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Abstract. This paper approaches the question of space dimensionality by discussing a neglected argument proposed by Hermann Weyl in 1955. In *Why is the World Four-Dimensional?* (1955), Weyl offered a different argument from the one generally attributed to him and presented in *Raum-Zeit-Materie*. In the first sections of the paper, this new argument and its features are spelled-out, and in the last section, I shall develop some useful remarks on the concept of topology of causation that can still inform our reflection on the dimensionality of the world.

1. Introduction

In the last decade, Ptkov, among others, suggested that of the most fundamental questions of the 21st century is the search for an explanation of the dimensionality of the world [1]. In his view, by considering the analysis of the kinematical effects of special relativity one can find the key to answer this question. Indeed, he thinks that these effects and experiments which confirmed them would be impossible if the world were three dimensional. Therefore, one has to consider the world as four dimensional. The way in which Petkov touches the question raises at least two important problems. First, one should define what one means by the word 'World' in such a discussion [2]. Second, given that both three- and four-dimensional representations of the physical world are consistent and allowed in the relativistic picture,[1] it results that Petkov does not explain *why* the world is four dimensional, he just shows that it *is* four dimensional and that this dimensionality correctly reflects the dimensionality of the world.

In this paper, I shall discuss another possible approach, one that explores the search for an explanation of the necessity of the four dimensionality of the world. Thus, this approach assumes the four dimensionality of space-time as given[2] and aims at explaining *why* the world has such a configuration and not another one. To reach this goal, not only philosophy and physics are needed, but a fundamental role is also played by mathematics.[3] In 1955, a few months before his

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[1] According to Ohanian, physics clearly points to the fact that the observable world is both 3D and 4D, because it can be considered as a manifold and as an embedded curved hypersurface [2].

[2] In other words, it already assumes it as a fact whose features are given by Minkowski's definition of the world.

[3] An approach that is similar (only methodologically) to the argument suggested by Weyl is offered by Tegmark [3]. Tegmark explored the possibilities entailed by PDEs solutions and spelled out how the problem of boundary conditions is of fundamental importance in judging the capacity of a scientific theory to explain the world-dimensionality problem.

death, in Washington D.C. Weyl gave a talk entitled *Why is the World four-dimensional?*[4]. The paper has been recently published in the Weyl collection *Levels of Infinity* edited by Peter Pesic and has been neglected for almost 60 years, even if in its title one can detect a timely question bringing with it the spirit of an old and complex problem, one that invests the foundations of our scientific theories. Both scientific and philosophical literature is replete of articles and books dealing with the problem of space dimensionality and the dimensionality of the world [1][2][5].[4]

Nowadays, the reflection on the question of the dimensionality of the world is often discussed in connection with the assumption of the Anthropic Principle, be it in its weak or strong form. However, I claim that from a rigourous philosophical standpoint one should avoid to ground the argument for the explanation of world dimensionality on the anthropic principle as this prevents us from endorsing a fruitful approach to this question. I shall discuss this point in the last sections of this paper. In offering a deep analysis of Weyl's arguement, I aim at generating an interest among physicists about the possibility of shaping the question of the world dimensionality in a way that is not entirely philosophical nor physical and foremost that is independent of any anthropic principle. Hermann Weyl's 1955 argument represents such an alternative and for this reason is worth being investigated. I will offer an enriched picture of his thought and will offer a new reading of his argument, at least new with respect to Barrow's and Tipler's interpretation of Weyls take on this question [8]. In *The anthropic cosmological principle*, they noticed:

> Weyl pointed out that only in (3+1) dimensional space-time can Maxwells theory be founded upon an invariant integral form of the action; only in (3+1) dimensions it is conformally invariant and this "does not only lead to a deeper understanding of Maxwells theory, but the fact that the world is four dimensional, which has hitherto always been accepted as merely accidental becomes intelligible through it" [8], 260-261).[5]

In what follows, I shall spell out the content of the 1955 argument and then show how it represents an alternative to other approaches to the explanation of the world-dimensionality that use the anthropic principle. Finally, I expound the notion of topology of causation emerging from Weyl's work and which sort of guiding questions it leaves open to future investigation.

