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Estimate for the neutrino magnetic moment from pulsar kick velocities induced at the birth of strange quark matter neutron stars

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In a minimal extension of the Standard Model, the neutrinos can have a non-vanishing magnetic moment. We estimate the value of this magnetic moment by computing the neutrino chirality flip rate that takes place in the core of a strange quark matter star during the first seconds of its evolution. This chirality flip allows neutrinos to escape anisotropically, thus inducing the star kick velocity. For simplicity, we consider the star core composed of strange quark matter. Assuming average values for the star radii, temperature, chemical potentials, magnetic field intensities and that the fraction of the total energy released as right-handed neutrinos is small compared to the energy carried away by left-handed neutrinos, we have estimated that the neutrino magnetic moment is $\mu_\nu \sim 3.6 \times 10^{-18} \mu_B$, where μ_B is the Bohr magneton. This value is more stringent than the bound for massive neutrinos in a minimal extension of the Standard Model.

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1. Introduction

In recent years, the study of the internal composition of white dwarfs and Neutron Stars (NS) has been subject to several and different analyses, due to the extremely high densities, temperatures, and magnetic fields that are found in these systems, which terrestrial laboratories [1] have little chance of reproducing. One of the aims of studying these kinds of systems is to probe the not yet understood phases of strongly interacting matter [2–4] in the regime of high density, low temperature, and large magnetic fields [5], which determine the proposed scenarios of the exotic phases of matter that compose the internal layers of NS [6–8].

NS are born after a massive star, with a mass larger than 8 solar masses, explodes as a Type II supernova [9, 10]. It has been realized that the properties of NS are related to the continuous neutrino emission during the first 100 years after NS formation, which rapidly cools down the star from dozens of MeV to only dozens of keV. If the emission of neutrinos takes place in an anisotropic way, it can be responsible for the so-called pulsar kicks [11].

Several scenarios have been proposed to explain the proper motion of pulsars [12–16], including an anisotropic neutrino emission [17–19]. It has been proposed that the kick takes place during the formation of the proto-NS, in which neutrinos are emitted in a preferred direction and hence provide the proto-NS with momentum in its opposite direction. This scenario is favored by Refs. [20, 21], where it has been shown that there is a peak in the electron neutrino luminosity of about 12 s after the core bounce stage of proto-NS formation. It has also been proposed in Refs. [22, 23] that the formation of quark matter during the core collapse could explain the observed NS kicks.

In a minimal extension of the standard model (SM), where neutrinos are massive and can have a non-vanishing magnetic moment [24–26]. If the neutrinos, due to their magnetic moment, interact in equilibrium with the NS medium, they can flip their chirality, becoming right-handed, and hence suppressing their interactions with matter. As the inverse process occurs out of equilibrium and the detailed balance is lost, the right-handed neutrinos cannot flip back onto the left-handed ones, making the no-go theorem non-applicable [27]. Furthermore, because matter effects dominate in the core of NS, the right-handed neutrinos cannot resonate back into the left-handed ones, as this process is suppressed as a consequence of the presence of magnetic fields.

A fraction of the emitted left-handed neutrinos can escape from NS as right-handed ones if the typical time that takes them to flip their chirality is smaller than the time needed to travel one mean free path, which is small compared with the NS core radius. This mechanism was implemented in Refs. [28, 29] and was used to establish a lower bound for the neutrino magnetic moment that, together with the most stringent upper bound for this magnetic moment [30, 31], allowed for setting the range $4.7 \times 10^{-15} \leq \mu_\nu/\mu_B \leq (0.1\text{–}0.4) \times 10^{-11}$. In this work, we complement that study by further elaborating on the idea that a neutrino chirality flip, produced by the existence of a neutrino magnetic moment, can explain the observed kick velocities. As supernova explosions are mainly driven by neutrino emission, not all of the neutrinos should flip their chirality; otherwise, the explosion itself would not take place. We show that even if a small percentage of neutrinos become right-handed from the original left-handed state, the observed kick velocities can be reproduced even for a neutrino magnetic moment of the order of SM bound for a neutrino mass of a few eV [24].

