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BEAM BREAK UP INSTABILITY ANALYSES FOR CAVITIES, LINACS, AND ENERGY RECOVERY LINACS*

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Abstract

This analyze argue that *BBU* instability both in separate cavities and in Linacs or *ERLs* is going due to the consequence of fundamental property of dipole modes. "Head-tail" bunch instability has also the same nature. New BBU instability testing methods are described and analytically proved in the article.

INTRODUCTION

Beam Break Up (*BBU*) instability of a beam in a cavity begins to develop when the beam current value becomes higher than some threshold value. Then a transversal beam oscillation appears with avalanche growth that has the frequency of a one of dipole modes in the cavity. The dipole mode amplitude there also becomes to avalanche growth. As a role the process continues up to the beam hit to the channel wall (the tail particles in the first instance) if the beam current source duration is long enough or if the focusing system cannot limit this oscillation. If the growth increment is high enough to have time to pick up BBU instability for a single bunch so this is known as "head-tail" instability.

THRESHOLD CURRENT OF BBU INSTABILITY

The dipole mode growth is attended by transferring of a part of beam kinetic energy to saved dipole mode field energy. Incidentally, some part of this energy disappears in the cavity wall. This energy transferring process is going on due to beam space-modulation (transversal oscillation) appeared with the dipole mode frequency as a result of the beam interaction with the dipole mode. This occurred only for those dipole modes that applies the energy brake to the beam. Only approximately half of all dipole modes in a cavity can do it. Such a ratio of stable and instable dipole modes remind of equal probability of some stochastic process. The reason will be clear then later in the pillbox cavity example.

To be a stable process, the transferred average power (P_{BBU}) must be lower then disappeared power (P_{dis}), i.e. $P_{BBU} \leq P_{dis}$. Such an approach has also considered by W.K.H. Panofsky in [1], but we have come to new conclusions.

On Fig.1 the example of the simplest dipole mode TM_{110} is shown in a pillbox cavity. Its resonance frequency (ω) is defined only by the cavity diameter (D) $\omega=0.82c\cdot D$ and is independent on its length (L).

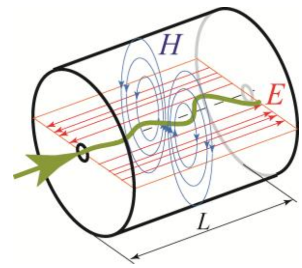

Figure 1: RF field distribution of TM_{110} dipole mode in a pillbox cavity with the view of a beam space-modulated trajectory.

We have to stress, only the average transferred power P_{BBU} plays a role in BBU instability. Each bunch separately hits to different phase of a dipole mode and so it can loss its energy or even to get it. So we can conclude that a bunch repetition frequency does not have an importance if dipole frequency is far enough from any harmonic of repetition frequency. Even any unbunched continuous beam can be instable.

BBU threshold current formula can be given if expressions for powers will be inserted to the above mentioned equation $P_{BBU} \leq P_{dis}$: $P_{BBU}=-E_{BBU}\cdot I$, where E_{BBU} is average energy gain of beam particles in the dipole mode in Volts; I is the average beam current. $P_{dis}=\omega\cdot U/Q$, where ω is circular resonance frequency of the dipole mode; U is saved dipole mode energy; Q is loaded quality factor of the dipole mode. After the conversion we get

$$I \cdot Q \leq I_Q = -\frac{\omega \cdot U}{E_{BBU}}. \quad (1)$$

The value of threshold current I_Q is the self-sufficing parameter for *BBU* instability as it follows from fundamental properties of dipole modes described further. Instability modes have positive value $I_Q>0$, for stability modes $I_Q<0$. Minus sign in *Eq.1* compensates the sign of energy loss of beam particles $E_{BBU}<0$.