2. Why is the World Four-dimensional? (1955)

Even if his paper starts with a question, Weyl immediately admits that he is unable to offer an answer and rather aims at showing why theoretical physicists have been unable to answer until then. First, he distinguishes the question of the dimensionality of the world, which includes time, from the question of the dimensionality of space. In order to do so he makes two premises. First, Weyl endorses Minskowski's definition of the world, namely the world is "the totality of space-time-points, of possible localizations in space and time", so that, having space 3 and time 1 dimension, one assumes the existence of the four-dimensional world (which is what we want to explain and therefore assume it as given). Then Weyl makes a second assumption, according to which there is a crucial difference between the representation of space-time in special relativity and that of space-time in general relativity:

> The metric field is revealed as a agent of formidable reality. Now something that acts upon matter in such forcible fashion has no claim to be enthroned in sphinx-like rigidity above the ever-changing world of matter; it should itself be flexible and respond to the changing distribution of matter; he who makes suffer must suffer itself ([4], 210).

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[4] One of the most discussed examples of arguments on space dimensionality is Kant's [6][7]. The latter is to be considered as alternative to teleological approaches to this question and their adaptation into modern versions of the anthropic principle.

[5] The reference is to [9], 284.

Weyl thus immediately highlights that to look at the metrical structure of space-time is not enough to answer the question of the world-dimensionality. This is because the notion of Riemannian manifold is not bound to the special dimensionality 4, and as far as this metrical structure goes, any other dimension would do just as well. He then suggests to look at all possible laws of the physical phenomena that take place on this stage and asks whether they can be carried out in the same unambiguous way from 4 to any other number of dimensions. They are all such as to allow generalization to n dimensions and therefore, according to Weyl:

> There is no reason, until we do not discover other physical laws that work differently in specific dimensionalities, to think that there is something special in the Creator or in the Nature's choice of modelling a 4 dimensional world ([4], 211).

Precisely in this passage, Weyl discards the possibility to appeal to any anthropic principle to justify the dimensionality of the world. The four-dimensionality of the world appears to be something contingent. One could expect to see Weyl appealing to the causal structure of spacetime in order to argue for its necessity, but this is not the case. Surprisingly, in 1955, he depicts causation as a necessary but not sufficient condition to determine dimensionality. Causation is a conformal structure of the world prior to the dimensionality of space and this structure preserves the laws of physics invariant under the modification of the metrical field. However, not all the physical systems seem to be reducible to conformal structures. Therefore, even if for Weyl spacetime can be treated as a structure originated by the conformal or causal structure of the world, yet this cannot explain why the world is four dimensional. Consider the following passage, where Weyl shows in which terms the causal structure is a necessary condition:

> Locally, that is to say in the infinitesimal neighborhood of a world point P, the equation of the light cone
> $$ds^2 = \sum_{i,j} g_{ij} dx_i dx_j = 0$$
> describes the worlds causal structure. Knowing the causal structure, we do not know the g_{ij} themselves but only their ratios. The causal structure is not changed when the g_{ij} defining the metric are replaced by $\lambda \cdot g_{ij}$ with a positive factor of proportionality λ which is an arbitrary function of P. [...] It would not seem unreasonable to assume that the world is endowed not with a metric, but with a conformal or causal structure only, in other words that only lengths of line elements at the same point are comparable to each other. Then the laws of nature would not be affected by the modification $g_{ij} \rightarrow \lambda \cdot g_{ij}$ of the metrical field. Now it is a fact that the equations characterizing a harmonic linear tensor formula are invariant under this substitution if and only if $n = 2e$, i.e. if the dimensionality n is twice the rank e of the electromagnetic tensor f. If $r = 1, e = 2$ we would thus obtain the desired $n = 4$. ([4], 212-213)

At this point Weyl introduces the reason why conformal structure is a necessary but not sufficient condition for originating spacetime dimensionality:

