods of initiated systems and stellar types of binary components at ZAMS, prior to the first RLOF in the system, which results in the formation of WD components, and prior to the second RLOF, which results in the formation of stripped He components. We note that some initiated systems have extremely large orbital eccentricities, but in the course of evolution they circularise. While the first RLOF for some systems proceeds stably, the second one almost always involves the formation of common envelopes.

Distributions of the parameters of the pairs of naked He-stars+WD after the second RLOF is shown as a grid in Fig. 2. The BPS results suggest that He stars are predominantly the lowest mass ones, from $M_{\rm min} \approx 0.32\,M_\odot$ to $\approx 0.4\,M_\odot$, but in some very rare cases they may be as massive as about $1.2\,M_\odot$. Accretor masses are predominantly in the $0.65\,M_\odot$ to $1\,M_\odot$ range. The presence of relatively massive donors implies that even in the case of not very efficient accretion, WDs may accumulate $M_{\rm Ch}$, as suggested by Solheim et al. (2005). The distribution is dominated by the binaries that formed with $P_{\rm orb} \lesssim 100$ min, but sometimes $P_{\rm orb}$ attain 150 min. In wider systems, He in the cores of stars may be exhausted before RLOF and they end their lives as WDs and merge with companions due to AML via GWR. In low-mass donors ($M_{\rm sdB,0} \lesssim 0.4$–$0.5\,M_\odot$) and more massive donors which overflow Roche lobes when He abundance in the cores is still $Y_c \gtrsim 0.3$, He burning is quenched almost immediately after RLOF and they transform into AM CVn stars. In more massive donors ($M_{\rm He,0} \gtrsim 0.55\,M_\odot$), if $Y_c \lesssim 0.3$ at RLOF, core He burning continues. When He in the centre is almost exhausted, the stars contract and burn remainders of He. Expansion after exhaustion of He results in a merger with their companions (for details, see the case of the $(0.65+0.8)\,M_\odot$, $P_{\rm orb,0} = 90$ min system in Yungelson 2008).

Figure 2 provides information on the space of the initial parameters of immediate progenitors of He-star AM CVn systems and their birthrate. We estimate the current Galactic birthrate of He-donor AM CVn stars as $\approx 4.6 \times 10^{-4}$ yr$^{-1}$. This number is commensurate with the value $2.7 \times 10^{-4}$ yr$^{-1}$ found by Nelemans et al. (2004) for the case of high-mass of the layer of accreted He, necessary for its detonation and prevention of formation of an AM CVn star (see further discussion below).

