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Radiative Transitions of Singly and Doubly Charmed Baryons in Lattice QCD

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We present the result of spin-$3/2 \to$ spin-$1/2$ radiative transitions of singly and doubly charmed baryons. We use pion mass of 156(9) MeV, 2+1 flavor lattices. The magnetic dipole, $M1$, and the electric quadrupole, $E2$, transition form factors are extracted. We have made a prediction of the decay widths and lifetimes of $\Xi_{cc}^{*+,++}$, Ω_{cc}^{*+} and Ω_c^* based on our results. Results we present here are especially evocative for experimental facilities to search for further states.

KEYWORDS: Charmed baryons, electric and magnetic form factor, lattice QCD

1. Introduction

Recently, there has been a important progress in charmed baryon sector. All the ground-state single-charmed baryons and some of the excited states have been confirmed. However observation of the doubly charmed baryons has been long overdue. First observed doubly charmed baryon was Ξ_{cc}^{+}, which was reported by SELEX collaboration in 2002 [1] and recently LHCb Collaboration discovered the Ξ_{cc}^{++} [2].

Electromagnetic properties of the spin-$3/2 \to$ spin-$1/2$ baryon transitions give information about their deformations and internal dynamics. Examining the radiative transitions of spin-$3/2 \to$ spin-$1/2$ charmed baryons is important to reveal the heavy-quark dynamics. Recently we have examined electromagnetic structure of the charmed baryons in lattice QCD [3–7]. In this work we widen our investigations to the singly and doubly charmed baryons' spin-$3/2 \to$ spin-$1/2$ electromagnetic transitions.

2. Lattice Formulation and Setup

Electromagnetic transition form factors for a $\mathcal{B}\gamma \to \mathcal{B}^*$ can be extracted by using baryon matrix elements written in the following form:

$$\langle \mathcal{B}^*(p', s')|\mathcal{J}_\mu|\mathcal{B}(p, s)\rangle = i\sqrt{\frac{2}{3}}\left(\frac{m_{\mathcal{B}^*}\, m_{\mathcal{B}}}{E_{\mathcal{B}^*}(\mathbf{p}')E_{\mathcal{B}}(\mathbf{p})}\right)\bar{u}_\tau(p', s')O^{\tau\mu}u(p, s), \tag{1}$$

where $\mathcal{B}$ and $\mathcal{B}^*$ are spin-$1/2$ and spin-$3/2$ baryons, respectively. p and p' are the initial and final four momenta of baryons and, s and s' denote the spins. $u(p, s)$ is the Dirac spinor and $u_\tau(p, s)$ is the

Rarita-Schwinger spin vector. Operator $O^{\tau\mu}$ given in terms of Sachs form factors as [8],

$$O^{\tau\mu} = G_{M1}(q^2)K^{\tau\mu}_{M1} + G_{E2}(q^2)K^{\tau\mu}_{E2} + G_{C2}(q^2)K^{\tau\mu}_{C_2}, \tag{2}$$

where G_{E2}, G_{C2} and G_{M1} represent the electric quadrupole, the electric charge quadrupole and the magnetic dipole transition form factors, respectively. Details of the kinematical factors are given in Ref. [9]. We use the following correlation functions to compute form factors,

$$\langle G^{\mathcal{B}^*\mathcal{B}^*}_{\sigma\tau}(t;\mathbf{p};\Gamma_4)\rangle = \sum_{\mathbf{x}} e^{-i\mathbf{p}\cdot\mathbf{x}}\Gamma_4^{\alpha\alpha'} \times \langle \text{vac}|T[\eta^\alpha_\sigma(x)\bar{\eta}^{\alpha'}_\tau(0)]|\text{vac}\rangle, \tag{3}$$