> I wish I could close here, but I am afraid that it would be thoroughly dishonest. It is true that the laws of electromagnetic field in empty space presuppose only a conformal structure of the world, they are invariant with respect to the replacement of g_{ij} defining the metric by $\lambda \cdot g_{ij}$. But when we pass from the non-homogenous Maxwell equations which describe the generation of such fields by matter, or to Einsteins laws of gravitation even in empty space, this is no longer the case. The field laws are not conformally invariant neither for the gravitational field nor for the electromagnetic field in the presence of electric charge and current. ([4], 213)

Therefore, even if is certainly important to highlight the fact that Maxwell equations for the electromagnetic field in empty space, in particular the laws for the propagation of light, are conformal-invariant if and only if $n = 4$, still this does not lead very far. Weyl wants a deeper reason *why* the world has a Riemannian structure and in order to reach this goal, he reshaped the question of dimensionality as follows:

- Why is the orthogonal group among all groups of homogenous linear transformations the one that is characteristic of the local metric of the world?
- Why is this metric Pythagorean?

Weyl thus advances here the proposal of an alternative approach to arguments appealing to the anthropic principle or to conformal structure only. For him, the development of mathematics and topology could lead us to an answer for the question of the dimensionality of the world, even if he was unable to give an answer to such long-standing question.

3. Weyl's view of topology

Before spelling out in detail Weyl's argument for the methodology to search for the explanation of the world dimensionality, let us clarify why for him topology can guarantee the sufficient condition for such explanation. In his view, both conformal structure and topology together can justify the actual dimensionality of the world, but topology is assumed by Weyl to be the essential ingredient to reach this goal. Why? In *The mathematical way of thinking* (1940), Weyl clarified what is for him the causal structure of the universe: "At the ground of the words past, present and future referring to time we find something more tangible than time, namely the causal structure of the universe. Events are localized in space and time; an event of small extension takes place at a space-time or world point, a here-now" ([10], 70-71). According to Weyl, the topological scheme is what allows us to embrace both special and general relativity:

> A certain 4-dimensional scheme can be used for the localization of events, of all possible here-nows; physical quantities which vary in space and time are functions of a variable point ranking over the corresponding symbolically constructed 4-dimensional topological space. In this sense, the world is a four-dimensional continuum. The causal structure will have to be constructed within the medium of this 4-dimensional world, i.e. out of the symbolic material constituting our topological space. Incidentally, the topological viewpoint has been adopted on purpose, because only thus our frame becomes wide enough to embrace both special and general relativity theory. The special theory envisages the causal structure as something geometrical, rigid, given once and for all, while in the general theory it becomes flexible and dependent on matter in the same way as the electromagnetic field ([10], 77).

The topology of causation does not only embrace special and general relativity theories, but also preserves the distinction between them. Furthermore, there is another element to consider. General relativity theory establishes laws of nature that connect the flexible causal structure with other flexible physical entities, distribution of masses, or the electromagnetic field:

> These laws in which the flexible things figure as variables are in their turn constructed by the theory in an explicit a priori way. Of course the topological structure cannot be flexible as the causal structure is, but one must have a free outlook on all topological possibilities before one can decide by the testimony of experience which of them is realized by our actual world. To that end one turns to topology ([10], 81).

Topology can offer a solution to the question of the dimensionality of the world, in the sense that it is able to offer the necessary and sufficient condition for its justification, and this sufficient

condition is embodied by the fact that topological possibilities can be tested via experiments. In analyzing a continuum, like space, Weyl wants to proceed in "a more general manner than by measurement of coordinates and adopt the topological viewpoint, so that two continua arising one from the other by continuous deformation are the same to us" ([10], 74).

In other words, it means that one can construct the connection between here-now points and one can construct such a connection in view of its systematic exposition, which can lead us to express it in the form of a physical law.

4. Weyl's argument revisited

It is now clear that for Weyl topology has the special status of a field chiefly able of developing mathematical possibilities, but at the same time is the key-element that enables them to be experimentally checked. We have now more elements to understand and discuss Weyl's 1955 argument that goes as follows.