BBU Fundamental Properties of Dipole Modes

One of unique property of dipole modes is the proportionality between values of average particle energy gain and dipole mode saved energy for beams propagating close enough to cavity axis:

$$E_{BBU} = -\frac{1}{I_Q} \cdot \omega U, \quad (2)$$

This proportionality is follows from the fact that rf longitudinal electric field of dipole modes (that caused the particle energy gain) is growth linearly with deviation (x) from

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axis. The proof is followed from the energy build-up equation *Eq.3* and of motion differential equation *Eq.4*. The averaged energy gain E_{BBU} in a longitudinal electric field $E_z(z,x)=x\cdot E'_z(z)$ for the particles moved along some trajectory $x= x(z,\varphi)$ defined by differential equation *Eq.4* is the integral along this trajectory (φ is the phase of the dipole fields at the coordinate $z=0$, then this energy gain is averaged on all φ phases):

$$E_{BBU} = \int_0^\pi \int_0^L E'_z(z) x(z,\varphi) \frac{\sin(2\pi z/\beta\lambda+\varphi)}{\pi} dz\, d\varphi \,. \qquad (3)$$

$$d(\gamma dx/dt)/dt = (e/m)\,\beta c B(z) \cos(\omega t + \varphi), \qquad (4)$$

Since the rf field in a cavity are proportional to square root of saved energy and particle deviations are also proportional to the square root of saved energy then the value after the integration is proportional to the saved energy. There can be shown [2, 3] that no initial deviation from the axis no initial inclination of beam do not influence to this conclusion if the trajectory lies close enough to the axis where there are linearity of E_z on x.

Equations *Eq.3* and *Eq.4* can be solved analytically for the simplest case of TM_{110} dipole mode shown in Fig.1. It is easy to show that the value of the threshold current periodically change their sign if the pillbox cavity length is elongated (see Fig.2), though the dipole mode fields there do not change. I.e. the dipole mode becomes periodically instable from stable one and then repeats back

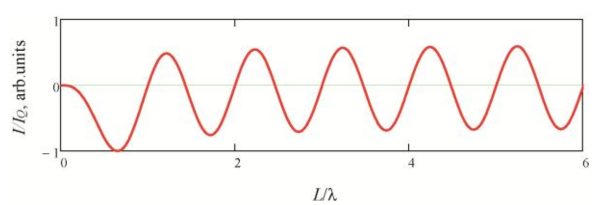

Figure 2: The dependency of backward value of BBU threshold current on the Pillbox cavity length.

The threshold current dependency on the beam energy can be obtained by using *Eq.1*, *Eq.3*, and the expression $U \sim L\lambda^2 B^2$, where λ=c/ω: $I_Q \sim (m/e)\omega^2\cdot\gamma/\beta^2$. One can see the low the frequency squared the low threshold current. The threshold current has at least value at the electron energy of *374 keV*. In connection with these we can introduce the new parameter for dipole modes depending only on their properties I_{QD} such that $I_Q = (\gamma/\beta^2)\cdot I_{QD}$. This I_{QD} may be named as Threshold Current of Dipole Modes, that uniquely determine potentially stable modes with $I_{QD}<0$, and instable one with $I_{QD}>0$.

From these we can conclude that BBU effect in cavities is the result of the fundamental property of dipole modes described by the new parameter I_{QD}.

The next unique property of dipole modes is that a space-modulated beam (transversally oscillated) can excite dipole modes independently on temporal distribution of bunches, initial deviation, and initial inclination if its trajectory lies close enough to the axis where longitudinal rf electric field is linear on deviation *x*. It is natural that this excitation going on due to transferring of kinetic beams energy to the saved dipole field energy.

To operate with polarized dipole modes let introduce the new vector parameter $\boldsymbol{F}$ such that $|\boldsymbol{F}| \equiv U^{1/2}$. The $\boldsymbol{F}$ has the direction of the force acting to the beam, i.e. it is transversally directed to the magnetic field vector $\boldsymbol{B}$ and to the axis.