### 3.2. Population of He-donor AM CVn stars detectable by LISA

Based on the information provided in Fig. 2, we computed a grid of 275 tracks with different combinations of $M_{\rm WD,0}$, $M_{\rm He,0}$, and $P_{\rm orb,0}$ with the STARS code. For computation of the $f - h_c$ relation, every computed system was assigned 20 000 random positions in the thin disk of the Galaxy. The $h_c$ obtained were then taken into account with the weight 1/20 000. Some cells shown in Fig. 2 contain up to five systems, and thus, altogether, we had 365 tracks. In the case of several systems in a cell, the
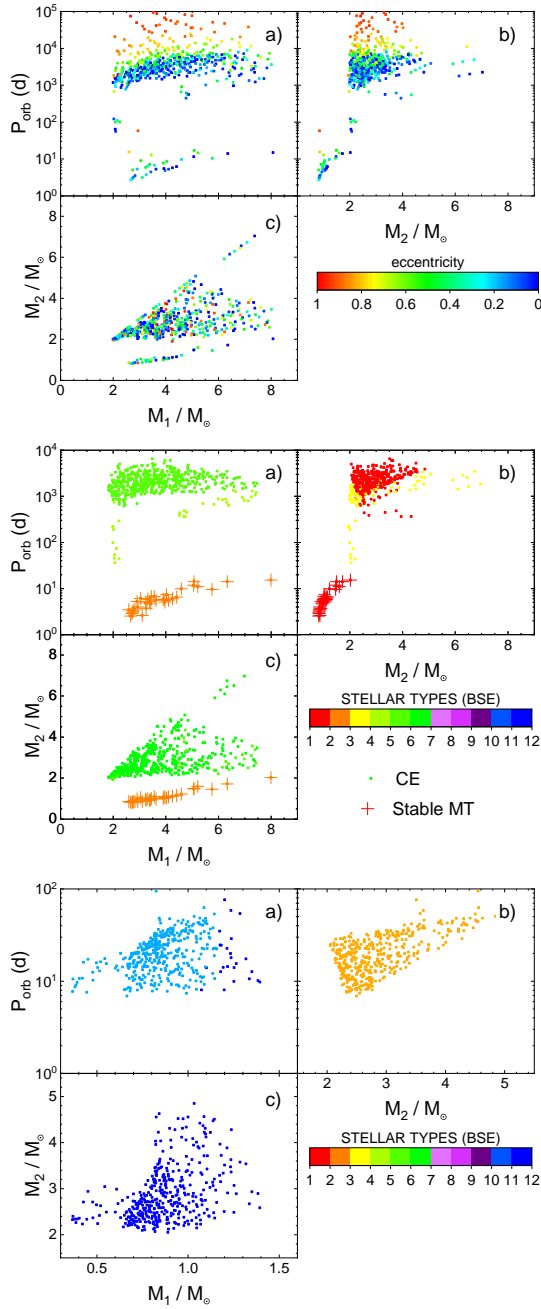
**Fig. 1.** Evolution of the relations between the parameters and types of components in the progenitors of AM CVn stars. *Upper panel*: precursors of AM CVns generated by BSE at ZAMS. (*a*) Relation between the masses of progenitors of WDs and initial $P_{orb}$; (*b*) relation between the masses of progenitors of He stars and $P_{orb}$; (*c*) relation between the masses of progenitors of WDs and He stars. *Middle panel*: the binaries immediately prior to the first RLOF. *Panels* (*a*)–(*c*): same as in the *upper panel*. Dots represent the binaries which pass through CE, crosses indicate the systems with stable RLOF. *Lower panel*: the binaries prior to the second RLOF. (*a*) Relation between the masses of WDs ($M_{WD}$) and $P_{orb}$; (*b*) relation between the masses of pre-He stars and $P_{orb}$; (*c*) relation between $M_{WD}$ and the masses of pre-He stars. The colour scale in the *upper panel* shows orbital eccentricities; in the *middle* and *lower panels*, colours indicate the types of systems according to BSE: 1 – MS star, 2 – Hertzsprung gap star, 3 – first-ascent red giant, 4 – core He burning giant, 5 – star at E-AGB, 6 – TP-AGB star, 7 – naked He star, 8 – He star in the Hertzsprung gap, 9 – naked He-(sub)giant, 10 – He WD, 11 – CO WD, 12 – ONe WD. In the lower panel (*c*) the colors of symbols are not related to BSE.
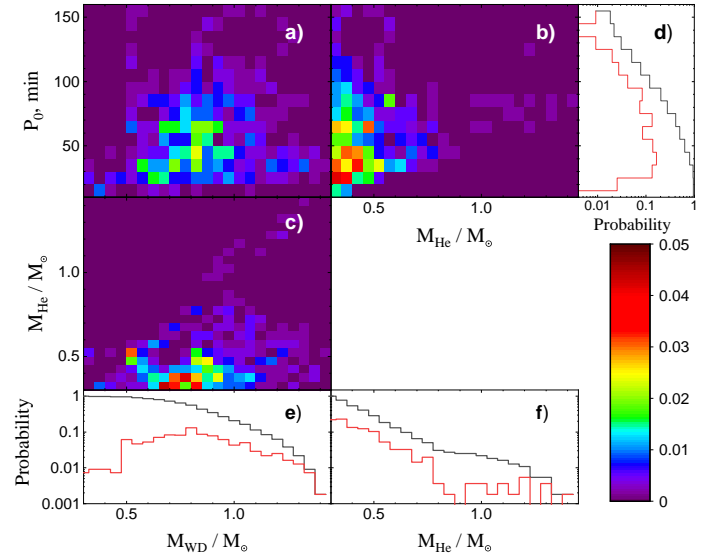


**Fig. 2.** Relations between the parameters of detached He-star+WD systems which will evolve into He-star AM CVns. The subpanels show the relations between the masses of WDs and post-CE $P_{orb}$ (*a*), between the masses of the nascent stripped He star and $P_{orb}$ (*b*), between the masses of WDs and stripped star components (*c*), the differential (red line) and cumulative (black line) distributions over $P_{orb}$ (*d*), the differential and cumulative distributions over $M_{WD}$ (*e*), and the differential and cumulative distributions over masses of He stars (*f*). The plots in the panels are normalised to unity.

computation of $h_c$ accounting for random positions was repeated appropriately.

Figure 3 shows the present-day distributions of detached pre-AM CVn and AM CVn systems in the $f − h_c$ plane. It is mainly defined by the distances to the objects and lifetimes of stars in the given frequency range. Evidently, the darkest shades are 'populated' by distant and long-living stars with low-mass donors. The possibility of detecting signals from most systems is limited by the sum of the 'confusion limit' that is formed by the signals of the population of unresolved detached close DWD and AM CVn stars (Evans et al. 1987; Bender & Hils 1997; Hils & Bender 2000; Nelemans et al. 2001b) and LISA instrumental noise, which prevents the detection of gravitational signals from binary WDs, unless they are very strong. We applied the 'observations-driven' confusion limit (Karnesis et al. 2021; Korol et al. 2022)[4], based on the results of the studies of the local DWD population using large samples of objects. The GW foreground dominates at $f \lesssim 1.8$ MHz, while LISA sensitivity (for $S/N > 5$) defines detectability at higher frequencies (see Fig. 4). The foreground is dominated by detached WDs and AM CVn stars do not play a noticeable role in its formation (Hils & Bender 2000; Nelemans et al. 2001b). This justifies the use of the confusion limit inferred from the observations of detached WDs only.

