

# Lessons from supersymmetric black holes

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**Abstract.** This paper studies the D4-D0 system in light of black hole puzzles. The properties of its microstates, that live in the associated quiver quantum mechanics [1, 2], are discussed. It is shown that the pure Higgs states of these quivers satisfy the needed properties to be (part of) the microstates of a single centered D4-D0 black hole.

## 1. Introduction

Black holes are arguably one of the most fascinating objects that are predicted by general relativity, see e.g. [3]. Since their introduction by Schwarzschild in 1915, they continued to challenge the understanding of physicists with their unconventional properties and behavior. Among them are the notorious black hole paradoxes that emerged in the beginning of the 70' of the last century. Despite the relentless effort of many physicists to demystify these paradoxes, there is no consensus about even the right approach to tackle them. However, such efforts led to many unexpected fertile ideas that have changed our way of perceiving simple notions in physics such as spacetime[4, 5].

The black holes' paradoxes are the consequence of the combination of classical results about black holes (no hair theorem [6, 7], singularity theorems [8], cosmic censorship conjecture [9], laws of black hole mechanics [10]) and the study of quantum fields on black hole backgrounds [11]. Our interested in this paper is the entropy paradox. The hope is that if we can figure out the microstates that give rise to such entropy, one can try to tackle the other two paradoxes (singularity and information loss paradoxes) by building a theory based on these microstates and symmetry considerations. Our approach will be based on string theory construction of BPS black holes. More precisely, we will be dealing with half BPS solutions of  $\mathcal{N} = 2$  four dimensional supergravity (see e.g. [12]). The reason is threefold. First of all, being BPS allows us to have a control over the number of states as we change the value of the coupling constant. Secondly, stringy constructions give us a way to characterize the states at weak coupling. Lastly, the class of solutions that we will deal with have special characteristics that allows us to distinguish between black hole and non black hole microstates [1, 2, 13, 14].

Our paper is organized as follows. We start by reviewing the black holes' paradoxes. Then move on and briefly describe how black holes emerge in sting theory putting emphasis on half-BPS black holes of  $\mathcal{N} = 2$  four dimensional supergravity. In the process, some of their most important properties are enumerated. After that, we give a short account of the gauge theory



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description of such systems. This turns out to be a quiver quantum mechanics theory [15]. Our brief discussion includes a description of: how to construct the associated quiver given D-brane charges, the two branches that appear in the theory, and the associated vacuum states. We follow that with a short study of the D4-D0 system that includes a description of: the type of quivers we need to deal with, and partial results about the properties of its microstates [16]. We finish our paper by a short discussion of our results and open questions. Some technical details are left to the appendix.

## 2. Black holes' paradoxes

In this section, we will briefly review the origin of black holes' paradoxes. We will first spell out the classical general relativity properties of black holes. After that, we will mention what new physics one gets once quantum field theory is put in a black background. Finally we will enumerate the black holes' paradoxes.

### 2.1. Black holes in general relativity

Classically, black holes are very boring objects: They are essentially a singularity covered by a horizon. The latter is a global property and cannot be detected by an infalling observer who is forced to crash into the black hole singularity without a way out. Such a singularity is conjectured to be always shielded by a horizon [9]. Moreover, asymptotically flat four dimensional black holes are "unique" (no hair theorem) [6, 7]:

*"A Black hole solution is unique if we fix its conserved charges: Mass, Angular momentum, and Maxwell Charge."*

Despite this, [10] managed to realize that black holes are not completely unrestrained as they should satisfy four laws that they dubbed the black hole laws of mechanics ([6, chapter 7]):

- 0<sup>th</sup> Law :  $\kappa$  the surface gravity is constant on the horizon.
- 1<sup>st</sup> Law: The parameters of a black hole satisfy the following relation:

$$\delta M = \frac{\kappa}{8\pi} \delta A_h + \Omega_h \delta J + \Phi_h \delta Q$$

where:

- Quantities measured at the boundary:  $M$  = Black Hole mass,  $J$  = angular momentum, and  $Q$  = electric charge.
- Quantities measured at the horizon:  $A_h$  = horizon area,  $\kappa$  = horizon surface gravity,  $\Omega_h$  = angular velocity, and  $\Phi_h$  = electric surface potential.
- 2<sup>nd</sup> Law : Horizon area  $A_h$  is a non decreasing function of time.
- 3<sup>rd</sup> Law: There is no physical process that allows us to decrease the surface gravity of a black hole until it vanishes.

These four laws strangely resemble the laws of thermodynamics if one identifies entropy with the black hole horizon area and temperature with the surface gravity. However, at that time, it was believed that this analogy is a mathematical one without any physical implications. This is because black holes do not "radiate". However, physical reality of the analogy was proved shortly after. This is the subject of the next section.