$$\langle G^{\mathcal{B}\mathcal{B}}(t;\mathbf{p};\Gamma_4)\rangle = \sum_{\mathbf{x}} e^{-i\mathbf{p}\cdot\mathbf{x}}\Gamma_4^{\alpha\alpha'} \times \langle \text{vac}|T[\eta^\alpha(x)\bar{\eta}^{\alpha'}(0)]|\text{vac}\rangle, \tag{4}$$

$$\langle G^{\mathcal{B}^*\mathcal{J}^\mu\mathcal{B}}_{\sigma}(t_2,t_1;\mathbf{p}',\mathbf{p};\mathbf{\Gamma})\rangle = -i\sum_{\mathbf{x_2},\mathbf{x_1}} e^{-i\mathbf{p}\cdot\mathbf{x_2}}e^{i\mathbf{q}\cdot\mathbf{x_1}}\mathbf{\Gamma}^{\alpha\alpha'}\langle \text{vac}|T[\eta^\alpha_\sigma(x_2)j_\mu(x_1)\bar{\eta}^{\alpha'}(0)]|\text{vac}\rangle, \tag{5}$$

where σ and τ denote the Lorentz indices of the spin-3/2 interpolating field, α, α' are the Dirac indices and σ_i are the Pauli spin matrices. Spin-1/2 baryon is created at $t = 0$, the external electromagnetic field is applied at time t_1. The interacted state propagates to t_2 where the spin-3/2 baryon is annihilated. The interpolating fields are chosen similar to Δ and N as

$$\eta_\mu(x) = \frac{1}{\sqrt{3}}\epsilon^{ijk}\left\{2[q_1^{Ti}(x)C\gamma_\mu q_2^j(x)]q_1^k(x) + [q_1^{Ti}(x)C\gamma_\mu q_1^j(x)]q_2^k(x)\right\}, \tag{6}$$

$$\eta(x) = \epsilon^{ijk}[q_1^{Ti}(x)C\gamma_5 q_2^j(x)]q_1^k(x), \tag{7}$$

where q_1 and q_2 denotes quark fields and i, j, k are the color indices. Charge conjugation matrix is defined as $C = \gamma_4\gamma_2$. For spin-1/2 baryons we employ the operators in Equation (7), we choose quark contents for $\Xi_{cc}^{++}, \Xi_{cc}^{+}, \Omega_{cc}^{++}, \Omega_c$ baryons as $(q_1 = c, q_2 = u)$, $(q_1 = c, q_2 = d)$, $(q_1 = c, q_2 = s)$ and $(q_1 = s, q_2 = c)$ respectively. For spin-3/2 baryons we employ the operators in Equation (6) with same quark contents mentioned above. We calculate the ratio using the two- and three-point functions in order to extract form factors,

$$R_\sigma(t_2,t_1;\mathbf{p}',\mathbf{p};\mathbf{\Gamma};\mu) = \frac{\langle G^{\mathcal{B}^*\mathcal{J}^\mu\mathcal{B}}_{\sigma}(t_2,t_1;\mathbf{p}',\mathbf{p};\mathbf{\Gamma})\rangle}{\langle \delta_{ij}G^{\mathcal{B}^*\mathcal{B}^*}_{ij}(t_2;\mathbf{p}';\Gamma_4)\rangle}\left[\frac{\delta_{ij}G^{\mathcal{B}^*\mathcal{B}^*}_{ij}(2t_1;\mathbf{p}';\Gamma_4)\rangle}{G^{\mathcal{B}\mathcal{B}}(2t_1;\mathbf{p};\Gamma_4)\rangle}\right]^{1/2}. \tag{8}$$

In the large Euclidean time limit, $t_2 - t_1 \gg a$ and $t_1 \gg a$, time dependence of the correlators are vanished. We can write the ratio in Equation (8) as

$$R_\sigma(t_2,t_1;\mathbf{p}',\mathbf{p};\Gamma;\mu) \xrightarrow[t_2-t_1\gg a]{t_1\gg a} \Pi_\sigma(\mathbf{p}',\mathbf{p};\Gamma;\mu). \tag{9}$$