As it is mentioned above, our evolutionary code does not allow one to compute the evolution of stars less massive than $(0.02 − 0.03) M_{\odot}$ ($P_{orb} \gtrsim (42−43)$ min). We find that the Galaxy harbours about 112 000 He-donor AM CVn stars with $P_{orb} \lesssim (42−43)$ min. However, as it is clearly seen in Fig. 3, for most

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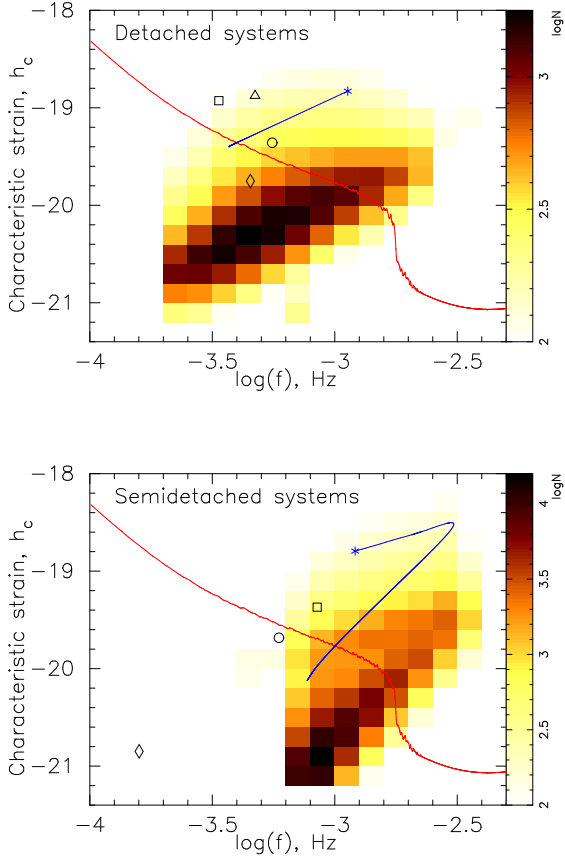[4] The estimate was carried out for LISA arms of 2.5 Gm.

**Fig. 3.** *Upper panel*: detached systems in the frequency – characteristic strain plane. Open circle, square, triangle, and diamond show positions of detached sdB+WD binaries PTF J2238+7430, HD265435, CD-30°11223, and OW J0815–3421 respectively. Solid red line shows confusion limit due to binary WD and LISA instrumental noise (Korol et al. 2022). Blue line shows the pre-contact evolutionary track for the system with initial parameters $M_{\mathrm{He}} = 0.43\,M_\odot$, $M_{\mathrm{WD}} = 0.87\,M_\odot$, $P_{\mathrm{orb}} = 90$ min, placed at $d = 1$ Kpc. Position of the system at RLOF is marked by an asterisk. Lower panel – semi-detached systems in the frequency – characteristic strain plane. Open circle, square, and diamond show positions of semi-detached sdB+WD systems ZTF J2130+4420, ZTF J2055+4651, J1920-2001. Blue line – continuation of the track shown in the *upper panel* in semi-detached stage. See the text in Sect. 4.3 for objects references.

systems the periods exceeding 42–43 min are longer than the periods at the confusion limit.

It is clear from Fig. 3 that, currently, most AM CVn stars should have masses of donors below $0.02\,M_\odot$ and relatively massive accretors ($\gtrsim 0.7\,M_\odot$, unless evolution is strongly non-conservative).

In order to illustrate our conclusions, we plotted in Fig. 3 an evolutionary track for the system with initial parameters $M_{\mathrm{He},0} = 0.43\,M_\odot$, $M_{\mathrm{WD},0} = 0.87\,M_\odot$, and $P_{\mathrm{orb},0} = 90$ min, assuming that its distance to the Sun is 1 Kpc. Such a system was chosen since its track both starts and ends below the foreground+sensitivity line. We split the track into 'detached' and 'semi-detached' parts. The system remained detached for about $\Delta t \approx 36$ Myr. The He star overflows its Roche lobe when $P_{\mathrm{orb}} \approx 20.2$ min. Before this, the binary might be observed as an sdB+WD system. Several such possible proto-AM CVn systems with well-measured parameters are known and indicated in Fig. 3. The minimum $P_{\mathrm{orb}}$ of the binary is 11.46 min, and it is reached at $t \approx 42$ Myr.

Our exemplary binary becomes undetectable in GW when its orbital frequency declines to 1 mHz ($P_{\mathrm{orb}} \approx 33.3$ min) at $t \approx 131$ Myr. At this $t$, the mass exchange rate is $\approx 1.4 \times 10^{-9}\,M_\odot$ yr$^{-1}$, meaning that the system should still be bright in optical. Later, $\dot{M}$ rapidly declines. The code breaks at $t \approx 496$ Myr after the formation of a He-star + WD system, when $P_{\mathrm{orb}} \approx 43.2$ min and $M_{\mathrm{He}} \approx 0.023\,M_\odot$.

## 4. Discussion and conclusions

We have presented the results of the first study of He-star AM CVns and their immediate precursors by the hybrid BPS. The advantage of the method is more precise tracking of semi-detached binaries than by analytic approximations. However, the physics implemented in the evolutionary code (opacity tables) restricts the range of orbital frequencies of AM CVns available for the study.

### 4.1. Comparison to other studies

We estimated that the birthrate of the Galactic thin disk He-donor AM CVn stars is $4.6 \times 10^{-4}$ yr$^{-1}$, and the number of objects with $P_{\mathrm{orb}} \lesssim (42-43)$ min is $\approx 112\,000$. We expect that about 500 of them may be discovered during a 4 yr long LISA mission.