### 2.2. Black holes and thermodynamics

It was the PhD student Bekenstein that first realized that black holes should have an entropy [17, 18]. His idea was simple and relies on the following thought experiment. Suppose that we have an object that carries some entropy which falls into a black hole. The moment this

object crosses the horizon, the whole universe will lose part of its entropy. This means that entropy of the universe can decrease with time: a clear contradiction with the second law of thermodynamics. The solution of Bekenstein is to associate to the black hole an entropy proportional to its horizon area. This was put on firmer grounds by Hawking [11] when he proved that black holes radiate at the Hawking temperature:

$$T_H = \frac{\kappa}{2\pi}$$

implying that the black hole entropy is given by the relation:

$$S_{BH} = \frac{1}{4} A_h$$

This was achieved by doing quantum field theory in a black hole background. These findings gave black holes a new identity: thermodynamical objects. Such an identity led to the black paradoxes that have been challenging the physicists till this day.

### 2.3. Black holes' paradoxes

We close this section by spelling out the black holes' paradoxes.

*2.3.1. The entropy paradox:* There is a clear contradiction between the no hair theorem and the fact that black holes have an entropy. This is because the entropy is supposed to reflect the number of states with the same macroscopic physical quantities. Hence, the question:

*“What does the entropy of a black hole count?”*

The other intriguing aspect of the black holes' entropy is that it is proportional to a surface and not a volume. Such a property seems to be in contradiction with the extensive nature of the entropy. Hence, the second puzzling question:

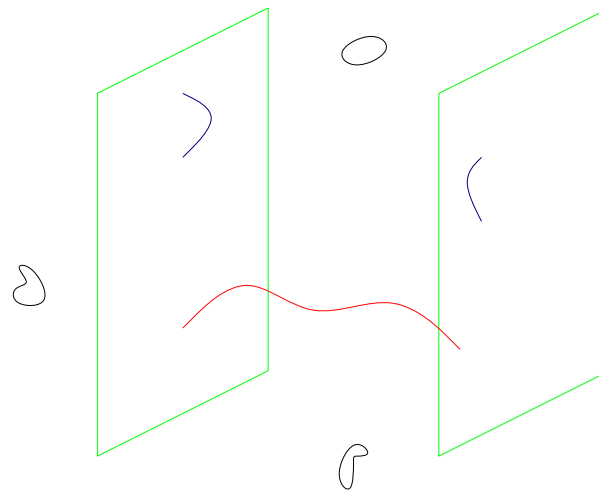
*“Why is the black hole entropy proportional to the area of its horizon?”*

We will try to answer the first question in this paper in a very simple set up. Unfortunately, we will have nothing to say about the second question in the present context.

*2.3.2. The information loss paradox:* We will just have a brief description of the paradox here. For more details check e.g. [19]. As with the previous paradox, there are two aspects to this one as well. Both aspects are related to the evaporation of black holes through Hawking radiation.

- The first aspect of the present paradox has to do with the information about the matter that formed the black hole. Since the Hawking radiation is a perfect thermal one [11, 20], then it cannot carry any type of information. This becomes a huge problem if the black hole completely evaporates destroying the unitarity of the physical process: formation and evaporation of a black hole.
- The second aspect of the present paradox, which is usually overlooked, has to do with the nature of the Hawking radiation. The latter is built from entangled particles with one escaping to the asymptotic infinity (measured radiation) and the other falling behind the black hole horizon. This means that after a certain time we will lose the information about this entanglement.

Unfortunately, there is no popular approach to solve this issue. For some proposals check e.g. [21, 22] and references therein.



**Figure 1.** A depiction of D-branes and different kinds of strings: open and closed.

*2.3.3. The singularity:* This last paradox is rarely discussed. There is an agreement in the community that the black hole singularity should not be there. This is because we cannot trust general relativity near it as the curvature of spacetime blows up. However, there is no consensus on how the singularity is resolved.

After this brief review of the black hole paradoxes, let us move on to the description of our system that we will use to address the black hole entropy paradox.

### 3. Half-BPS solutions of $\mathcal{N} = 2$ four dimensional supergravity

We are ready to shift gears and discuss briefly black holes in string theory. This section is meant to give a simple picture of how black holes are described in string theory. We will first give an oversimplified description of how black holes emerge in string theory. Then, we will move on and briefly describe our class of black holes of interest to us.

#### 3.1. D-branes and black holes

String theory, as its name indicates, is a theory of vibrating strings. These are one-dimensional extended objects. In addition to them, there are other extended objects called D-branes, see e.g. [23]. These are hypersurfaces that open strings can end on. See figure fig.1 for an example. These branes have both mass and charge which is pivotal in our constructions of black holes. Let us describe this in a very simple set up.



**Figure 2.** A simplified depiction of black holes in string theory. On the left, our empty spacetime and on the right the emergence of a non-trivial gravitational field due to the presence of a D1-brane wrapping the internal circle  $S^1$ . The ball on the right will be attracted to the black hole.

Let us suppose that we live in a two-dimensional spacetime  $\mathbb{R}^2$  whereas string theory lives in one dimension higher. We suppose the latter to be a circle  $S^1$ , see picture fig.2 for more details.

A D1-brane can wrap around the circle  $S^1$  whose size will change due to the mass and charge of the D1-brane. This is because gravitational attraction due to mass will try to shrink this circle, whereas the Coulomb repulsive force will try to increase its size. The neighboring circles will react to this by changing their sizes as well. This is a result of smoothness requirement of spacetime geometry. As a result, we end up with the impression that we live in a non-trivial gravitational field sourced by a charged black hole sitting somewhere in our spacetime. This simple story generalizes to the full string theory, where now, the extra dimensions can be very complicated allowing for different kinds of charged black holes. Let us now move on to theory we are interested in.