2. Neutron star kick velocity

The produced kick velocity for the NS star can be computed from Refs. [18, 32, 33] as

$$dv = \frac{\chi}{M_{NS}}\frac{4}{3}\pi R^3 \epsilon dt, \tag{1}$$

where χ is the electron spin polarization (a function only of temperature, density, and the magnetic field intensity after the neutrinos flip their chirality); ϵ is the right-handed neutrino emissivity; and M_{NS} and R are the mass and radius, respectively, of the NS core.

The fraction of neutrinos asymmetrically emitted is equal to the electron spin polarization, given by

$$\chi^{-1} = 1 + \frac{2\left(\int_1^\infty dx \, \frac{x^2}{\exp\left(\frac{m_e}{T}\sqrt{x^2+1}-x_e\right)+1} + \frac{b^2 m_e}{24T}\int_0^\infty dx \, \frac{\sec^2\left(\frac{m_e}{T}\left(\sqrt{x^2+1}-x_e\right)\right)}{\sqrt{x^2+1}}\right)}{\int_0^\infty dx \, \frac{1}{\exp\left(\frac{m_e}{T}\sqrt{x^2+1}-x_e\right)+1}}, \tag{2}$$

where x is a dimensionless variable, m_e is the electron mass, $x_e = \mu_e/m_e$ is the electron chemical potential, normalized to the electron mass, T is the temperature and $b = B/B_e^c$, with $B_e^c \equiv m_e^2/e$ the electron critical magentic field. To obtain the expression for χ, we have made use of the Euler-McLaurin formula [34], which assumes that the magnetic field strength is small compared to the electron critical field. This quantity was better studied in [33].

On the other hand, the right-handed neutrino emissivity can be expressed as

$$\epsilon = g\tau\Gamma\varepsilon, \tag{3}$$

where Γ is the total reaction rate for the chirality flip, τ is the emission time-scale, and g is a factor that accounts for the fraction of total energy carried away by the right-handed neutrinos. When the emissivity changes with temperature, the cooling equation can be used, namely,

$$-\epsilon = g\tau\Gamma\frac{dU}{dt} = g\tau\Gamma\frac{dU}{dT}\frac{dT}{dt} = g\tau\Gamma C_v\frac{dT}{dt}, \tag{4}$$

where U is the internal energy density and C_v is the heat capacity. Therefore, the kick velocity is given by

$$v = -\frac{g}{M_{NS}}\frac{4}{3}\pi R^3 \tau \int_{T_i}^{T_f} \Gamma\chi \, C_v dT. \tag{5}$$

This velocity can be written in the following form

$$v = -g \; 804 \; \frac{\text{km}}{\text{s}}\left(\frac{1.4 M_\odot}{M_{NS}}\right)\left(\frac{R}{10\text{ km}}\right)^3\left(\frac{\tau I}{\text{MeV fm}^{-3}}\right), \tag{6}$$

where

$$I = \int_{T_i}^{T_f} \Gamma\chi \, C_v dT. \tag{7}$$

The latter integral depends on Γ, C_v, and χ, which, in turn, are given in terms of B, μ_i, and T. The electron spin polarization and the heat capacity were derived and studied in Ref. [33].

Assuming that for each fermion species f, the quantity $2|e_f|B^c f b_f$ is small, then we can resort, once again, to the Euler-McLaurin formula [34] and obtain the heat capacity for each fermion species