In addition, we found that within 1 Kpc around the Sun, there might be approximately 130 He-star AM CVns with $P_{\mathrm{orb}} \lesssim (42-43)$ min (assuming that they follow space distribution (Eq. (3)). This is the lower limit of the number of He-star AM CVn systems, since we were not able to trace evolution for $P_{\mathrm{orb}} \gtrsim 43$ min. In fact, we lost the majority of the systems, since evolution slows down. One hundred and thirty stars correspond to the space density of $3.1 \times 10^{-8}$ pc$^{-3}$. This number may be compared to a $2\sigma$ limit on the space density of AM CVns based on *Gaia* DR2 data $\rho > 7 \times 10^{-8}$ pc$^{-3}$ (Ramsay et al. 2018). Keeping uncertain selection effects in mind, serendipitous discoveries of AM CVn stars, taking into account, on the one hand, that He-star AM CVn stars may evolve much longer than we can account for, but, on the other hand, WDs might explode without leaving bound remnants or disrupting the binary, we deem that this result does not contradict the finding of only three candidate He-donor AM CVns in the sample of known objects.

Published estimates for the population of AM CVns were obtained under different assumptions. The most important computed parameters are distributions of the initial binaries over the IMF of the primaries, mass ratios of components, orbital separations and eccentricities, the spatial distribution of binaries, star formation history (SFH), the CE formalism, the efficiency and consequences of accretion, and the treatment of the evolution of a He star. In addition, the estimates of the number of AM CVn stars that might be detected by LISA vary as the project itself and suggested data processing evolve. We list below the results of some computations with available data on assumed detection details.

Tutukov & Yungelson (1996) found the birthrate of He-donor AM CVn stars $\nu_{\mathrm{He-AM}} = 4.9 \times 10^{-3}$ yr$^{-1}$ for CE efficiency $\alpha_{ce}=1$ (when considering common envelopes, they set the donor envelope binding energy factor $\lambda = 1$). The birthrate declined to 0 for $\alpha_{ce}=0.1$. The reason is clear: future He-donors are formed via CE. Roughly, post- and pre-CE separations are related as $a_f \propto \alpha_{ce} \times a_i$ (see below). For small $\alpha_{ce}$, all possible precursors merge in CE. If $\alpha_{ce}$ exceeds some limit, all post-CE systems are so wide that He in the cores of potential donors is exhausted before RLOF. The evolution of He-donor AM CVn stars was integrated, assuming constant $\dot{M} = 3 \times 10^{-8}\,M_\odot$ yr$^{-1}$. For $\alpha_{ce}=1$,
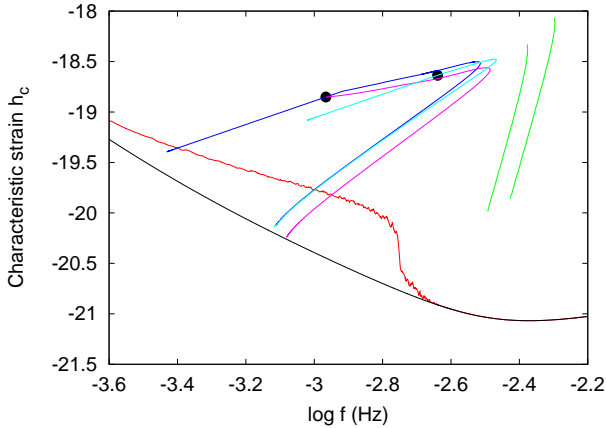
**Fig. 4.** Comparison of the tracks computed in the present study and the tracks computed using $M - R$ relations, as in Nelemans et al. (2001a). The blue line represents the track for $(M_{He,0} + M_{WD,0}, P_0) = (0.43+0.87) M_\odot$, 90 min; the magenta line is the part of the track for the same system under the assumption of non-conservative mass exchange; and the cyan line represents the track for $(M_{He,0} + M_{WD,0}, P_0) = (0.33+0.72) M_\odot$, 35 min. Black dots mark RLOF. Green lines indicate the tracks computed using analytic $M - R$ relations for the systems $(M_{He,0} + M_{WD,0}, P_0) = (0.33+0.72) M_\odot$, 35 min and $(M_{He,0} + M_{WD,0}, P_0) = (0.43+0.87) M_\odot$, 90 min (*left to right*, respectively). The black line is the LISA noise line, and the red line indicates the gravitational waves' foreground+LISA noise.

the number of objects was estimated as $N_{He-AM} = 4.9 \times 10^5$ to $1.9 \times 10^5$. The reason for the rather small number of objects was the assumption that, depending on $M_{WD}$ and $\dot{M}$, the accreted layer may either detonate after the accumulation of $0.2 M_\odot$ of He and destroy the WD in a supernova (SN) or it may strongly expand and an R CrB star may form.

Nelemans et al. (2001a, 2004), who used code SEBA, varied SFR(t), several assumptions on stellar evolution, the Galactic model, and age. As opposed to most other studies, the CE formalism based on the conservation of angular momentum was used for stars with non-compact components. Most important, it was assumed that an accreted He layer may detonate after the accretion of either $0.15 M_\odot$ or $0.30 M_\odot$. Within the range of different assumptions, $\nu_{He-AM}$ varied from $0.27 \times 10^{-3}$ to $1.6 \times 10^{-3}$, $N_{He-AM}$ varied from $1.1 \times 10^7$ to $3.1 \times 10^7$, and surface density $\rho$ exceeded $2 \times 10^{-5}$ pc$^{-3}$.