### 3.2. $\mathcal{N} = 2$ four dimensional supergravity and its half-BPS black holes

The compactification of type IIA string theory on a Calabi-Yau threefold leads to  $\mathcal{N} = 2$  four dimensional supergravity theory [24]. Such a theory has a special class of solutions called half-BPS solutions because they preserve half of the supersymmetry charges the theory offers. These solutions are bound states of black holes. From the point of view of string theory, these solutions are the manifestation of D-branes wrapping different cycles in the internal Calabi-Yau manifold. See the simple example discussed earlier. Hence, the solutions will be parameterized by vectors  $\Gamma_a = \{p^0, p^A, q_A, q_0\}$  in the even cohomology lattice of the compactification Calabi-Yau that encodes the  $\{D6, D4, D2, D0\}$  brane charges [12]. The most general solution was found in [25, 26, 27] and it is defined by the positions  $\vec{r}_a \in \mathbb{R}^3$  and the charges  $\Gamma_a$  of the black hole centers. The positions  $\vec{r}_a$  should satisfy the integrability/bubble equations:

$$\sum_{b \neq a} \frac{\langle \Gamma_a, \Gamma_b \rangle}{r_{ab}} = \langle h, \Gamma_a \rangle; \forall a,$$

where  $\langle \cdot, \cdot \rangle$  is the symplectic pairing of electric and magnetic charges:

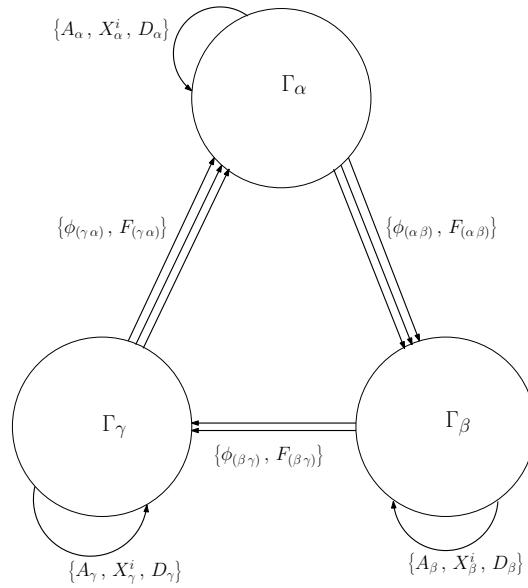
$$\langle \Gamma_{(1)}, \Gamma_{(2)} \rangle = -p_{(1)}^0 q_0^{(2)} + p_{(1)}^A q_A^{(2)} - q_A^{(1)} p_{(2)}^A + q_0^{(1)} p_{(2)}^0,$$

and  $h$  in a polyform in the even cohomology of the compactification Calabi-Yau that fixes the moduli at infinity. A necessary condition for the bubble equations to have solutions is:

$$\sum_a \langle h, \Gamma_a \rangle = 0.$$

We will always assume that this is the case. Furthermore, these solutions manifest the following important properties:

- Some solutions to the bubble equations will not exist for all values of the moduli  $h$ . This is because the  $r_{ab}$  are distances and hence should be positive and satisfy the triangle inequalities. As one varies the moduli  $h$ , such solutions decay at codimension one hypersurfaces in the moduli space. This is the celebrated wall crossing phenomenon, see the review [28] and references therein for more details.
- The single centered black hole solutions exist only in the scaling regime of charges. Furthermore, they exist for all value of the moduli  $h$ .
- Some class of multi-centered black hole solutions do exist for all values of the moduli  $h$ . They are dubbed the scaling solution as the distances  $r_{ab}$  can be made arbitrary small [29, 1, 30]. Of course, such solutions exist only for special values of the charges  $\Gamma_a$ .
- Some multi-centered black hole solutions have more entropy than a single centered solution with the same total charges [1]. Hence, we should characterize the black hole states exactly if we want to explain its entropy.



**Figure 3.** A three nodes quiver with the different bosonic fields that enter in the Lagrangian.

The next step after constructing solutions is to count the number of physically distinct ones. The authors of [1] proposed that this number is the same as the multiplicity of the solution's angular momentum:

$$\vec{\mathcal{J}} = \frac{1}{2} \sum_{\alpha < \beta} \frac{\langle \Gamma_\alpha, \Gamma_\beta \rangle \vec{r}_{\alpha\beta}}{r_{\alpha\beta}}.$$

Notice that such angular momentum depends on the solution to the bubble equations. Another proposal was put forward by the authors of [31], where phase space quantization techniques (see e.g. [32, 33, 34]) were used to directly quantize the space of solutions of supergravity. This was the gravity description. String theory offers a different but equivalent description of the solutions above. The idea is to look at the underlying D-brane system at weak coupling. Supersymmetry then give us the means to make sure that the number<sup>1</sup> of quantum states will not change in the process.

