Simulating relativistic binaries with `Whisky`

Luca Baiotti

Department of Earth Science & Astronomy, College of Arts and Science, University of Tokyo, 3-8-1 Komaba, Meguro-ku, Tokyo, 153-8902, Japan

Abstract

We report about our first tests and results in simulating the last phase of the coalescence and the merger of binary relativistic stars. The simulations were performed using our code `Whisky` and mesh refinement through the `Carpet` driver.

1 Introduction

Despite the presence of numerous works in the literature [1, 2, 3, 4, 5], the relativistic *binary neutron star problem* still poses a fundamental challenge in general relativity and in theoretical and observational astrophysics, as well as in numerical relativity. Furthermore, binary systems of compact objects are considered one of the most important sources for gravitational-wave emission and are thought to be at the origin of some of the most violent events in the Universe: (short) γ-ray bursts.

Among the additional motivations that make this problem so interesting, there is surely the investigation of gravitational waves, of their consistency with Einstein's theory and of their detectabilty in the now-operating gravitational-wave detectors. Detection of gravitational waves from relativistic-star binaries will provide a wide variety of physical information on the component stars, including their mass, spin, radius and equation of state.

As said, the study of relativistic-star binary systems is also finalized to the understanding of the origin of some type of γ-ray bursts, because the short rise times of the bursts imply that their central sources have to be highly relativistic objects [6]. After the observational confirmation that γ-ray bursts a have cosmological origin, it has been estimated that the central sources powering these bursts must provide a large amount of energy ($\sim 10^{51}$ ergs) in a very short timescale, going from one millisecond to one second (at least for a subclass of them, called *short γ-ray bursts*). It has been suggested that the merger of relativistic-star binaries could be a likely candidate for the powerful central source. The typical scenario is based on the assumption that a system composed of a rotating black hole and a surrounding massive disc is formed after the merger. If the disc had a mass is $\gtrsim 0.1 M_\odot$, it could supply the large amount of energy by neutrino processes or by extracting the rotational energy of the black hole.

In our previous work [7, 8, 9, 10], we have described how we can perform - with our code `Whisky`, mesh refinement (through the `Carpet` driver [11]) and without excision - accurate three-dimensional relativistic simulation of rotating relativistic-star collapse and how we can extract the (weak) gravitational signal emitted until and past the newly formed black-hole ring-down phase. We have now started to apply the `Whisky` code to investigate the binary problem (where the gravitational-wave signal is expected to be much stronger) and we report here our initial setup.

Hereafter, unless explicitly shown otherwise for convenience, we use a system of units in which $c = G = M_\odot = 1$.

2 Basic equations and their implementation

The `Whisky` code solves the general-relativistic hydrodynamics equations on a three-dimensional numerical grid with Cartesian coordinates [12]. The code has been constructed within the framework of the `Cactus` Computational Toolkit (see [13] for details). While the `Cactus` code provides at each time step and on a spatial hypersurface the solution of the Einstein equations $G_{\mu\nu} = 8\pi T_{\mu\nu}$, where $G_{\mu\nu}$ is the Einstein tensor and $T_{\mu\nu}$ is the stress-energy tensor, the `Whisky` code provides the time evolution of the hydrodynamics equations, expressed through the conservation-equations for the stress-energy tensor $T^{\mu\nu}$ and for the matter current density J^μ