Nissanke et al. (2012), using the updated version of SEBA (Toonen et al. 2011) and varying assumptions on the occurrence of double detonations and the Galactic model, estimated $N_{He-AM}$ as 0 to $1.12 \times 10^7$. The number of detectable He-star AM CVn systems in the most optimistic case (5 Mkm detector) was only about 80.

The basic difference between the present study and the studies involving SEBA is the treatment of the evolution of He stars. In SEBA, an analytic approximation to the $M - R$ relation was used, minimum masses of the donors were $0.007 M_\odot$, and the Galactic age was set to 13.5 Gyr. The $M - R$ relation roughly approximated the post-period minimum part of the donor track for the system $(0.5+1.0) M_\odot$ in which the donor overfilled the Roche lobe almost unevolved (Tutukov & Fedorova 1989). For the sake of comparison, we computed the evolution of the $(M_{He,0} + M_{WD,0}, P_0) = (0.43 M_\odot+0.87 M_\odot$, 90 min) and $(M_{He,0} + M_{WD,0}, P_0) = (0.33 M_\odot+0.72 M_\odot$, 35 min) systems using this $M - R$ relation. Computations were continued until $M_{He} \approx 0.007 M_\odot$, as in Nelemans et al. (2001a); readers can refer to Fig. 4 for more information. This figure clearly shows

the major difference to the tracks computed by a full evolutionary code: a shift to larger frequencies and strain. The lifetime of the systems in the region of the $f - h_c$ diagram above the LISA sensitivity line is 41.4 Myr and 67.5 Myr, by about 30% and 50% longer, respectively, than that of the systems computed by an evolutionary code. These factors, along to the higher birthrate and different Galactic model, may be partially responsible for the higher number of potentially detectable systems and their higher frequencies in the models of Nelemans and his coauthors.

Other studies of AM CVn stars and their detection neglected He-star systems, as they have been deemed unimportant sources for LISA compared to detached binary WDs and DD AM CVn stars. We only list below some estimates of detection rates. Nelemans et al. (2004) estimated the total number of detectable DD AM CVns as $\approx 11\,000$ for a 5 yr mission and $S/N > 5$; Ruiter et al. (2010), assuming a 1 yr long mission, $S/N > 5$ and a 5 Gm arm length for the detector found $N = 5300$ sources; Nissanke et al. (2012) estimated the number as $N \lesssim 2000$; Yu & Jeffery (2013) found 8010, 19 820, and 3840 objects for quasi-exponential, constant, and instantaneous SFH, respectively, after a 1 yr of integration and $S/N > 3$; Kremer et al. (2017) found 2700 sources, requiring $S/N > 5$ and negative chirp $<0.1$ yr$^{-2}$; Breivik et al. (2018) provide $N \sim 3000$ as an average of different assumptions on common envelope parameters for a 4 yr long mission. Keeping all of the uncertainties in BPS in mind, as well as different detection criteria, all estimates are, in fact, in the same range.

Götberg et al. (2020), using a simplified BPS and a grid of tracks (Götberg et al. 2019), estimated the number of stripped $(0.3 - 2.5 M_\odot)$ He stars with WD companions in the Galaxy as $\sim 90\,000$ and suggested that 15% of these systems are currently in the mass-transfer stage. Götberg et al. (2020) do not present $M_{WD} - M_d$ relation for the binaries that started mass transfer, but a naked-eye comparison of Fig. 2 from the present paper and Figs. 2 and 3 from Götberg et al. (2020) suggests that no more than about 30% of binaries ($\approx 4000$) from the 'interacting' sample will experience stable mass transfer ($M_d \lesssim 1 M_\odot$, $P_{orb} \lesssim 1$ h). This number is still commensurate with our estimate of $\simeq 14\,800$ pre-AM CVn stars, keeping differences as to assumptions about the initial distribution of binaries over masses, periods, CE parameters, and evolutionary codes in mind. Götberg et al. (2020) estimate that under extremely favourable assumptions, LISA will be able to detect, within a 10 yr mission, about 100 He-star+WD systems with $S/N > 5$. However, realistic assumptions suggest numbers $\lesssim 10$.

### 4.2. Dependence on assumptions

While the existence of DD AM CVn stars is beyond a doubt, there are some questions concerning their formation and fate. This is associated, foremost, with the orders of magnitude discrepancy of observed and predicted space densities of the objects $\rho$. Observations suggest $\rho > 7 \times 10^{-8}$ pc$^{-3}$ (Ramsay et al. 2018), which is at least an order of magnitude less than the numbers listed above. Among the major unsolved problems is the stability of mass exchange immediately after RLOF by He WDs since, in the case of inefficient spin-orbit coupling, most systems should merge (Nelemans et al. 2001a; Marsh et al. 2004; Brown et al. 2016). Existing observational data on the slow rotation of two AM CVn stars (Kupfer et al. 2016) are too scarce for any conclusions to be drawn.