#### 4. The dual gauge theory description

In the weak coupling regime, the half-BPS states, we are interested in, are effectively described by  $\mathcal{N} = 4$  quiver quantum mechanics. The latter is obtained through reducing the system of supersymmetric branes wrapping different cycles in the internal Calabi-Yau to one dimension [15]. Roughly speaking, we associate to each D-brane of charge  $\Gamma_\alpha$  a node with gauge group  $U(N_\alpha)$ , where  $N_\alpha$  is the number of D-branes that source this charge. We connect the nodes  $\Gamma_\alpha$  and  $\Gamma_\beta$  with  $|\langle \Gamma_\alpha, \Gamma_\beta \rangle|$  chiral fields. These chiral fields sit in a bifundamental representation of  $U(N_\alpha) \times U(N_\beta)$ . They are depicted pictorially as arrows with the direction fixed by the sign of  $\langle \Gamma_\alpha, \Gamma_\beta \rangle$ , if it is positive then the arrow starts at node  $\alpha$  and finishes at node  $\beta$  and vice versa. See fig.3 for an example.

To be able to describe the BPS states we are after, we need to be more precise<sup>2</sup>. To each node  $\alpha$ , we associate a vector multiplet  $(A_\alpha, X_\alpha^i, D_\alpha)$ ;  $i = 1, 2, 3$ , with the gauge group  $U(N_\alpha)$ . To

<sup>1</sup> Strictly speaking it is the index that do not change.

<sup>2</sup> We will omit the fermions as they do not play an important role in our discussion

each arrow  $(\alpha\beta) : \alpha \rightarrow \beta$  we associate a chiral multiplet  $(\phi_{(\alpha\beta)}, F_{(\alpha\beta)})$  that transforms in the  $(N_\alpha, \bar{N}_\beta)$  of  $U(N_\alpha) \times U(N_\beta)$ . Remember that there are  $|\langle \Gamma_\alpha, \Gamma_\beta \rangle|$  of them. All fields  $D_\alpha$  and  $F_{(\alpha\beta)}$  are auxiliary fields and satisfy the following matrix equations:

$$m_\alpha D_\alpha = \theta_\alpha \mathbb{I}_{N_\alpha} + \sum_{a:\beta \rightarrow \alpha} \phi_{(\beta\alpha)}^a \phi_{(\beta\alpha)}^{a\dagger} - \sum_{a:\alpha \rightarrow \beta} \phi_{(\alpha\beta)}^{a\dagger} \phi_{(\alpha\beta)}^a,$$

$$F_{(\alpha\beta)}^a = - \frac{\partial \mathcal{W}}{\partial \phi_{(\alpha\beta)}^a},$$

where the superpotential  $\mathcal{W}$  exists only if there is a closed loop in the quiver,  $\theta_\alpha$  are the Fayet-Iliopoulos parameters, and  $\sum_{a:\alpha \rightarrow \beta}$  means sum over all arrows  $a$  that start at node  $\alpha$  and end at node  $\beta$ . The first equation above is called the D-term equation, whereas the second one is called the F-term equation.

The BPS states in this description are related to the vacuum of the theory. It turns out that there are two branches [15] that we will briefly describe below.

#### 4.1. The Coulomb branch

In this branch, the chiral fields  $\phi$  vanish in the vacuum:

$$\phi_{(\alpha\beta)}^a = 0, \quad \forall \alpha, \beta, a.$$

This implies, thanks to the D-term and F-term equations, the vanishing of the  $D$  and  $F$  fields. Hence, the only remaining fields to consider are  $X_\alpha^i$  which turn out to be diagonal matrices:

$$X_\alpha^i = x_\alpha^i \times \mathbb{I}_{N_\alpha},$$

where  $x_\alpha^i$  are some real numbers. As was discovered in [15] this is not the end of the story. Adding quantum corrections force the real numbers  $x_\alpha^i$  to satisfy the constraints [15]:

$$\sum_{\beta \neq \alpha} \frac{\langle \Gamma_\alpha, \Gamma_\beta \rangle}{x_{\alpha\beta}} = 2\theta_\alpha; \quad \forall \alpha.$$

where  $x_{\alpha\beta} = |\vec{x}_\alpha - \vec{x}_\beta|$ . This is reminiscent of the bubble equations in supergravity.

#### 4.2. The Higgs branch

In this branch, the fields  $\phi_{(\alpha\beta)}^a$  are non-trivial. It turns out that the fields  $X_\alpha^i$  take the form [15]:

$$X_\alpha^i = x^i \times \mathbb{I}_{N_\alpha},$$

where  $x^i$  are the same real numbers for all nodes. On top this, one gets the following matrix equations [15]:

$$\theta_\alpha \mathbb{I}_{N_\alpha} = \sum_{a:\alpha \rightarrow \beta} \phi_{(\alpha\beta)}^{a\dagger} \phi_{(\alpha\beta)}^a - \sum_{a:\beta \rightarrow \alpha} \phi_{(\beta\alpha)}^a \phi_{(\beta\alpha)}^{a\dagger},$$