The space-modulated beam can be described by *Eq.3* if there will be replaced $E_{BBU}\rightarrow E_{MOD}$, where E_{MOD} is the energy gain of space-modulated beam, and $x=r\cdot\sin(2\pi z/\lambda+\varphi+\Phi)$, where Φ is the phase difference between the beam oscillation and the dipole mode B field; r is the amplitude of the beam oscillation. So we can write

$$E_{MOD} = -\sqrt{\omega\,(R_{II}/Q)/4}\cdot \cos(\Phi)\cdot(\bar{r}\cdot\bar{F}), \quad (5)$$

where the oscillatory and field polarizations there are taken into account by the scalar product of vectors $\boldsymbol{r}$ and $\boldsymbol{F}$; R_{II}/Q is the conventional coupling impedance of the dipole mode in Ohm/m^2. The specific coupling impedance for a space-modulated beam is less exactly by factor 4 then the conventional one, so in *Eq.5* the conventional coupling impedance is used with the factor of ¼.

On the basis of law of conservation of energy there is true the differential equation: $-E_{BBU}\cdot I - E_{MOD}\cdot I = P_{dis} + \Delta U/\Delta t$. After inserting to this the mentioned expression for P_{dis}, *Eqs.2, 5*, we get:

$$\boldsymbol{F}(t) = \boldsymbol{F}_0\cdot \exp\frac{(I/I_Q-1/Q)\omega t}{2} + \frac{\boldsymbol{r_0}}{2}\cos(\Phi)\frac{I\sqrt{(R_{II}/Q)/\omega}}{I/I_Q-1/Q}\times \left(1-\exp\frac{(I/I_Q-1/Q)\omega t}{2}\right), \quad (6)$$

where $\boldsymbol{F_o}$, and $\boldsymbol{r_o}$ is the complex values of initial dipole mode field and beam oscillation correspondingly represents the polarization vectors in the complex space.

Let us consider some interesting cases of *Eq.6* applications. Without of the beam ($I=0$) *Eq.6* describes usual process of damped oscillations in a cavity. If the dipole mode is instable ($I_Q>0$), and the beam current there are more than threshold one ($I/I_Q-1/Q>0$) then there begins the avalanche growth of the dipole mode with the increment $\delta=(I/I_Q-1/Q)\omega$, the more the beam current the more the growth rate. Only stable dipole modes with any beam currents gives always damped oscillations

BBU EFFECT IN LINACS

If the cavities are installed one by one like in a Linac ($\Phi=0$), they have the same dipole modes, and the beam current is more than the threshold one $I/I_Q-1/Q>0$ then the growth rate and its increment in each next following cavity is more than it has been in previous one since the initial amplitude r_o becomes more and more. The numerical solving of the differential equation for the case with oscillating growth $r_o\equiv r_o\cdot\exp[(I/I_Q-1/Q)\omega t/2]$ in the first cavity have shown the increment growth in the second cavity by the factor 1.34, and in the third cavity by the factor of 1.6, and etc.

The last example demonstrates the importance of a proper beam focusing to suppress the BBU grows rate in

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Linacs – focusing systems must support oscillation r_0 values as less as possible ($r_0 \to 0$).

BBU EFFECT IN ERLS

In this case, the same beam with different energies comes through each cavity twice. The first passage usually has no space-modulation. The second one can be space-modulated due to BBU instability problems through the passing cavities.

On the basis of law of conservation of energy we can write: $-E_{BBU1} \cdot I - E_{BBU2} \cdot I - E_{MOD2} \cdot I = P_{dis} + \Delta U/\Delta t$, where the index *1* corresponds to the first beam passage, and index *2* – to the second one. After inserting to this the expressions for E_{BBU1}, E_{BBU2} E_{MOD2}, P_{dis} , U, and take into account $I_{Q1}=I_{QD} \cdot \gamma_1$, and $I_{Q2}=I_{QD} \cdot \gamma_2$. After integrating, there can be obtained the same equation of *Eq.6* but only there is the characteristically important replacing:

$$1/I_Q = 1/I_{Q1} + 1/I_{Q2}. \quad (7)$$

So the threshold current for ERLs is less than the least of them among the cavities that occurs in a cavity with the lowest value of beam energy.