$$\nabla_\mu T^{\mu\nu} = 0 \ , \qquad \nabla_\mu J^\mu = 0 \ . \tag{1}$$

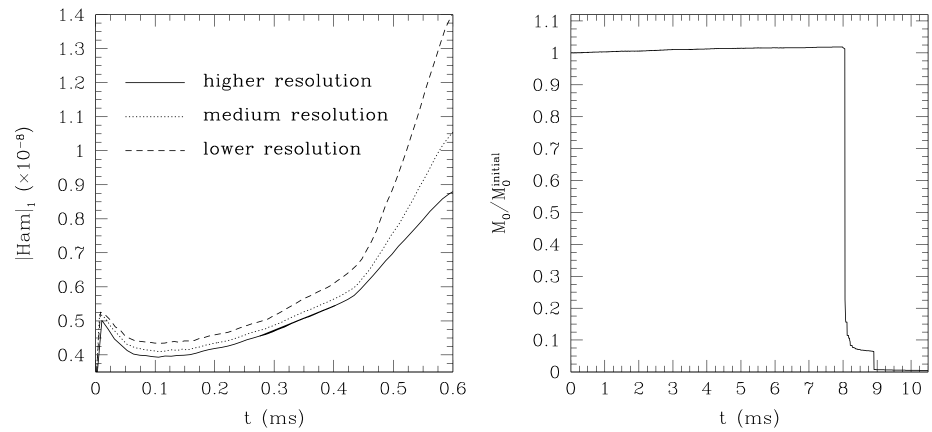

Figure 1: LEFT: Comparison of the Hamiltonian constraint violation for three resolutions. The spacing for the finest grid of the three resolutions are $0.016M$, $0.02M$ and 0.025. RIGHT: conservation of baryon mass.

Details on the system of field equations we use are given in [14] and in the previous `Whisky` articles [7, 8, 9, 10]. The code is designed to handle arbitrary shift and lapse conditions, which can be chosen as appropriate for a given spacetime simulation. More information about the possible families of spacetime slicings which have been tested and used with the present code can be found in [14, 15].

The singularity-avoiding properties of the above gauge choices have proved equally good both when using excision, as we did in [7] and [8], and when not using excision. In the latter case, these gauge choices are essential to "freeze" the evolution in those regions of the computational domain inside the apparent horizon, where the metric functions experience the growth of very large gradients. Furthermore, in this case a small additional dissipation in the metric and gauge terms is also necessary to obtain long-term stable evolutions [9].

An important feature of the `Whisky` code is the implementation of a *conservative formulation* of the hydrodynamics equations [16, 17, 18], in which the set of equations (1) is written in a hyperbolic, first-order and flux-conservative form of the type

$$\partial_t \mathbf{q} + \partial_i \mathbf{f}^{(i)}(\mathbf{q}) = \mathbf{s}(\mathbf{q}) \, , \tag{2}$$

where $\mathbf{f}^{(i)}(\mathbf{q})$ and $\mathbf{s}(\mathbf{q})$ are the flux-vectors and source terms, respectively [2]. Note that the right-hand side (the source terms) must not depend on derivatives of the stress-energy tensor.

Additional details on the formulation we use for the hydrodynamics equations can be found in [2]. We stress that an important feature of this formulation is that it has allowed to extend to a general-relativistic context the powerful numerical methods developed in classical hydrodynamics, in particular High-Resolution Shock-Capturing schemes based on exact [19, 20, 21] or approximate Riemann solvers (see [2] for a detailed bibliography). Such schemes are essential for a correct representation of shocks, whose presence is expected in several astrophysical scenarios.

3 Initial data and evolution

As initial data for relativistic-star binary simulations we use the ones produced by the group working at the Observatoire de Paris-Meudon [22, 23]. These data, which we refer to also as the *Meudon data*, are obtained under the simplifying assumptions of quasi-equilibrium and of conformally-flat spatial metric. These initial configurations are computed using a multi-domain spectral-method code named `LORENE`, which is a free software under the GNU General Public License; a specific routine then converts from spherical coordinates to a Cartesian grid of the desired dimensions and shape.