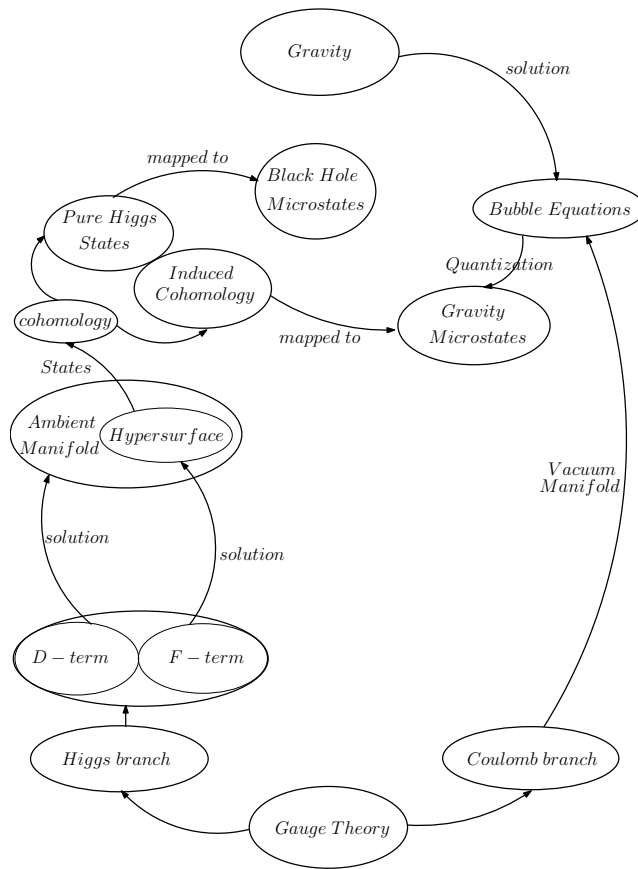
$$0 = \frac{\partial \mathcal{W}}{\partial \phi_{(\alpha\beta)}^a},$$

which are called respectively the *D-term* and *F-term* constraints. Notice that a condition for the existence of a non-trivial solution is that:

$$\sum_\alpha N_\alpha \theta_\alpha = 0.$$

This can be interpreted as the decoupling of the center of mass degrees of freedom.

A natural question to ask at this point is: *are the descriptions described earlier completely equivalent?* After all, they are valid in different regimes of parameters. We will give a brief description of the answer that seems to emerge in the next section.



**Figure 4.** A schematic depiction of the relation between the half-BPS microstates of the different descriptions of  $\mathcal{N} = 2$  four dimensional supergravity.

### 5. Gravity vs gauge theory

After, we gave different descriptions for the same D-brane system in the previous two section. Let us now describe how are their microstates interrelated. The actual understanding of these relations is given in picture fig.4 above.

The easiest descriptions to compare are the supergravity and the Coulomb branch ones. As already noticed before, the vacuum manifold is defined by similarly looking equations: the bubble equations. So the two descriptions are equivalent, at least classically. This survives after adding quantum corrections due to the combined effect of a non-renormalization theorem<sup>3</sup> [15] and the fact that we can use the same procedure to quantize the space solutions of both descriptions [31]. Hence, the supergravity and the Coulomb branch are equivalent and we will not make a distinction between them in this paper.

What about the Coulomb branch and the Higgs branch? To answer this question, we need first discuss the nature of BPS states in the Higgs branch. Typically, they are defined by the cohomology of a complete intersection manifold defined by the F-term constraints inside an ambient manifold defined by the D-term constraints [1]. It turns out that in the absence of the superpotential  $\mathcal{W}$ , the number of BPS states in the Coulomb and Higgs branches agree [15]. However, when the superpotential is present the story is more involved [1]. In the non-scaling regime where one can tune the moduli to separate the centers, the number of the BPS states

<sup>3</sup> See also the quick discussion in [31] in section-4.1 and also the reference [35].

on the Higgs and Coulomb branches agree. This equality breaks down in the scaling regime: the number of BPS states in the Higgs branch is bigger than the corresponding number in the Coulomb branch. To be more precise, the cohomology of the complete intersection manifold has two components [2, 13, 14]. A part that is inherited from the ambient manifold, that we will call the induced cohomology, and a part that is inherent to the complete intersection manifold. We will call the latter the singular/primitive cohomology. It is conjectured, and actually checked in several cases, that the BPS states that are described by the induced cohomology are mapped in a one-to-one manner to the Coulomb branch BPS states, whereas the ones associated to the singular cohomology exist only on the Higgs branch [2, 13, 14]. The map between the induced cohomology and the Coulomb branch ground states is even stronger as the BPS states of both branches sit in similar spin multiplets. In this paper we will give another evidence to this picture. In the remaining of this paper, we will summarize some of the results of [16]. We will be very brief without delving into the technical details.