The transversal beam oscillation and the dipole mode excited in n-th passage ($t=nT$) acts on the dipole mode in the next passage ($t=(n+1)T$) according to *Eq.6,* where $F_0 \equiv F(nT) \cdot cos(\omega T)$, and $r_o \equiv æF(nT) \cdot cos(\omega T)$. There the phase incursion for a one passage is taken into consideration by the factor of $cos(\omega T)$; $æ = |æ| e^{j\theta}$ is the complex coefficient granting the linear relation between the dipole mode and the oscillation; θ is the polarization tern angle of the beam oscillation due to the beam focusing and transportation along the *ERL* ring:

$$\dot{F}\big((n+1)T\big) = F(nT)cos(\omega T)e^{\frac{(I/I_Q - 1/Q)\omega T}{2}} +$$

$$+\frac{æ \cdot F(nT)}{2} cos(\omega T) \frac{I\sqrt{(R_{II}/Q)/\omega}}{I/I_Q - 1/Q} \cdot \left(1 - e^{\frac{(I/I_Q - 1/Q)\omega T}{2}}\right). \quad (8)$$

The stability *BBU* condition in *ERL* is following: $|\boldsymbol{F}((n+1)T)| / |\boldsymbol{F}(nT)| < 1$. To carry out this there don't have enough stability in cavities. Let us find the minimal *ERL* threshold current (I_{QERL}) in the least favorable case when an integer number of oscillations will go into the *ERL* length, i.e. $cos(\omega T)=1$, and the polarization doesn't change, i.e. $\theta=0$. From *Eq.8* it is follows:

$$\frac{-æ}{2} \cdot \frac{I\sqrt{(R_{II}/Q)/\omega}}{I/I_Q - 1/Q} \le 1. \quad (9)$$

If we replace in *Eq.9:* $I \cdot Q = I_{QERL}$, then will get for instable modes ($I_Q>0$) the *ERL* threshold current always less than the cavity one:

$$\frac{I_Q}{1 + æ\sqrt{(R_{II}/Q)/\omega \cdot I_Q}} \le I_{QERL} \le I_Q. \quad (10)$$

And for the stable cavity modes ($I_Q<0$):

$$I_{QERL} \le \frac{|I_Q|}{æ\sqrt{(R_{II}/Q)/\omega} \cdot |I_Q| - 1}, \quad (11)$$

i.e. even stable cavity modes can be instable in *ERL*. At a sufficiently small æ value the denominator of *Eq.11* becomes negative. So the beam will be absolutely stable at any current if

$$-1/æ\sqrt{(R_{II}/Q)/\omega} < I_Q < 0. \quad (12)$$

At this condition there are enough power transporting from the saved dipole energy to kinetic beam energy to suppress the *BBU* instability in *ERL*.

NEW METHODS OF BBU TESTING

There are new testing methods for detection of instable dipole modes and definition of its threshold currents. If we compare two decrements (δ and δ') of damped oscillations in a cavity, and one of them (δ') is going with any electron beam passing through the cavity then the decrement of stable modes will be grown involving the beam current I but for instable modes it will be fall:

$$\delta'(I) = \delta - I/I_Q. \quad (13)$$

$$I_Q = \frac{I}{\delta - \delta'(I)}. \quad (14)$$

The next method utilizes space-modulated beams. If we compare two fields in a cavity excited by space-modulated beam with different average currents, $F=F(I)$, $F'=F(I')=F(n \cdot I)$, and at the same modulation amplitudes ($r'=r$) then the threshold current can be calculated as

$$I_Q = I \cdot Q \cdot \frac{n(1 - F'/F)}{n - F'/F}. \quad (15)$$

This method can be used for testing of trapped dipole modes that cannot be seen by any test probes in the cavity. In this case the F and F' signals in *Eq.15* are the beam oscillation amplitude signals taken from special probes.

Mentioned above methods can be applied both for a special stand and for a Linac itself or ERL. In this case the accelerator beam itself can be the instrument of the testing.

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