Journal of Physics: Conference Series **2377** (2022) 012050 doi:10.1088/1742-6596/2377/1/012050

Quantum Dynamics of Two Nitrogen-Vacancy Center Ensembles Coupled to a Driven Superconducting Quantum Circuit

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Abstract. We theoretically model and simulate the dynamics of a hybrid quantum system consisting of two non-local ensembles of nitrogen-vacancy center and a superconducting transmon qubit mediated by two transmission line resonators. We apply a time-dependent external field to enhance this system's speed and fidelity to function as a controlled-phase gate. Our simulation result shows that a high-fidelity entangled state of two non-local NV spins is 92%. It is achievable under realistic parameter regimes within a timescale of 1.1 nanoseconds. Our result paves the way to improving potential quantum computing and sensing applications.

1. Introduction

A hybrid quantum system is the best way to produce optimal quantum technology. We can build it through a combination of several quantum systems such as atoms, photons, nuclear magnetic resonances (NMR), diamond nitrogen-vacancy (NV) center, cavity quantum electrodynamics (QED), quantum dots, and superconductor [1]. The advantages and disadvantages of each of these quantum systems are the main reasons for the emergence of hybrid quantum systems.

Atomic systems, spins, and superconducting quantum circuits are systems that can be utilized for future quantum technologies. Atoms have weak interactions with the environment so they can achieve long coherence times, which are useful for storing quantum information [2]. Spins can also be used for storage or processing quantum information [1]. Then the dynamics of the electronic states of atoms or electric charges in superconducting elements can realize fast processing of information encoded in their quantum states [3].

One of the most promising hybrid quantum systems for quantum technology is combining the spin system in a nitrogen vacancy ensemble (NVE) with a new superconducting circuit called transmon [4]. NVE is a composite of many NV centers within a diamond consisting of a nitrogen atom substitution that replaces a carbon atom and an adjacent vacancy [5]. NVE has a long coherence time and can operate at room temperature [6]. While transmon is superior to other superconducting circuits because it has anharmonicity and charge dispersion of the energy levels which can reduce charge noise [7].

Table 1 compares different systems between electron spin and superconducting qubits. The system has its advantages and disadvantages to be exploited in quantum technology. Macroscopic systems, such as superconducting qubits have physical characteristics such as flexibility, scalability, and strong coupling to external fields, but have short coherence times and generally cannot be produced identically. In contrast microscopic spin systems can naturally be easily produced as identical qubits with long coherence times, but they operate slowly due to their weak connection to external fields and have limited scalability [1]. It is difficult to control the many atoms acting as individual qubits. In overcoming this problem, a promising idea emerged by combining these two different systems and constructing a new hybrid quantum structure.

Table 1. Comparison between quantum systems of electron spin and superconducting qubit [8].

	Electron Spin	**Superconducting Qubit**
Size	~10^{-10} m (impurities) ~10^{-8} m (quantum dot)	~10^{-6} m
Energy gap	1-10 GHz	1-20 GHz
Frequency range	Microwave	Microwave
Operating temperature	~100 mK (quantum dot), room temperature (NV center)	~10 mK
Single-qubit gate operating time	~10 ns	~1 ns
Two-qubit gate operating Time	~0.2 ns	~10-50 ns
Coherence time	ms to s	~10-50 μs
Coupling type	Magnetic or electric	Magnetic or electric
Coupling strength with the Cavity	>MHz (quantum dot) ~100 Hz (impurities)	~0.1-1 GHz

Qubits consisting of two or more types of quantum systems have in decades been studied extensively. For example, in 2016, Yu et al. proposed a hybrid quantum system in which highly excited atoms interact strongly with a superconducting LC oscillator through the electric field of an electric motor [9]. In 2018, Zhang et al. proposed a quantum coupling system between two nanomechanical resonators with superconducting qubits [10]. In 2013 Lu et al. studied how to generate high-fidelity quantum storage using two flux qubits coupled with a nitrogen vacancy ensemble [11].

2. Method

We consider a hybrid quantum system model as shown in Figure 1. There are two Transmission Line Resonators ($TLR_{a(b)}$) given a time-dependent external field where there is an NVE in each TLR. Both are mediated by transmons so that two non-local NVE can be connected.