## 6. The D4-D0 system: I- The associated quiver

After we briefly reviewed the general story of half-BPS solutions of  $\mathcal{N} = 2$  four dimensional supergravity, let us now concentrate on a specific set of solutions that is believed to include the D4-D0 black hole. Our discussion generalizes what was done in [31, 36]. To be more precise, we will be interested in a system with charges  $(\Gamma_1, \Gamma_2, N\Gamma_*)$  such that:

$$\langle \Gamma_1, \Gamma_2 \rangle = \kappa, \quad \langle \Gamma_*, \Gamma_1 \rangle = \langle \Gamma_2, \Gamma_* \rangle = \eta,$$

where  $\kappa$  and  $\eta$  are some positive integers. In the case of the D4-D0 system of [31, 36], we have the identification:

$$\Gamma_1 = \bar{\Gamma}_6, \quad \Gamma_2 = \Gamma_6, \quad \Gamma_* = \Gamma_0.$$

The idea [16] is to allow for a generic partition of  $N$  to account for all BPS states:

$$N = \sum_{\mu} \mu N_{\mu}.$$

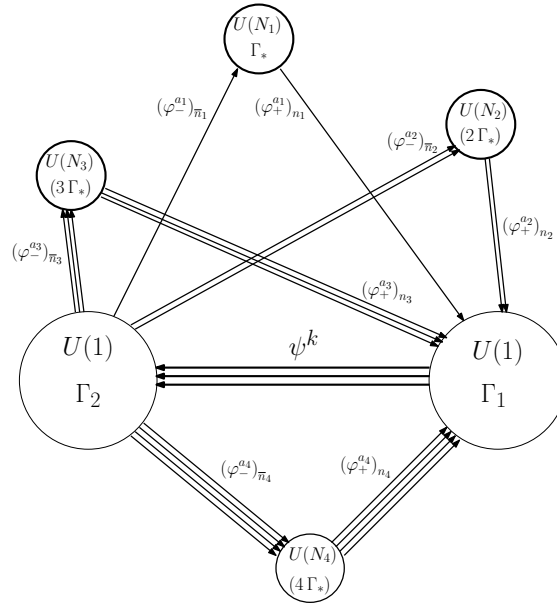
At the end we need to sum over all possible partitions of  $N$ . We will be calling the quivers associated to such partitions the *starfish quivers*, see fig.5 for an example. In the quivers of interest to us, the nodes associated to the charges  $\Gamma_1$  and  $\Gamma_2$  are  $U(1)$  whereas the nodes associated to the charges  $(\mu\Gamma_*)$  are generically non-Abelian:  $U(N_{\mu})$  for the node  $(\mu\Gamma_*)$ . There are arrows between the nodes  $\Gamma_1$  and  $\Gamma_2$ ,  $(\mu\Gamma_*)$  and  $\Gamma_1$ , and  $\Gamma_2$  and  $(\mu\Gamma_*)$ , but no arrows between the different  $(\mu\Gamma_*)$  nodes. We will use in what follows the shorthand notations:

$$\mu\eta = q_{\mu}, \quad \mathcal{Q} = N\eta = \sum_{\mu} q_{\mu} N_{\mu}, \quad \mathfrak{N} = \sum_{\mu} N_{\mu}^2.$$

The scaling regime in this case corresponds:

$$\kappa < 2\mathcal{Q} - \mathfrak{N}$$

The simplest case of a three nodes quiver with all of its nodes are  $U(1)$  was studied in detail in [2]. The present work is a slight generalization where we allow for non-Abelian nodes and more nodes, as described earlier. However, what we are doing is not the most general case. In the remaining of this paper, we will spell out some of the results derived in [16].



**Figure 5.** An example of a starfish quiver with  $\mu = 1, 2, 3$  and  $4$  depicted. The pairing between charges is such as:  $\langle \Gamma_1, \Gamma_2 \rangle = 3$ , and  $\langle \Gamma_*, \Gamma_1 \rangle = \langle \Gamma_2, \Gamma_* \rangle = 1$ . The different chiral fields that are involved in the discussion of the Higgs branch are also depicted.

### 7. The D4-D0 system: II- The Coulomb branch

We are going to adopt the gravity description as it identical to the Coulomb branch. Following the approach developed in [31, 36] in the case where the moduli satisfy:

$$\langle h, \Gamma_* \rangle = 0, \quad \langle h, \Gamma_1 \rangle = -\langle h, \Gamma_2 \rangle = h_*$$

we find two chambers.

*7.0.1. The threshold chamber* This corresponds to  $h_* > 0$ . The microstates sit in  $SO(3)$ -spin multiplets with spin  $j$  given by equation:

$$j = \frac{\kappa - 1}{2} - \sum_{\mu} \sum_{i=1}^{N_{\mu}} \left( m_i^{\mu} + \frac{1}{2} \right).$$

The degeneracy of each spin multiplet is given by the number of  $N_{\mu}$ -multiplets of integers  $\{m_i^{\mu}\}$  satisfying the inequalities:

$$0 \leq m_1^{\mu} < m_2^{\mu} < \dots < m_{N_{\mu}}^{\mu} < q_{\mu}, \quad \sum_{\mu} \sum_{i=1}^{N_{\mu}} \left( m_i^{\mu} + \frac{1}{2} \right) \leq \frac{\kappa - 1}{2}.$$

Furthermore, in the non-scaling regime, one finds the following degeneracy:

$$\mathcal{N} = (\kappa - \mathcal{Q}) \prod_{\mu} \binom{q_{\mu}}{N_{\mu}}$$

### 7.1. The scaling chamber

This chamber corresponds to  $h_* < 0$  and exists only in the scaling regime. The microstates sit in SO(3)-spin multiplets with spin  $j$  given by equation:

$$j = \frac{2\mathcal{Q} - \kappa - 1}{2} - \sum_{\mu} \sum_{i=1}^{N_{\mu}} \left( m_i^{\mu} + \frac{1}{2} \right).$$

The degeneracy of each spin multiplet is given by the number of  $N_{\mu}$ -multiplets of integers  $\{m_i^{\mu}\}$  satisfying the inequalities:

$$0 \leq m_1^{\mu} < m_2^{\mu} < \dots < m_{N_{\mu}}^{\mu} < q_{\mu}, \quad \sum_{\mu} \sum_{i=1}^{N_{\mu}} \left( m_i^{\mu} + \frac{1}{2} \right) \leq \frac{2\mathcal{Q} - \kappa - 1}{2}.$$

Let us turn our attention to the Higgs branch.