We extracted the Sachs form factors choosing applicable combinations of projection matrices Γ and Lorentz direction μ. In this work we fix the kinematics for spin-3/2 at rest as

$$G_{M1}(q^2) = \Pi_1 - \Pi_2, \quad G_{E2}(q^2) = \Pi_1 + \Pi_2, \tag{10}$$

where $\Pi_1 = C(\mathbf{q}^2)\frac{1}{|\mathbf{q}|}\sum_{k,l}\Pi_l(q_k,\mathbf{0};\Gamma_k;l)$ and $\Pi_2 = C(\mathbf{q}^2)\frac{1}{|\mathbf{q}|}\sum_{k,l}\Pi_k(q_k,\mathbf{0};\Gamma_l;l)$ and $C(\mathbf{q^2}) = 2\sqrt{6}\frac{E_{\mathcal{B}}m_{\mathcal{B}}}{m_{\mathcal{B}^*}+m_{\mathcal{B}}}\left(1+\frac{m_{\mathcal{B}}}{E_{\mathcal{B}}}\right)^{1/2}\left(1+\frac{\mathbf{q}^2}{3m^2_{\mathcal{B}^*}}\right)^{1/2}$. For real photons, only G_{M1} and G_{E2} contribute. We extract the $M1$ and $E2$ form factors for this work.

We have run our simulations on $32^3 \times 64$ lattices with 2 + 1 flavors of dynamical quarks. Gauge configurations have been generated by the PACS-CS collaboration [10]. They have a lattice spacing of $a = 0.0907(13)$ fm and hopping parameters for sea quarks are $\kappa_{ud}^{\rm sea} = 0.13781$, $\kappa_s^{\rm sea} = 0.13640$. Details of the gauge configurations are given in Ref. [9]. We use 146, 163 and 194 configurations, respectively for Ω_c, Ξ_{cc} and Ω_{cc}.

Since our results are obtained with almost-physical quark masses, we do not make any chiral extrapolations. We use a simple scaling method to get the form factors at zero momentum as in Ref. [11]. With the help of this simple scaling, $G_{M1}(0)$ is estimated by

$$G_{M1}^{s,c}(0) = G_{M1}^{s,c}(q^2)\frac{G_{E0}^{s,c}(0)}{G_{E0}^{s,c}(q^2)}. \tag{11}$$

Since the charge form factor contributions have different scales, we calculate quark contributions separately. We employ the Tsukuba action for the charm quark [12]. Details of the action are given in Ref [9]. In this work statistical errors are estimated by jackknife analysis. We use wall-source/sink method [13]. Since wall-source/sink method depends on gauge, we fixed it to Coulomb.

Table I. Extracted Ω_c,Ω_c^*,Ξ_{cc}, Ξ_{cc}^*, Ω_{cc}, and Ω_{cc}^* masses together with other lattice collaborations and experiments.

	This work	PACS-CS [14]	ETMC [15]	Brown et al. [16]	RQCD [17]	Experiment [18]
m_{Ω_c} [GeV]	2.707(11)	2.673(13)	2.643(14)(19)(42)	2.679(37)(20)	2.642(07)(18)	2.695(2)
$m_{\Omega_c^*}$ [GeV]	2.798(24)	2.738(13)	2.728(16)(19)(26)	2.755(37)(24)	2.709(11)(21)	2.766(2)
$m_{\Omega_{cc}}$ [GeV]	3.719(10)	3.704(17)	3.658(11)(16)(50)	3.738(20)(20)	3.713(06)(10)	—
$m_{\Omega_{cc}^*}$ [GeV]	3.788(11)	3.779(18)	3.735(13)(18)(43)	3.822(20)(22)	3.785(06)(10)	—
$m_{\Xi_{cc}}$ [GeV]	3.626(30)	3.603(22)	3.568(14)(19)(1)	3.610(23)(22)	3.610(09)(12)	3.62140(72)(27)(14)
$m_{\Xi_{cc}^*}$ [GeV]	3.693(48)	3.706(28)	3.652(17)(27)(3)	3.692(28)(21)	3.694(07)(11)	—

3. Results and Discussion

We extract the masses using the two-point correlation functions in Equations (3) and (4). We give our results for the masses in Table I, with comparison to experimental values and results from other lattice collaborations. Our results show that mass splittings are too small for any strong decay occur, the radiative channels are dominant.