$$C_{vf} = \frac{d_f}{4\pi^2 t^2}\left\{\frac{m_f^5}{|e_f|B_f^c}\int_0^\infty dx\,\frac{\left(x^2-\frac{b_f}{m_f^2}|e_f|B_f^c\right)\left(\sqrt{x^2+1}-x_f\right)^2}{1+\cosh\left(\frac{\sqrt{x^2+1}-x_f}{t}\right)}+\frac{|e_f|B_f^c b_f^2 m_f}{3}\right.$$
$$\left.\times\int_0^\infty dx\,\frac{\left(\sqrt{x^2+1}-x_f\right)}{\sqrt{x^2+1}}\,\frac{\left[-1-\cosh\left(\frac{\sqrt{x^2+1}-x_f}{t}\right)+\frac{\sqrt{x^2+1}-x_f}{2t}\sinh\left(\frac{\sqrt{x^2+1}-x_f}{t}\right)\right]}{\left(1+\cosh\left(\frac{\sqrt{x^2+1}-x_f}{t}\right)^2\right)}\right\}, \tag{8}$$

where d_f is a degeneration factor, which for quarks $d_f = 3$ and for leptons $d_f = 1$.

We consider the core of the NS as a plasma made out of magnetized strange quark matter, namely, a gas composed of quarks u, d and s as well as electrons and neutrinos in the presence of a magnetic field. We also impose conditions that are believed to exist in the core of NS, such that the three quark species present in the core are in β equilibrium; among them, the core must maintain a neutral charge, and there should be a conservation of the baryon number and a fixed fraction of electrons plus neutrinos to baryons of $Y_L = 0.4$ [35, 36]. These conditions are referred to as stellar equilibrium conditions.

3. Right-hand neutrino emissivity

Following the procedure described in Refs. [28, 30, 31], to compute the chirality flip rate, we assume that when the neutrinos are being produced, the plasma is in thermal equilibrium at a temperature T and with an electron chemical potential μ_e such that $T, \mu_e \gg m_e$. The production rate of right-handed neutrinos, with energy p_0 and momentum $\mathbf{p}$ is given by

$$\Gamma(p_0) = \frac{\tilde{f}(p_0)}{2p_0}\,\mathrm{Tr}\left[\not{P} R\,\mathrm{Im}\,\Sigma\right], \tag{9}$$

where $\tilde{f}(p_0)$ is the Fermi-Dirac distribution for right-handed neutrinos, which are assumed massless for the sake of simplicity, such that $P_\mu = (p_0, \mathbf{p})$. $R = \frac{1}{2}(1+\gamma_5)$ is the right-chirality projector and Σ is the neutrino self-energy in the medium. The self-energy includes the interaction between the medium and neutrinos through thermal photons, which is modeled as a magnetic dipole interaction, proportional to the neutrino magnetic moment, μ_ν. Hence, the production rate is proportional to μ_ν^2.

The total reaction rate is obtained as the integral of $\Gamma(p_0)$ over the available phase space, that is

$$\Gamma = V\int\frac{d^3p}{(2\pi)^3}\Gamma(p_0) = \frac{V}{2\pi^2}\int_0^{p_0^{max}} dp_0\, p_0^2\,\Gamma(p_0), \tag{10}$$

where V represents the volume in which the chirality flip process occurs and the upper limit of the integral over the energy is determined by the maximum energy allowed for the neutrino in the beta decay process. We fix $p_0^{max} = 1.2$ MeV, which represents the maximum value for the energy of the massless neutrino beta decay in a vacuum. This is due to the fact that the process is mediated by a W boson, whose mass is significantly larger than the typical energy scales that are present during the formation of a proto-NS.

4. Estimate for the neutrino magnetic moment

From Eqs. (2), (8) and (10), we can compute the value of the induced kick velocity. Since Γ is proportional to μ_ν^2, then this velocity is determined by the neutrino magnetic moment. Therefore, we can use typical values of NS kick velocities, together with average values of NS mass, radii, magnetic field intensities, temperature, and chemical potentials at birth, to find an estimate for the neutrino magnetic moment. To this end, we assume that the volume where the chirality flip takes place is a cylinder whose height is a neutrino mean-free path, $\lambda \sim 1$ cm [37], and the transverse size is determined by the radius R of NS. The process needs to take place within the first 10–30 s after the core collapse, otherwise, the left-handed neutrino mean free path becomes larger than the radius of the star and the neutrinos escape, producing the SN explosion. Therefore, for $R \sim 10$ km, the total volume where the reaction takes place is