## 8. The D4-D0 system: III- The Higgs branch

This section relies heavily on some notions that are summarized in Appendix A. We encourage the reader to check it before reading this section.

We have as well two chambers in this branch with identical existence regimes as in the gravity description. However, we need to deal with two types of microstates: the induced one and the singular one. It turns out that [16] the induced states sit in Lefschetz spin multiplets with the same characterization as their counter parts on the gravity side. This is true for both chambers. What about the total degeneracy? To evaluate it, we calculate the total Euler number.

### 8.1. The threshold chamber

We find in this chamber that the total Euler number is given by the contour integration [16]:

$$\chi(\mathcal{M}_t) = \frac{1}{N_{\mu}!} \oint dh \left( \frac{1+h}{h} \right)^{\kappa} \left( \prod_{i=1}^{N_{\mu}} \left[ dx_i \left( \frac{1+x_i}{x_i} \right)^{q_{\mu}} \right] \right) \prod_{m=1}^{N_{\mu}} \left( \frac{h+x_m}{1+h+x_m} \right)^{q_{\mu}} \prod_{k \neq \ell} \left( \frac{x_k - x_{\ell}}{1+x_k - x_{\ell}} \right)$$

We can show [16] that in the non-scaling regime, the total Euler number is given by:

$$\chi(\mathcal{M}_t) = (\kappa - \mathcal{Q}) \prod_{\mu} \binom{q_{\mu}}{N_{\mu}}$$

### 8.2. The scaling chamber

One finds the following expression for the total Euler number in this chamber [16]:

$$\chi(\mathcal{M}_s) = \frac{1}{p_{\mu}!} \oint dh \left( \prod_{i=1}^{p_{\mu}} dz_i \right) \left( \frac{1+h}{h} \right)^{q_{\mu} - \kappa} \prod_{j=1}^{p_{\mu}} \left( \frac{1+z_j}{z_j} \right)^{q_{\mu}} \prod_{a=1}^{p_{\mu}} \left( \frac{h+z_a}{1+h+z_a} \right)^{q_{\mu}} \prod_{k \neq \ell} \left( \frac{z_k - z_{\ell}}{1+z_k - z_{\ell}} \right)$$

where:

$$p_{\mu} = q_{\mu} - N_{\mu}$$

Let us turn out attention to the properties of pure Higgs states

## 9. Properties of the pure Higgs states

Following the conjecture proposed by [2, 13, 14], the pure Higgs states should be the microstates of a single centered black hole with the same total charges. For this to hold, the pure Higgs states should satisfy the properties that a single centered black hole satisfies. We will enumerate these properties in the following [16].

1. *Non existence in the non-scaling regime:* The pure Higgs states of our system do not exist in the non-scaling regime. This is because of the equality between degeneracy of induced states and the total degeneracy in this regime.
2. *Stability under wall crossing:* The pure Higgs states do not disappear as we move between the two chambers: scaling and threshold. This is shown by evaluating the jump in the degeneracy of induced states and the total degeneracy. We find for both cases:

$$\delta\mathcal{N} = (\kappa - \mathcal{Q}) \prod_{\mu} \binom{q_{\mu}}{N_{\mu}}$$

This is in agreement with the degeneracy of states in the non-scaling regime.

3. *Entropy:* The final test that the pure Higgs states should pass is that their degeneracy should reproduce the entropy of the associated single centered black hole. Unfortunately, we have not been able to check this.

## 10. Conclusions and outlook

We have reviewed in this paper the origin of the black hole paradoxes. After that, we discussed a stringy inspired set up where one can hope to explain the entropy of black holes by counting the underlying microstates. In a relatively simple case, we checked that the conjectured microstates (pure Higgs states) share the same properties as their black hole counterparts. However, we are still missing the last bit that has to do with reproducing the entropy of these black holes. The techniques developed in [37] might be of use here.

Another thing that we hope to achieve, is to compare the behavior of the number of pure Higgs states with the other states in order to understand the entropy enigma [1]. We would like also to generalize our techniques to other types of black holes.

On the long run, the information paradox should be tackled. For this, one needs to generalize the ideas here to more realistic non-supersymmetric black holes. However, we do not know what will replace the quiver quantum mechanics in this case. After all, the emergence of the quiver quantum mechanics and the beautiful mathematics we used relied heavily on the fact that we are dealing with BPS states. Hence, going beyond supersymmetric black holes is not as trivial as one might hope.