First, let us consider that to any group of transformations one can ascribe a structure, which is represented by the corresponding abstract group. The structure of the orthogonal group turns out to be quite different for different dimensions n. All orthogonal groups except that of dimension 4 have the structural property of being simple[6] and only to the 4-dimensional orthogonal group one can ascribe a more complex structure, being the direct product of two single groups.[7]

Weyl notices at this point of his argument that the orthogonal group for a positive definite quadratic form is closely related to the notion of sphere. Therefore, topology will reveal the elementary and highly significant differences in the behavior of spheres of different dimensions. Weyl was led to this insight by also considering Witold Hurewicz's studies on the homotopy group. By improving these studies, Weyl believed that important hints for the explanation of the world-dimensionality could come from the consideration that there are elementary and highly significant differences in the behaviour of spheres of different dimensions concerning the continuos mappings $S_n \to V$ of an n-dimensional sphere S_n into a given manifold or variety V. Such a mapping is considered trivial if one can deform it continuously into one that maps S into a single point of V. In the case where V is also a sphere, one obtains a sphere S_{n-d} of lower dimension $n - d$ than the first S_n. Now, the question is whether all mappings $S_n \to S_{n-d}$ are trivial or not and the answer depends only on the difference d of dimensions, at least as soon as $\geq 2(d-1)$. Therefore, one obtains the following answers:

- yes for $d = 4$ and 5
- no for $d = 1, 2, 3, 7$ (non-trivial mappings)
- no for $d = 2p - 3$ where p is a prime number.

That a simple property of a spherical space varies from dimension to dimension in a "strange way" and thus it is not insensitive to dimension points to the existence of a deeper level that pertains to mathematics, but that could also be expressed by a physical law. Unfortunately, Weyl notices that the research of this deepest level in which properties of spheres change from dimension to dimension is still unable to provide a definite answer. His hope was that:

> One day physics will discover laws of nature, the mathematical formulation of which takes into account of such structural features as are highly sensitive to dimension. Only this will allow us to explain and understand the specific character of the world actual dimensionality 4 ([4], 215).

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[6] A simple group has no normal subgroups other than itself and the identity, thus it cannot be split into smaller elements.

[7] In discussing this point, Weyl ironically points out that this fact is against the trivial idea that nature is simple.

5. Weyl's argument and the alternative to the Anthropic Principle

Weyl's treatment of the problem of the world-dimensionality implies a genuine philosophical question. To recognize that among different possible metrics only the Pythagorean metric embodies real physical space means to attribute to it a necessary status. However, if this representation of space-time is necessary, there must be a deep reason for it. And Weyl thought that a clue could be offered by the understanding of the uniqueness of the Lorentz group.

Weyl admits the possibility of recognizing a double status to the Pythagorean metric, namely it can be necessary and contingent at the same time. It is an "essence" that could be replaced by another one. To attribute the status of contingent necessity or necessary contingency to the dimensionality of the world does not rule out in principle theological and teleological arguments for the explanation of its dimensionality, but can lead us to the possibility of rethinking the conditions under which natural laws "generate" dimensionality.

This is the most difficult point of Weyl's approach and it goes beyond any analysis of Weyl's or any other argument on dimensionality. I refer here to what I call the *Problem of Generation* (PoG). It arises when one wants to define the principle or the rules according to which, independently from the generalization of the laws of physics to n-dimensions, a certain index and thus the four-dimensionality of the world arises. Which characters do we have to attribute to such a dimensionality generation? Does this generation imply the passage from the mere possibility or mathematical reality to existence? The PoG reveals that there is a profound link between the way in which we shape the strategy to explain world-dimensionality and the content of our scientific theories. And once this link is disclosed, also the effectiveness of mathematics and the question about our ability to produce physical theories describing and explaining the world receive a more refined answer.

Let us now go back to the question oulined at the beginning of this section. How could, in Weyl's view, the four dimensionality described by necessary physical laws be also contingent? The answer is simple, but it has important consequences on our way of conceiving of physical theories. It might be found indeed that there is a physical theory that is able to use other than the Lorentz group to represent features of space-time.[8]

This new mathematical entity would possess some new properties that enable the effectiveness of new physical laws in their application to the world and in explaining phenomena (which would render the Lorentz group applied to physics contingent), but it also entails the conservation of former essential properties of the Lorentz group and therefore the capacity of representing fundamental properties of the world (which still renders this group necessary).