TLR is a harmonic oscillator so the Hamiltonian TLR_a and TLR_b can be written as

$$H_a = \hbar\omega_a a^\dagger a \quad (1)$$

$$H_a = \hbar\omega_b b^\dagger b \quad (2)$$

Here, a (b) is the annihilation operator of TLR_a (TLR_b) and $\omega_{a(b)}$ is the frequency of TLR_a (TLR_b).

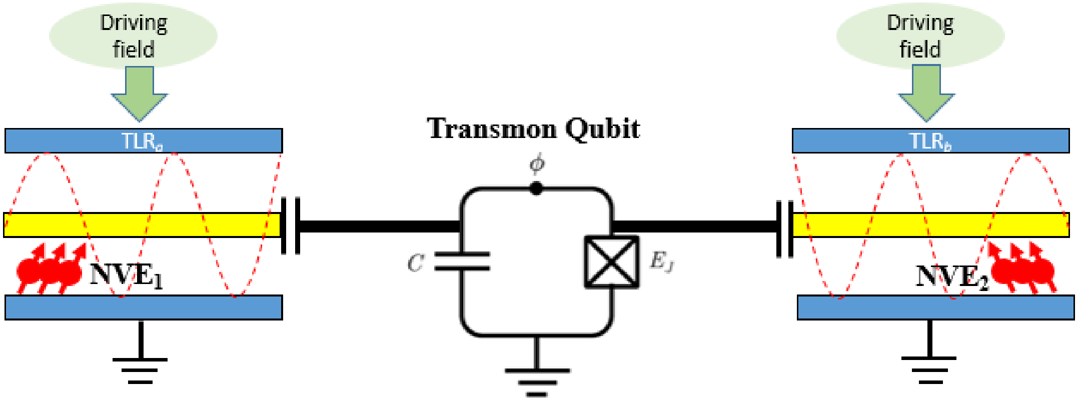

Figure 1. Setup of two non-local NVE coupled to two non-local transmission line resonators with driving field mediated by a transmon qubit [4].

We apply a driving field to each TLR (a and b). A classical driving field of frequency ω_d, amplitude $\Omega_{A(B)}$, and phase ϕ_0 can be modelled by including an additional term in the Hamiltonian as:

$$A(t) = \Omega_A \sin(\omega_d t + \phi_0) \tag{3}$$

$$B(t) = \Omega_B \sin(\omega_d t + \phi_0) \tag{4}$$

When the driving field interacts with each of the TLRs, the system interaction equation can be written as

$$H_{da} = \hbar\omega_a a^\dagger a + \Omega_A \sin(\omega_d t + \phi_0)\,(a^\dagger + a) \tag{5}$$

$$H_{db} = \hbar\omega_b b^\dagger b + \Omega_b \sin(\omega_d t + \phi_0)\,(b^\dagger + b) \tag{6}$$

By applying a rotating wave approximation, the Hamiltonian in interaction picture can be written as

$$H_{da}^{RWA} = \hbar(A(t)a^\dagger e^{i\delta_a t} + A(t)^* a e^{-i\delta_a t}) \tag{7}$$

$$H_{db}^{RWA} = \hbar(B(t)b^\dagger e^{i\delta_b t} + B(t)^* b e^{-i\delta_b t}) \tag{8}$$

Here, $\delta_a = \omega_d - \omega_a$ and $\delta_b = \omega_d - \omega_b$. By considering the two lowest energy levels as shown in Figure 2, the Hamiltonian of the transmon qubit can be expressed as

$$H_q = \hbar\omega_q \sigma^+ \sigma^- \tag{9}$$

where ω_q is the transition frequency between the ground state and the exited state from transmon qubit. $\sigma^+$is raising operator and σ^- is lowering operator.

In this system, the NVE has an energy level configuration as shown in Figure 3. The spin-spin interactions lead to an equal energy separation between the states |m$_s$=0⟩ and |m$_s$=±1⟩ from NVE. When an external magnetic field is applied along the NVE axis of symmetry, the degenerated state |m$_s$ = ±1⟩ is split, which causes level separation $D_{ue} = \gamma_u B$ [12]. Where γ_u is the gyromagnetic ratio of an electron. We label the status |m$_s$=0⟩, |m$_s$=1⟩, and |m$_s$=+1⟩ from NVE as |g⟩, |e⟩, and |u⟩.