Except where explicitly indicated, the simulations we discuss here refer to evolutions of equal-mass irrotational initial data having the following properties: initial orbital period T=3.395 ms; rest mass of a star $M_0 = 1.625 M_\odot$; gravitational mass of a star $M = 1.456 M_\odot$; radius of a star R=13.68 km; coordinate distance between stellar

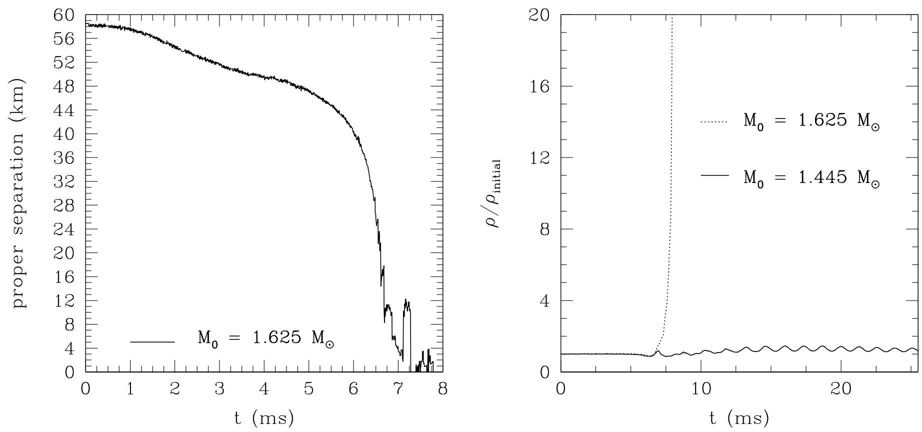

Figure 2: LEFT: Time evolution of the proper distance between the maximum–rest-mass-density points of the stars. RIGHT: Time evolution of the maximum of the rest-mass density, in the case of a prompt formation of black hole (dotted line) and in the case of the formation of an oscillating relativistic star.

centres 45km = 41 M_0 = 0.19 $\lambda_{\rm GW}$ = 3.4 R; compactness of a star M/R = 0.14; ratio of the polar to the equatorial radius of a star 0.93; polytropic exponent $\Gamma = 2$.

We performed evolutions with 8 refinement levels. In the case of the highest-resolution simulation, the finest grid covered only the interior of the stars, with a resolution of $\Delta x \simeq 0.016M$ and the coarsest one had the outer boundary at $\simeq 175M$. This is still work in progress, but the presently available data for the initial part of the time evolution seems to suggest that these resolutions are sufficient for reliable evolutions, as the time evolution of the rest mass also shows (right panel of Fig. 1). The rest mass should in principle be constant and the data represented in the figure indicate that the violation of the conservation, due to numerical errors, is less than 0.1%, until horizon formation, when this measure ceases to be meaningful. The convergence rate of the code for this kind of simulations, as measured through the norm of the Hamiltonian constraint violation, is 1.5. This is also shown in the left panel of Fig. 1.

One positive remark we can make at this point is that in our evolutions for the above-mentioned initial data we can follow the orbit of the stars for several periods, before the beginning of the plunge, as is shown in Fig. 2, which reports the time evolution of the proper distance between the maximum–rest-mass-density points (*i.e.* the centres) of the stars. Such a proper measure shows an approximately constant decrease of the distance, only slightly modulated by residual eccentricity. At the time of the beginning of the plunge, at around $t = 6$ ms, the proper distance between the stars is about 5 times the total initial ADM mass of the system. During the merger, the stars bounce shortly before collapsing as a single object to a black hole. This phase is illustrated in the figure by the spikes present between 6.5 and 8 ms. Even if for space reasons it is not reported in the present article, we can also simulate the ring-down phase of the newly formed black hole and extract the complete gravitational-wave signal.

The right panel of Fig. 2 shows the time evolution of the maximum of the rest-mass density. This illustrates two very different possible outcomes of the merger, namely the prompt collapse to a black hole, about 1 ms after the merger, and the formation of a compact object without (apparent) horizons. Such a compact object oscillates violently, emitting a persistent gravitational radiation of amplitude similar to the one emitted during the last phases of the coalescence. After dissipative effects (depending strongly on the equation of state) like shock heating and gravitational-radiation emission reduce the pressure and the angular momentum of the compact star, a delayed collapse to black hole is possible. Indeed, we observe this behaviour in some simulations now under investigation (not reported here).

In the near future, we plan to carry out a detailed study of relativistic-star mergers, investigating in particular the different dynamics and gravitational waveforms obtained with different initial data (masses) and different equations of state.

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