Shen (2015) noted a problem common to other CVs as well (see also Metzger et al. 2021; Shen & Quataert 2022). In the initial stages of RLOF, H-rich matter is transferred, leading to the

classical novae outbursts. In the AM CVn stars, a later transfer of He may cause outbursts. If these events result in the formation of envelopes that engulf both components, dynamical friction may shrink the orbits, enhance the mass-transfer rate, and finally lead to the merger of components. Just-formed 'stripped stars' possess H-rich envelopes (Ziółkowski 1970). Thus, they may first experience a series of H novae, if $\dot{M}$ is appropriate and, later, outbursts of He burning, also accompanied by the formation of common envelopes. However, these inferences should be confirmed by modelling WD motion inside postulated envelopes.

We assumed that mass exchange in pre-AM CVn and AM CVn systems is conservative. This is a certain simplification, since it is known that He accretion onto WDs at the rates close to the range of expected accretion rates in AM CVn stars may result in thermonuclear outbursts of a different strength in the layer of accreted He (e.g. Taam 1980; Nomoto 1982a,b; Iben 1991; Woosley & Kasen 2011; Piersanti et al. 2014), ranging from weak flashes to detonations, potentially initiating sub-Chandrasekhar SNe Ia via a mechanism of double-detonation (Livne 1990) or, for example, faint peculiar SN Iax due to deflagration in a He layer (Justham et al. 2009). If SN disrupts the binary, a single helium-rich object may be formed. Geier et al. (2015) suggested that runaway helium star US 708 is a remnant of a binary disrupted by a double-detonation sub-Chandrasekhar SN. Two further candidates of the same class were recently suggested by Neunteufel et al. (2022). We note, however, that an attempt to model a population of high- and hypervelocity He stars in the latter paper hints to a very low rate of events that disrupt He-star+WD binaries.

In the strong flashes, accretor loses accumulated He layer partially or completely. However, the issue of possible detonations and matter retention efficiency in flashes has not been solved yet, especially since the character of thermonuclear flashes depends on the rotation of the accretor. In the sample of pre-AM CVn stars generated by BPS, we found no system satisfying all conditions for accumulation of the He layer prone to detonation of rotating WDs formulated by Neunteufel et al. (2019). However, as an illustration of the possible influence of the loss of matter, we present in Fig. 4 the track for the $(M_{\text{He,0}} + M_{\text{WD,0}}, P_0) = (0.43 M_\odot + 0.87 M_\odot, 90\,\text{min})$ system computed under an extreme assumption that all accreted matter is lost by WD by isotropic re-emission. The experiment shows two effects. First, a decrease in the total mass of the system results in reduced strain (see Eqs. (1) and (2)), but the difference is not significant. Second, since isotropic re-emission slows down widening of the system, the non-conservative binary spends about 60 Myr above confusion limit, which is by 50% longer than the time spent by its conservative counterpart. The combined effect would be an increase in the number of AM CVn stars above the confusion limit, but with slightly weaker signals.

To compare this with the $(M_{\text{He,0}} + M_{\text{WD,0}}, P_0) = (0.43 M_\odot + 0.87 M_\odot, 90\,\text{min})$ system discussed above, we plotted in Fig. 4 the track for a more typical system $(M_{\text{He,0}} + M_{\text{WD,0}}, P_0) = (0.33 M_\odot + 0.72 M_\odot, 35\,\text{min})$, also assuming the distance of 1 Kpc. The system is observable as a detached He-star+WD binary for 8.9 Myr. It spends about 89 Myr above foreground, that is 3 times longer than the more massive system. However, at a given frequency, the signals are quite comparable. At the orbital frequency $f \approx 10^{-3}$ Hz, characteristic strain $h_c$ becomes lower than the foreground level. The mass of the donor at this instant declines to $\approx 0.045 M_\odot$, and the mass exchange rate becomes $1.7 \times 10^{-9}$ $M_\odot$ yr$^{-1}$, that is to say the star should still be relatively bright. Further evolution of the system as an AM CVn star, which we were able to trace with STARS,

lasted for $\Delta t \approx 334$ Myr. The mass of the last donor model is $\approx 0.026 M_\odot$, and the orbital period of the system is 43.7 min.

The greatest uncertainty in the BPS is the treatment of common envelopes (see Ivanova et al. 2020, for the latest review). It is still an unsolved 3D problem and, therefore, simple energy or angular momentum balance considerations were applied. Since the post-CE separation of components $a_f$ was evaluated using the energy balance,

$$\frac{a_f}{a_i} = \frac{M_c M_2}{M_d} \frac{1}{M_2 + 2M_e/(\alpha_{ce}\lambda r_L)}, \qquad (5)$$