Since for Weyl the particular representation of important features of the world, i.e. four dimensionality expressed via physical theories arises from the capacity of the Lorentz group of representing and capturing these essential features, to use another formalism or mathematical entity would automatically make the feature of four dimensionality contingent with respect to the new formalism and the new physical theory.

Thus the contingent character of the four dimensionality of the world would be just epistemological rather than an ontological one. This, of course, originates an immediate association to some versions of the anthropic principle, and apparently would seem to fit nicely with them. We have seen in the introduction that Barrow and Tipler did not take into account Weyl's late view of the question of the dimensionality of the world. Nevertheless, we are accostumed to consider their study of the anthropic principle as being independent from arguments, such as Weyl's 1955. Let me however ask the following question: does Weyl's 1955 argument challenge Barrow's and Tipler's view of the anthropic principle?[9] Barrow and Tipler

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[8] Notice that for Weyl mathematics is the realm of the open possibilities, whereas physics concers actuality. Mathematics is portrayed by Weyl as a constructive (creative) activity necessary to discover patterns and in this respect it is irreducible to physics.

[9] I shall not discuss the logical inconsistency of Barrow's and Tipler's argument, for such a discussion and its

considered different versions of the Anthropic Principle, and today the less problematic and acceptable one is called *Weak Anthropic Principle* (WAP):

> WAP: The observed values of all physical and cosmological quantities are not equally probable, but they take on values restricted by the requirement that there exist sites where carbon-based life can evolve and by the requirement that the Universe be old enough for it to have already done so ([8], 15).

Barrow and Tipler considered this as a restatement of the bare fact that in scientific practice it is essential to take into account the limitations of the measuring apparatus when interpreting observations ([8], 23). And, therefore, they considered our existence as generating a selection effect in assessing the probability of the observed features of the universe.

Weyl's 1955 approach to the question of the dimensionality of the world represents a challenge to WAP, and in a profound way. Indeed, the argument proceeds completely a priori by following a heuristic rule and what he called the topological scheme. Weyl's approach can be classified as relying on one premise that is very different from the postulate assumed by the anthropic principle. The latter presupposes taking the world as a whole [11], whereas we have seen that Weyl presupposes Minkowski's definition of the world as a totality of space-time-points, but meant to be only possible localizations in space and time. Starting from a completely different premise, Weyl's argument assumes an internalist perspective (internalist with respect to a scientific theory, namely relativity theory) and searches for an explanation that is to be found outside this theory by means of mathematics. He tried to assess whether the features of the Lorentz group governing the behaviour and the properties of the constitutive structure of spacetime can offer such an explanation, but he found that topology was not yet able to provide it, and therefore he had to limit his analysis in describing why scientists could not achieve an explanation for the four-dimensionality of the world.

Supporters of WAP devise a different strategy and methodology. Their argument is meant to justify observed values of physical and cosmological quantities and not to explain them. Therefore, basic features of the universe (shape, size, age etc.) must be observed to be of a type that allows the evolution of the observers, because our existence acts as a selection effect in assessing these features [8]. Therefore, given that on the ground of our existence and faculties we have to expect to observe a world possessing precisely three spatial dimensions, supporters of WAP are led to conclude that to search for an explanation of the world dimensionality is meaningless. One will notice that this argument is circular and fallacious. It is circular because the ground for the features to be observed as they are is also their consequence. It is fallacious and weak due to the necessary teleological link established between observed physical quantities and the evolution of the observers. Indeed, none of the physical theories used to interpret observations of the features of the universe needs the theory of evolution to be universally valid. More importantly, none of these theories has laws that only allow the observation of features or quantities directly connected to our evolution. What is then the justification for thinking that observed quantities should allow the evolution of observers? We can summarize the main tenets of WAP as follows:

- WAP assumes a premise according to which we cannot know the world-system as a whole in its totality on the base of observation only.
- WAP uses the existence of human beings and their intelligence to justify this premise, but in so doing, it pretends to exclude alternative ways of investigating basic features of the universe, including its dimensionality.
- WAP reverses the status of consequences and makes them appear as a ground and it does so without any justification.

formal reconstruction, see [11].