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## Appendix A. Some results from algebraic topology

A summary of some results from algebraic geometry is given in this appendix. In the following, we denote by  $\mathfrak{M}$  a Kähler manifold, and by  $\mathcal{E}$  a complex vector bundle. Its base will be denoted by  $\mathcal{M}$  whose complex dimension is  $\mathfrak{d}$ . The rank of the fiber of  $\mathcal{E}$  will be denoted by  $n$ .

### Appendix A.1. The Betti numbers

As was briefly mentioned in section-5, the BPS microstates in the Higgs branch are associated to the cohomology of the vacuum manifold. Remember that the cohomology of a manifold is a set of special forms (these are the natural objects that can be integrated on cycles) [38]. They can be seen as a vector space (more precisely a lattice). The first thing we are interested in is counting states, which is just the dimension of the associated cohomology. This dimension is known as the Betti number and denoted by  $b^m(\mathfrak{M})$ , with  $m$  the degree of the cohomology.

### Appendix A.2. The Lefschetz spin

There is a more refined characterization of the cohomology of a Kähler manifold. These numbers are called Hodge numbers and denoted by  $h^{(p,q)}(\mathfrak{M})$ . They encode the holomorphicity of the forms associated to the cohomology of  $\mathfrak{M}$ . We have the following properties:

$$b^n(\mathfrak{M}) = \sum_{p+q=n} h^{(p,q)}(\mathfrak{M})$$

$$h^{(p,q)}(\mathfrak{M}) = h^{(q,p)}(\mathfrak{M}) = h^{(\mathfrak{d}-p,q)}(\mathfrak{M})$$

Due to this properties, one can define an  $SL(2)$  action on the cohomology of  $\mathfrak{M}$ , called the Lefschetz  $SL(2)$  action, as follows:

$$\mathcal{L}_+ = \mathcal{J} \wedge, \quad \mathcal{L}_- = \iota_{\mathcal{J}}, \quad \mathcal{L}_0 = \frac{1}{2} (deg - \mathfrak{d}),$$

where  $\iota_{\mathcal{J}}$  is the contraction with  $\mathcal{J}$  the Kähler two form of  $\mathfrak{M}$ , and  $deg$  is the degree of the form. The existence of such action allows us to split the cohomology of  $\mathfrak{M}$  into spin multiplets.

### Appendix A.3. The induced and singular cohomology

The vacuum manifold of the Higgs branch is a complete intersection manifold  $\mathcal{M}$  (defined by the F-term equations) that lives inside an ambient manifold  $\mathfrak{M}$  (defined by the D-term equations). To describe the BPS states in this case, we need to figure out how to find the cohomology of  $\mathcal{M}$ . It turns out that one can describe the manifold  $\mathcal{M}$  as a vanishing locus of a complex vector bundle  $\mathcal{E}$  over  $\mathfrak{M}$  [16]. This allows us to use the following Lefschetz-Sommers theorem [39, theorem 7.1.1]:

**Theorem Appendix A.1** (Lefschetz-Sommers). *Let  $\mathcal{M}$  be the vanishing section of the complex vector bundle  $\mathcal{E}$  of rank  $n$ . We have the following relation between Hodge/Betti numbers:*

$$h^{(p,q)}(\mathcal{M}) = \begin{cases} h^{(p,q)}(\mathfrak{M}) & \text{if } (p+q) < \mathfrak{d} - n, \\ h^{(p,q)}(\mathfrak{M}) + \beta^{(p,q)} & \text{if } (p+q) = \mathfrak{d} - n, \end{cases}$$

where  $\beta^{(p,q)}$  are some positive integers and  $\mathfrak{d}$  is the complex dimension of  $\mathfrak{M}$ . The remaining of the Hodge diamond of  $\mathcal{M}$  is completed using the properties of the Hodge numbers.

The numbers  $\beta^{(p,q)}$  are the ones that are associated to the pure Higgs states. But how do we evaluate them? This is the subject of the last subsection.

### Appendix A.4. The pure Higgs states

From the discussion above, it is clear that if we can somehow get the total number of states then subtract from them the number coming from the induced cohomology, we can find the number of pure Higgs states. This can be done through the Gauss-Bonnet theorem. We find [16]:

$$\chi(\mathcal{M}) = \sum_{i=0}^{2(\mathfrak{d}-n)} (-1)^i b^i(\mathcal{M}) = \int_{\mathfrak{M}} \mathfrak{c}(\mathfrak{M}) \cdot \left( \frac{\mathfrak{c}_n(\mathcal{E})}{\mathfrak{c}(\mathcal{E})} \right).$$

where  $b^i(\mathcal{M})$  the Betti numbers of  $\mathcal{M}$ ,  $c(\mathfrak{M})$  is the total Chern class of the ambient manifold  $\mathfrak{M}$ ,  $c(\mathcal{E})$  is the total Chern class of complex vector bundle  $\mathcal{E}$ ,  $c_n(\mathcal{E})$  its top Chern class, and  $\chi(\mathcal{M})$  is the Euler number of  $\mathcal{M}$  [38]. We assumed here that  $\mathcal{M}$  is described in the same way as in the previous subsection. Now using the Lefschetz-Sommes theorem given by thm-Appendix A.1 one can easily evaluate the unknown  $\beta$ , the number of pure Higgs states.

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