We apply the rotating-wave approximation of the couplings between the TLRs with the transmon qubit and the couplings between the NVEs with the corresponding TLR, so the equation can be written as

$$H_{aq} = g_a\left(a\sigma^+ e^{i\delta_{aq}t} + a^+\sigma^- e^{-i\delta_{aq}t}\right) \tag{10}$$

$$H_{bq} = g_b\left(b\sigma^+ e^{i\delta_{bq}t} + b^+\sigma^- e^{-i\delta_{bq}t}\right) \tag{11}$$

$$H_{a1} = g_1\left(aS_{1,ge}^+ e^{i\delta_a^{1ge}t} + a^+ S_{1,ge}^- e^{-i\delta_a^{1ge}t}\right) + g_1\left(aS_{1,gu}^+ e^{i\delta_a^{1gu}t} + a^+ S_{1,gu}^- e^{-i\delta_a^{1gu}t}\right) \tag{12}$$

$$H_{b2} = g_2\left(bS_{2,ge}^+ e^{i\delta_b^{2ge}t} + b^+ S_{2,ge}^- e^{-i\delta_b^{2ge}t}\right) + g_2\left(bS_{2,gu}^+ e^{i\delta_b^{2gu}t} + b^+ S_{2,gu}^- e^{-i\delta_b^{2gu}t}\right) \tag{13}$$

Through merging between multiple quantum systems, the total Hamiltonian of the hybrid system in the interaction picture can be described as

$$H_t = H_{da} + H_{db} + H_{aq} + H_{bq} + H_{a1} + H_{b2} \tag{14}$$

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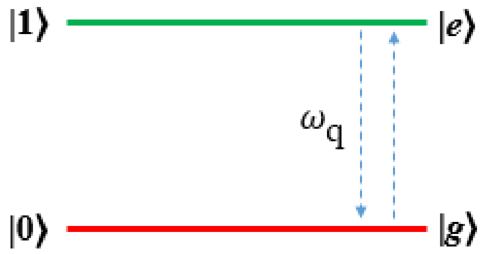

Figure 2. Transmon Qubit Energy Level Configuration Diagram

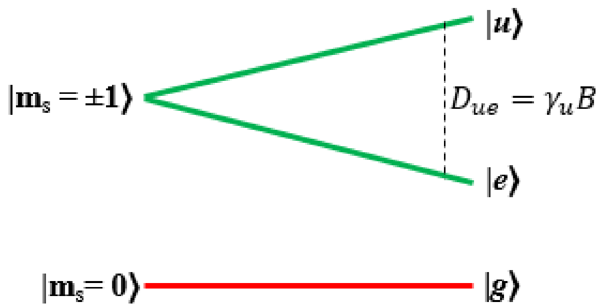

Figure 3. NV Center Energy Level Configuration Diagram Under External Magnetic Field

3. Result and Discussion

In quantum information processing, the transfer of the quantum state from one location to another is an important task and it is the premise of the realization of large-scale quantum computing and quantum networks. In this research, hybrid quantum system composed of two non-local NVEs, two TLRs, and the transmon qubit for the state transfer is initially in the superposition state as

$$|\psi_0\rangle = (\cos\theta_1|e\rangle_1 + \sin\theta_1|u\rangle_1) \otimes (\cos\theta_1|u\rangle_1 + \sin\theta_1|u\rangle_1) \otimes |g\rangle_q \otimes |0\rangle_a \otimes |0\rangle_b \tag{15}$$

Then we apply the c-phase gate on two non-local NVE to its state evolution

$$U_{c-phase} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{16}$$

so that the final state becomes an entangled state which is written as

$$|\psi_f\rangle = (\alpha_1|e\rangle_1|g\rangle_2 - \alpha_2|e\rangle_1|u\rangle_2 + \alpha_3|u\rangle_1|g\rangle_2 + \alpha_4|u\rangle_1|u\rangle_2) \otimes |g\rangle_q \otimes |0\rangle_a \otimes |0\rangle_b \tag{17}$$