- WAP uses the existence of human beings to justify the observation of basic features of the universe by using principles that are valid outside the domain of the scientific theories that are supposed to explain these observed features.

The hidden premise of WAP is of a teleological nature and assumes that there must be a necessary and actual means-to-end relationship (that actually is just purely thought by us) between actual observations of physical quantities and biological evolution of the observers. This amounts to admit a sort of "driving force" linking the existence of intelligent beings and the features of the universe. This attitude inevitably leads to teleological arguments, one that assumes the formal characters of the kind used by Paley in the 19th century [12].

If one accepts to break the alleged mutual influence of the premises and consequences of WAP, due to the impossibility of explaining something existent on the ground of a possibility or a reality entailed in some scientific theories (not all of them), then one can easily see that the quest for the explanation of why and how does the world have such a dimensionality is still open and worth being investigated. One thus can and should possibly avoid to search for this explanation just on the ground of WAP.

6. When philosophy, mathematics and physics meet: Weyl's legacy

Weyl's 1955 argument in this sense provides a valuable hint as to the way to proceed. Furthermore, it shows that the problem is not to prove whether 3D and/or 4D are correct descriptions of the world or whether they are compatible with each other. As Weyl underlined, the crucial question is to explain the uniqueness of the four-dimensional Lorentz group, by looking for physical laws justifying it.[10] Weyl urged that the distinctive character of the four-dimensional Lorentz group, either as a group of linear transformations or as an abstract group, must have been clarified. Thus, when looking for an explanation of the dimensionality of the world, a first step can be a sort of search for a mathematical explanation. I say "sort of" because Weyl himself took it as something contingent, as something that should serve as a guiding principle in the investigation. The problem and the answer, indeed, are not only mathematical: Weyl asks whether another group can replace the four-dimensional Lorentz group in the construction of natural laws by also sharing with the known natural laws those features that are essential for the constitution of physical world and for describing the homogeneity of the four-dimensional world.

In other words, a second step in explaining the world dimensionality is necessary and cannot be considered as a mere analysis of the mathematical and formal properties of the group considered *per se*. This second step consists in finding out what is essential to the properties of the Lorentz group that allows physical laws of a scientific theory to be applicable to the actual world. This second step basically enables the transition from the level of pure possibility to the level of physical reality. And this is why for Weyl one cannot say to have an explanation for the dimensionality of the world if one cannot answer this fundamental question: "one cannot claim to have understood nature unless one can establish the uniqueness of the four-dimensional Lorentz group" ([13], 23). To answer this question also allows us to grasp which kind of features in group theory count in order to be able to recognize them in the future and to use them when giving birth to a new scientific theory.[11]

Before concluding, I would like to remark that, when elaborating his 1955 argument, Weyl was working at the Institute of Advanced Study in Princeton and could witness the debates surrounding difficulties, exhaltation and great results of both relativity and quantum mechanics. He was one of the few mathematical physicists who had access to the formalisms of both theories

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[10] Weyl defined the four-dimensional Lorentz group as 'basic group pattern of the universe' ([13], 22).

[11] It would be interesting to compare Weyl's argument with current studies on emergence of a 4D World and causal quantum gravity [14].

and mastered them. He therefore had the privilege to describe at a deep level how scientific explanation is a complex process and moreover a process constituted by degrees or levels. The latter also depend on the intrinsic historical dimension of mathematics that is never complete. This is the reason why Weyl identified a proper heuristic role of mathematical explanation in the making of a scientific theory and as a step to be included within the process of scientific explanation. These questions, above all that on the nature of scientific explanation, are widely discussed among philosophers of science and mathematics, but somehow the debates focus more on the analysis of the scientific theories at hand, rather than on the processes at stake when constructing a theory. The latter is something left to historians of science for reconstruction. However, in more recent times, we witnessed an exception in the debates surrounding the attempts at constructing a theory of quantum gravity or when exhamining the search for heuristic tools relying on the study of the properties instantiated by mathematical objects. If one wants to apply Weyl's methodology to current philosophy of physics and to the epistemology and methodology of theory construction, then there appear to be some guiding questions:

- Is it possible to trace the topological and differential structures of spacetime back to something that could be interpreted as a physical law?
- Can we understand the role of mathematics without taking into account physics?
- How should we conceive of mathematics within physics when looking for explanation?