Using two- and three-point correlation functions, we construct the ratio given in Equation (8). Once getting the Π, it is straightforward to compute G_{M1} and G_{E2} using Equation (10). Our results for form factors are given in Table II lowest allowed four-momentum transfer ($Q^2 = 0.181\ GeV^2$) results are result of LQCD first principle calculation, zero momentum transfer results computed using Equation (11).

The decay width is given by [18]

$$\Gamma = \frac{\alpha}{16}\frac{(m_{\mathcal{B}^*}^2 - m_{\mathcal{B}}^2)^3}{m_{\mathcal{B}}^2 m_{\mathcal{B}^*}^3}\{3|G_{E2}(0)|^2 + |G_{M1}(0)|^2\}. \tag{12}$$

Our results for decay widths are $\Gamma_{\Omega_c\gamma\to\Omega_c^*} = 0.096(14)$,$\Gamma_{\Omega_{cc}^+\gamma\to\Omega_{cc}^{*+}} = 0.120(8)$,$\Gamma_{\Xi_{cc}^+\gamma\to\Xi_{cc}^{*+}} = 0.123(31)$ and $\Gamma_{\Xi_{cc}^{++}\gamma\to\Xi_{cc}^{*++}} = 0.178(73)$ in the unit of keV which are estimations based on a combined use of LQCD and other models.

In summary, we have computed radiative transition of singly and doubly charmed baryons in Lattice QCD. We have extracted the magnetic dipole and electric quadrupole form factors. We have found that light quarks dominate the $M1$ form factors. Our results quantitatively disagree around 1 or greater than 1 order of magnitude with other approaches, which needs more investigations to resolve. The results are especially suggestive for experimental facilities to search for further states.

Table II. Results for G_{M1} and G_{E2} at the lowest allowed four-momentum transfer and zero momentum transfer. Quark sector contributions are given separately weighted with number of valance quarks.

	Q^2[GeV^2]	$G^{\ell}_{M1}(Q^2)$	$G^{c}_{M1}(Q^2)$	$G_{M1}(Q^2)$	$G^{\ell}_{E2}(Q^2)$	$G^{c}_{E2}(Q^2)$	$G_{E2}(Q^2)$
$\Omega_c\gamma \to \Omega_c^*$	0.180	1.456(102)	-0.209(30)	-0.625(43)	-0.195(11)	0.010(23)	0.059(43)
	0	1.748(122)	-0.215(31)	-0.725(50)	-0.234(13)	0.010(24)	0.071(52)
$\Omega_{cc}^{+}\gamma \to \Omega_{cc}^{*+}$	0.181	-2.138(78)	0.511(19)	1.054(33)	-0.034(30)	0.002(13)	0.013(14)
	0	-2.567(91)	0.564(20)	1.218(39)	-0.040(36)	0.003(14)	0.015(16)
$\Xi_{cc}^{+}\gamma \to \Xi_{cc}^{*+}$	0.180	-2.012(349)	0.470(67)	0.984(128)	0.069(301)	-0.005(71)	-0.026(108)
	0	-2.536(431)	0.493(70)	1.174(153)	0.087(380)	-0.006(75)	-0.033(133)
$\Xi_{cc}^{++}\gamma \to \Xi_{cc}^{*++}$	0.180	-2.012(349)	0.470(67)	-1.028(226)	0.069(301)	-0.005(71)	0.043(210)
	0	-2.536(431)	0.493(70)	-1.362(299)	0.087(380)	-0.006(75)	0.054(269)

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