where $M_d$ is mass of the donor, $M_2$ is the mass of the accretor, $M_c$ is the mass of the donor core, $M_e$ is the mass of the donor envelope, $\alpha_{ce}$ is the so-called common envelope efficiency, $\lambda$ is the binding energy parameter of the donor envelope, and $r_L$ is the Roche lobe radius. The most problematic term in Eq. (5) is $\alpha_{ce} \times \lambda$. It is evident that $\alpha_{ce} \leq 1$, unless highly uncertain additional energy sources (see Ivanova et al. 2020) are invoked and $\alpha_{ce}$ should be 'individual' for all binaries. In its own turn, $\lambda$ depends on the evolutionary stage of the star, core-boundary definition, possible account of terms, other than gravitational binding energy. Since $a_f/a_i \propto \alpha_{ce} \times \lambda$, the attempts to evaluate $\alpha_{ce}$ from the empirical data, in fact, provide this product, but not $\alpha_{ce}$, unless some assumptions as to $\lambda$ were made. Theoretically, the run of $\lambda$ along evolutionary tracks may be evaluated for certain sets of assumptions. As shown, for example, by Dewi & Tauris (2000) and Loveridge et al. (2011), for intermediate-mass stars, precursors of components in He-star AM CVns, $\lambda$ is about 0.2–0.4 in the RGB stage and it becomes closer to 1 in the AGB stage. Keeping in mind that the formation of He-star AM CVns invokes two common envelope episodes (Fig. 1) and uncertainties in the derivation of $\alpha_{ce}$ and $\lambda$, we consider $\alpha_{ce} = 1$ as a reasonable and conservative assumption.

As a test, we performed three runs of BSE for $10^6$ initial systems, assuming $\alpha_{ce} = 0.5$, 1, and 4. The obtained numbers of precursors of He-star AM CVns were 275, 5296, and 21 670. This is understandable: by virtue of the relation $a_f/a_i \propto \alpha_{ce} \times \lambda$, more systems merge in common envelopes for small $\alpha_{ce}$ and many more survive for, probably nonphysical, high $\alpha_{ce}$. We note that even for $\alpha_{ce} = 0.5$, the number of those potentially observed by LISA stars would be small, but not negligible (in our computations for $10^5$ initial binaries, there were 524 progenitors for $\alpha_{ce} = 1$).

### 4.3. Observed sdB+WD binaries

Along to the He-donor AM CVn stars, we estimated the number of their immediate precursors – detached stripped He-star+WD binaries. It is clear that the lifetime in the pre-AM CVn stage is short, and the number of precursors is small, close to 14 800. About 80 of them may be detected by LISA and they should be among the strongest sources (Fig. 3). In observations, they should be identified with detached sdB+WD systems.

We plotted in Fig. 3 positions of well-studied sdB+WD binaries with estimated distances. It is worth considering their destiny and possible relation to the AM CVn stars.

Detached binary CD-30°11223 (Vennes et al. 2012; Geier et al. 2013) has $M_{\text{sdB}} \approx 0.51 M_\odot$, $M_{\text{WD}} \approx 0.75 M_\odot$, and $P_{\text{orb}} = 70.5$ min; PTF J2238+7430 (Kupfer et al. 2022) has $M_{\text{sdB}} \approx 0.383 M_\odot$, $M_{\text{WD}} \approx 0.725 M_\odot$, and $P_{\text{orb}} = 76.3$ min; and OW J08153241 (Ramsay et al. 2022) has $M_{\text{sdB}} \approx 0.343 M_\odot$, $M_{\text{WD}} \approx 0.707 M_\odot$, and $P_{\text{orb}} \approx 73.7$ min. Neunteufel et al. (2019) have shown that in the systems with combinations of donor and rotating WD masses, as in these systems, detonation of an

accreted He layer does not occur. Rather, deflagrations and ejection of some matter may be expected. Thus these binaries may become AM CVn stars. For PTF J2238+7430, our conclusion is in disagreement with that of Kupfer et al. (2022), who expect double-detonation after the accumulation of 0.17 $M_\odot$ of He, but they did not take rotation effects into account.

HD265435 is a massive ($M_{\rm sdB} \approx 0.62\,M_\odot$, $M_{\rm WD} \approx 0.91\,M_\odot$), wide ($P_{\rm orb} \approx 90.1$ min) system (Pelisoli et al. 2021). Large $P_{\rm orb}$ suggests that the RLOF by sdB will happen when He in its core will be considerably exhausted. Because of the high mass of the donor, the presumably low abundance of He in the core, and the expected continuation of He burning after RLOF, we expect that HD265435 will not become an AM CVn star, but its donor will merge with the companion, possibly, with a SN Ia. A similar conclusion was also reached by Pelisoli et al. (2021).

As described in Sect. 3, post-RLOF evolution of He stars depends on their mass and degree of exhaustion of He in their cores. An additional factor, which defines the possible outcome of accretion of He onto WDs, is rotation.

The semi-detached system ZTF J2055+4651 (Kupfer et al. 2020a), with $M_{\rm sdB} \approx 0.4\,M_\odot$ and $M_{\rm WD} \approx 0.68\,M_\odot$, has large $P_{\rm orb} = 56.35$ min. The presence of H in the spectrum means that the subdwarf overflowed the Roche lobe close to the current $P_{\rm orb}$. This implies that He in the core of the subdwarf is almost exhausted and it will evolve into a hybrid WD and merge with the companion, as also suggested by Kupfer et al. (2020a). The system ZTF J2130+4420 (Kupfer et al. 2020b) is similar to ZTF J2055+4651: $M_{\rm sdB} \approx 0.337\,M_\odot$, $M_{\rm WD} \approx 0.545\,M_\odot$, and $P_{\rm orb} = 39.34$ min. There is H in the spectrum too and it may be expected that its evolution must be similar to that of ZTF J2055+4651.