Here, $\alpha_1 = \cos\theta_1\cos\theta_2$, $\alpha_2 = \cos\theta_1\sin\theta_2$, $\alpha_3 = \sin\theta_1\cos\theta_2$, and $\alpha_4 = \sin\theta_1\sin\theta_2$. Then, to demonstrate the feasibility of the scheme for the construction of the gate, we numerically calculate the fidelity of the gate using the master equation because this system is an open system, so the equation can be written as [4]

$$\frac{d\rho}{dt} = -i[H,\rho] + \kappa_a D[a]\rho + \kappa_b D[b]\rho + \gamma D\left[S^-_{1_{ge}} + S^-_{1_{gu}}\right]\rho + \gamma D\left[S^+_{1_{ge}} + S^+_{1_{gu}}\right]\rho + \gamma D\left[S^-_{2_{ge}} + S^-_{2_{gu}}\right]\rho + \gamma D\left[S^+_{2_{ge}} + S^+_{2_{gu}}\right]\rho + \gamma D[\sigma^+]\rho + \gamma D[\sigma^-]\rho \tag{18}$$

where $D[L] = \rho = (2L\rho L^+ - L^+L\rho - \rho L^+L)/2$. L is collapse operator. κ_a (κ_b) is the decay rate of the TLR_a (TLR_b). While γ is the energy relaxation rate and dephasing rate of the transmon qubit and NVE. Then, we calculate the fidelity of the gate which is defined as [13]

$$F = Tr(|\sqrt{\rho_f}\rho(t)\sqrt{\rho_f}|) \tag{19}$$

where ρ_f is the density operator of the final state $|\psi_f\rangle$, while $\rho(t)$ is the realistic density operator after the c-phase gate operation in the initial state $|\psi_0\rangle$ with the realistic Hamiltonian H_t.

In this simulation we take $\theta_1 = \theta_2 = {}^{\pi}/_{4}$. By taking the numerical parameters, we set the frequencies as $\omega_d = \omega_a = \omega_b = \omega_q = \omega_d = \omega_{ge} = 4\pi$ GHz and $\omega_{gu} = 5.6\pi$ GHz. Then for the coupling strength are $g_{a(b)} = 0.52\pi$ GHz, $g_1 = 14\pi$ GHz, and $g_2 = 8$ GHz. The energy relaxation rates and the dephasing rates of the transmon qubit and the NVEs are $\kappa^{-1} = \gamma^{-1} = 2$ ns. In this simulation we use the Quantum Toolbox in Python (QuTIP). We use QuTIP because it has the advantage of being open source. Several previous studies have used QuTIP to perform quantum simulations [14].

Since TLR has an infinite state, we try to limit it by conditional values of numbers of TLR. We first simulate the effect of numbers of TLRs *N* on dynamical quantum evolution. By increasing *N* in TLRs we can find out whether dynamical quantum evolution has the same trajectory or not. This situation leads us to choose a low value of *N* in conducting state transfer simulations to be effective and minimize error value.

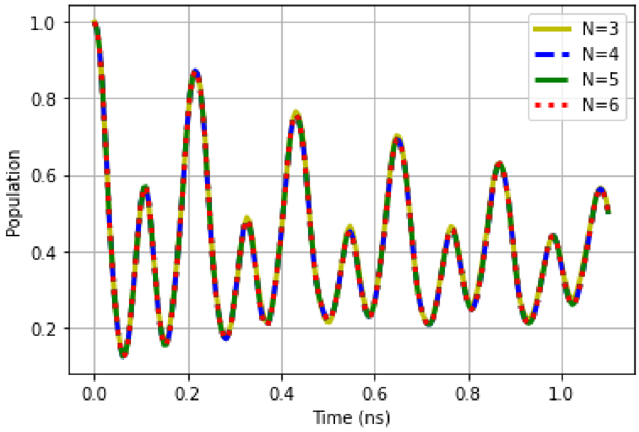

Figure 4. The population in the ground state of the transmon qubit varies with numbers of TLRs (*N*) without driving field during the c-phase gate operation.