$$V = \pi R^2 \lambda \approx 3 \times 10^{51}\ \text{fm}^3. \tag{11}$$

Additionally, the time interval where the flip takes place is taken as the time for the neutrinos to travel one mean free path at the speed of light, $\tau = \lambda/c \sim 3 \times 10^{-11}$ s. We assume that, at birth, the temperature ranges between $T = 30$–50 MeV, the neutrino chemical potential is $\mu \sim 300$ MeV, and the electron chemical potential is $\mu_e \sim 275$ MeV. We estimate the fraction of the total energy carried away by the right-handed neutrinos g as the fraction of the product of the volume times the emission time where the flip takes place $V\tau \sim 9 \times 10^{40}$ fm^3 s, to the total NS core volume times the total neutrino emission time $V_{NS}\tau_{NS} \sim 1.2 \times 10^{59}$ fm^3 s, giving

$$g \sim 7.5 \times 10^{-19}, \tag{12}$$

which means that, if the energy released in the left-handed neutrinos is around 10^{53} erg, then the energy released in the right-handed neutrinos is around 10^{34} erg. Using these estimates, we find that the NS kick velocity is

$$v \sim 3.1 \times 10^{37} \left(\frac{\mu_\nu}{\mu_B}\right)^2\ \text{km s}^{-1}. \tag{13}$$

Recall that the average observed birth velocity for pulsars whose characteristic age is less than three million years is $v \approx 400$km s^{-1} [11]. Using this value, we estimate that the neutrino magnetic moment is of the order

$$\mu_\nu \sim 3.6 \times 10^{-18} \mu_B. \tag{14}$$

On the other hand, the minimal extension of the SM expression for the neutrino magnetic moment, for neutrino with mass m_ν, at the one-loop level, is given by [24]

$$\mu_\nu = \frac{3eG_F m_\nu}{8\pi^2\sqrt{2}} = \frac{3G_F m_e m_\nu}{4\pi^2\sqrt{2}}\mu_B. \tag{15}$$

By considering the parameters obtained from the solar, atmospheric, and reactor neutrinos, an upper bound for the previous expression is [38]

$$\mu_\nu \geq (4 \times 10^{-20})\mu_B. \tag{16}$$

We therefore, find that our estimate for the neutrino magnetic moment is more stringent than the SM one.

5. Summary and conclusions

In this work, we study the neutrino chirality flip during the birth of an SN, produced by the possible existence of a neutrino magnetic moment. The flip is caused by the interaction of the neutrino with electrons of the medium in the core of a magnetized strange quark matter NS. The calculation is performed using average values of NS mass, radii, magnetic field intensities, temperature, and chemical potentials at the birth of NS. We compute the kick velocity induced by this anisotropic right-handed neutrino emission. By also resorting to average values of the known kick velocities, we estimate the value $\mu_\nu \sim 3.6 \times 10^{-18}\mu_B$ for the neutrino magnetic moment. This estimate is more stringent than the SM bound obtained from bounds on the neutrino mass from solar, atmospheric, and reactor neutrinos. We point out that the mechanism we hereby put forward does not discard other mechanisms as viable explanations for the observed kick velocities. In fact, it may happen that different mechanisms coexist and are more or less important, depending on the given range of parameters describing NS, such as the mass, radius, temperature, and field strength. However, we also believe that the mechanism we put forward in this work can be at play, requiring only a relatively small neutrino magnetic moment. In this sense, this work is also a motivation to continue searching for such a neutrino property. Also, notice that in the calculation, the assumption is implicit that the angle between the pulsar angular velocity and the magnetic field is small. This seems to be supported by the findings in Refs. [39–41]. We also point out that the inclusion of a neutrino magnetic moment and thus, the possibility of chirality flip within the first seconds of the evolution of NS, can have also other important consequences that should be explored in order to have a better characterization of these objects.

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