Especially this last question touches the methodology of natural science that Weyl conceived in terms of symbolic construction. I won't enter into details here, because it would be further the aims of this paper, but I would like to offer the following quote as a stimulous for further reflection on our current views of the relationship between mathematics and physics and on the different ways in which human beings search for explanations:

> Not content with an answer to the question 'How is it?' we wish to know 'How did it come to be so?' Man, wherever he awakens to ponder the riddles of existence, is prone to expect *evolution* to enlighten him about the essence of things. [...] The experience of science accumulated in her own history has led to the recognition that evolution is far from being the basic principle of world understanding; it is the end rather than the beginning of an analysis of nature. Explanation of a phenomenon is to be sought not in its origin but in its immanent law. Knowledge of the laws and of the inner constitution of things must be far advanced before one may hope to understand or hypothetically reconstruct their genesis [15], 285-286.

Therefore, if Weyl's 1955 argument for the dimensionality of the world is to be classified, it should be recognized as an argument endowed of an internalist perspective, one that looks at mathematics as the most appropriate source for the internal analysis of physical reality.

7. Conclusion

Even if it was not possible for him to offer an explanatory answer to the question of the world dimensionality, Weyl believed that it was just a matter of waiting for the development of mathematics, and of topology in particular, to enlarge our understanding of the essence of space-time structure. His reflection clearly invest the foundations of physics, encourage a dialogue between philosophers and physicists in reflecting upon the relationship between mathematics and physics, on the character of scientific explanation and above all it forces philosophy to rethink of old problems, such as that of the contingent necessity of a being that could lead to the formulation of new philosophical concepts and therefore 'tools' to understand and give a meaning to the world. Thus Weyl's approach to the problem of the dimensionality of the world should be first of all understood as a methodology to obtain a deeper understanding of

the space-time structure according to physical laws, or better, by recalling Galileo's words, his approach aimed at encouraging us in spelling out the letters of the alphabet through which we can read the book of the universe.

Acknowledgments

References

[1] Petkov V 2007 Relativity, dimensionality, and existence *Relativity and the Dimensionality of the World* ed V Petkov (Dordrecht: Springer) pp 115-135

[2] Ohanian H C 2007 The Real World and Space-Time *Relativity and the Dimensionality of the World* ed V Petkov (Dordrecht: Springer) pp 81-100

[3] Tegmark M 1997 On the dimensionality of spacetime *Class. Quantum Grav.* **14** L69

[4] Weyl H 2013 Why is the world four-dimensional? *Levels of infinity: selected writings on mathematics and philosophy* ed P Pesic (NY: Dover) pp 203-216

[5] Callender C 2005 *Stud. Hist. Philos. Sci.* **36** 113

[6] Kant I 2012 Thoughts on the true estimation of living forces *Immanuel Kant: Natural science* ed E Watkins (Cambridge: Cambridge University Press) pp 1-55

[7] De Bianchi S, Wells J D 2015 *Synthese* **192** 287

[8] Tipler F J, Barrow J 1986 *The anthropic cosmological principle* (NY: Oxford University)

[9] Weyl H 1952 *SpaceTimeMatter* (Translated from the 4th German edition) ed H Brose (NY: Dover)

[10] Weyl H 2013 The mathematical way of thinking *Levels of infinity: selected writings on mathematics and philosophy* ed P Pesic (NY: Dover)

[11] Craig W L 1988 *Br. J. Philos. Sci.* **39** 389

[12] Paley W 1802 *Natural theology* (Philadelphia: H Maxwell)

[13] Weyl H 1948 *Similarity and congruence: a chapter in the epistemology of science* ETH Bibliothek, Hochschularchiv Hs 91a:31 23 Bl.

[14] Ambjørn J, Jurkiewicz J, Loll R 2004 *Phys. Rev. Lett.* **93** 131301

[15] Weyl H 1949 *Philosophy of mathematics and natural science* (Princeton: Princeton University Press)