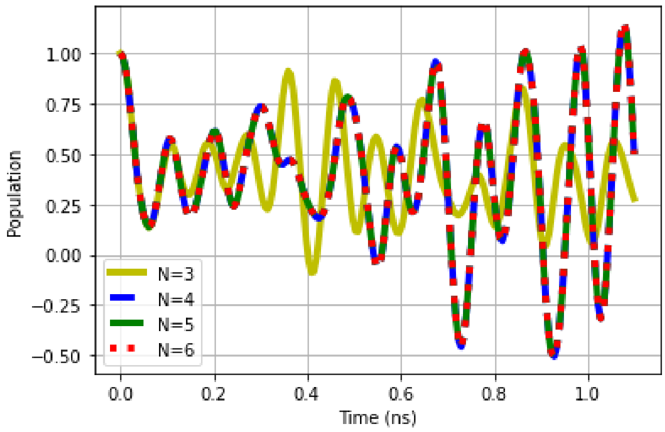

Figure 5. The population in the ground state of the transmon qubit varies with numbers of TLRs (*N*) with driving field during the c-phase gate operation.

Based on the initial simulation results for TLRs without the driving field effect, there is no significant difference in the time evolution trajectory so that we can choose any value of *N* to determine the gate fidelity value (Figure 4). However, for TLRs with the driving field effect there is a significant effect when we take the value of $N = 3$ with $N = 4$. After that by increasing the value of *N* to 6, the results show that there is no significant difference (Figure 5), so we use the value of $N = 6$ to perform the fidelity simulation.

The driving field applied to the TLR allows us to control the desired frequency. The driving field applied to each TLR has a significant effect on the quantum gate fidelity value as shown in Figure 6. By applying the driving field to the TLRs we get a 92% fidelity value. Note that although this fidelity value is less than that obtained by the previous research that is 97%, we can get this condition within 1.1 ns, which is much faster compared to their result that is 101 ns [4]. In the quantum dynamics simulation, the initial and final population states have the same expected values for TLRs under the driving field effect with those not under the driving field effect as shown in Figure 7.

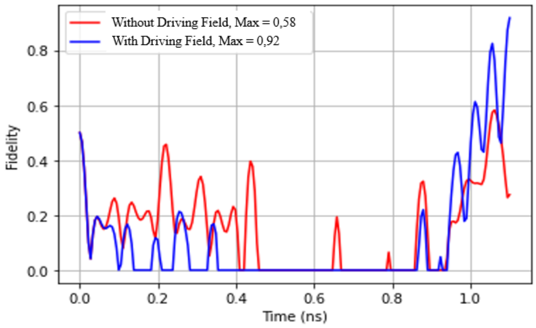

Figure 6. The fidelity of the c-phase gate varies with time (without)

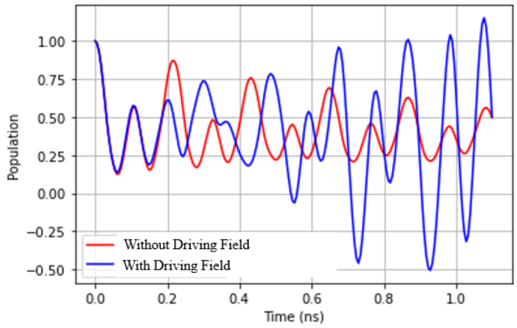

Figure 7. The different population in the ground state of the transmon qubit varies with time during the c-phase gate operation with driving field and without driving field.

4. Conclusion

In conclusion, we have proposed a quantum hybrid system between superconducting circuits connected with two non-local NV ensembles. Two NVEs are used to create a c-phase quantum gate so that we can get an entangle state. The simulation results show that the fidelity value reaches 92% within 1.1 ns by applying the driving field to the TLRs. Furthermore, we conjecture that this result may be significantly improved with the application of a pulse optimization algorithm with the CRAB algorithm [15]. Our result provides initial steps of the practical applications of quantum computing in the realization of large-scale quantum memory devices.

5. Acknowledgement

We thank Universitas Indonesia for supporting this research through PUTI Research Grant No. NKB-478/UN2.RST/HKP.05.00/2022.

6. References

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