The most recently discovered system J1920-2001 (Li et al. 2022), with $M_{\rm sdB} \approx 0.337\,M_\odot$ and $M_{\rm WD} \approx 0.545\,M_\odot$, differs from the above-mentioned systems by a large $P_{\rm orb} = 3.4946$ h. The position of the subdwarf in the $T_{\rm eff} - \log g$ diagram suggests that it is in the He-shell burning stage. After completion of this stage, it will turn into a WD and merge with its companion within $\simeq 1$ Gyr, as estimated by Li et al. (2022).

### 4.4. The number of stripped He stars in the Solar vicinity

At the suggestion of the referee, we compared the model number of stripped He stars in the 1-Kpc vicinity of the Sun with the number of objects in the same region in the catalogue of known hot subdwarfs (Geier 2020). In fact, the solution to such a problem requires full population synthesis of subdwarfs and is far beyond the aim of the present paper.

The stars in Geier's catalogue have parallaxes $\pi$ from *Gaia* DR2. To obtain the distances to the stars $d$, we cross-correlated this catalogue with that of Bailer-Jones et al. (2018), which provides *Gaia* distances corrected for non-linearity of $\pi \to d$ transformation.

Assuming Galactic SFR after Yu & Jeffery (2011) and using the same assumptions as for modelling an AM CVn population, we estimated, using BSE, the birthrate $\nu$ and current Galactic number $N$ of detached He-star+WD systems ($\nu \approx 5.8 \times 10^{-4}$ yr$^{-1}$ and $N \approx 40$), He-star+MS star binaries ($\nu \approx 1.9 \times 10^{-3}$ yr$^{-1}$ and $N \approx 130$ for $M_{\rm He} \leq 1.5\,M_\odot$), and single sub-dwarfs that formed by the merger of He WDs ($\nu \approx 9.4 \times 10^{-5}$ yr$^{-1}$, $N \approx 35$). The number of obtained model objects ($\approx 200$) is 5 times lower than the number of objects within 1 Kpc from the Sun in the subdwarfs catalogue.

However, observed hot subdwarfs are a mixture of objects (see, e.g. comprehensive review by Heber 2016), possibly forming also via channels different from those listed above. We note that we did not consider a possible merger of red giants and low-mass main-sequence stars in the common envelopes. As suggested by Politano et al. (2008), such mergers may result in the formation of single hot subdwarfs. The solution to the problem requires 3D modelling, which is still beyond our current capabilities. In our model, such mergers occur at the rate $\nu = 1.8 \times 10^{-2}$ yr$^{-1}$. If this channel really produces predominantly hot subdwarfs and if their typical lifetime is (200 – 300) Myr, as that of the subdwarfs that formed via 'stripping' channel, this scenario may occur to be the main route for formation of single hot subdwarfs. Keeping in mind that within 1 Kpc from the Sun reside about 0.1% of all hot subdwarfs formed via the 'stripping' channel, formation via merger may also resolve the problem of 'deficiency' of model stars respective to the catalogued ones.

We also remind readers that there is still an unresolved hypothesis about single subdwarfs precursors – 'hot flashers' (see, e.g. D'Cruz et al. 1996, and references therein) – or suggestions that subdwarfs may be formed due to an interaction of red giants with brown dwarfs (Nelemans 2010) and even planets (Soker 1998).

### 4.5. Conclusions

To summarise, we explored the formation of short and moderate period ($P_{\rm orb} \lesssim 43$ min) AM CVn stars with He donors, conjoining fast BPS for their progenitors with detailed evolutionary computations for the AM CVn stage itself. We found that the number of such systems in the Galaxy – if their components do not merge due to tidal friction in the novae envelopes in the early stages of mass transfer and if they are not destroyed by He detonations – may amount to $\simeq 112\,000$. About 500 of them may be detected by LISA with $S/N > 5$ during a 4-yr mission. In addition, LISA may detect up to 80 of their immediate precursors.

Helium-star AM CVns were modelled separately from DD in several papers using code SEBA (Nelemans et al. 2001a, 2004) and its clones only. Since we did not cover the entire range of $P_{\rm orb}$, it is impossible to compare predicted numbers directly. Nevertheless, it is clear that we expect a much lower number of AM CVns, mostly because of the difference between the results of evolutionary computations for binaries and results, based on analytic $M - R$ relations. A low rate of predicted detections of AM CVns in GWR in our study may be justified since the main limiting factor is the confusion limit.

The scarcity of He-star AM CVns in the total sample of the observed AM CVn stars may mean that unrecognized selection effects still exist. As well, the problem of possible merger of components due to tidal friction in the envelopes ejected in strong flashes of nuclear burning of accreted H and He still remains open. The same concerns possible explosions of accreting WDs as SNe or the decay of binaries due to mass ejection in the outbursts of He burning. Possible non-detection of these stars by LISA may confirm these assumptions. The deficiency of He-donor AM CVn stars may also point to a low value of the product of 'common envelope efficiency' and binding energy parameter $\alpha_{ce} \times \lambda$. A much lower number of model hot subdwarfs within 1 Kpc from the Sun compared to the number of them with *Gaia* distances, catalogued by Geier (2020), may indicate that there are other scenarios of formation of single subdwarfs, apart from the merger of WDs or the disruption of binaries. This problem definitely deserves a dedicated study.

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