

Relativistic Mean Field theory with density dependent meson couplings

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Abstract. Relativistic mean field theory with density dependent coupling constants is used to investigate ground state properties and properties of excited configurations in nuclei far from stability. It turns out that it is very important to consider also a density dependence the isovector channel. This provides not only an improved description of the equation of state for neutron matter and asymmetric nuclear matter but also for isovector properties of finite nuclei far from stability such as the neutron skin thickness, and the isoscalar and isovector giant resonances.

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1. Introduction

Properties of nuclei far from the valley of stability are presently in the center of many experimental and theoretical investigations. Experiments with radioactive nuclear beams have disclosed a wealth of structure phenomena in exotic nuclei with extreme isospin values, and the next generation of radioactive-beam facilities will present new exciting opportunities for the study of the nuclear many-body systems with extreme isospin. Because the properties of these nuclei determine the conditions for their formation and for their decay in many astrophysical processes such investigations have important applications in nuclear astrophysics.

Mean Field theories based on density functionals play an important role in this context. The density dependence of these functionals has two sources, (i) the bare three-body forces lead on the mean field level to density two-body forces and, more important, (ii) the pion exchange force with its large tensor term, produces in the nuclear medium an large effective density dependent two-body interaction. In recent years several investigations have been devoted to the microscopic derivation the density dependence of the effective interaction in the nuclear interior. Brueckner theory has been used to derive the density dependence of effective coupling constants in relativistic Hartree models from the bare two-body interaction [1, 2, 3]. So far, the results of such investigations cannot be used to calculate properties of realistic nuclei with a satisfying accuracy. Therefore all the successful density functionals contain phenomenological parameters, which are adjusted to experimental data.

2. General considerations on the parameters in covariant density functional theory

Relativistic density functionals contain only a relatively small number of phenomenological parameters and they provide a universal scheme. They take into account not only the usual translational and rotational symmetry but in addition Lorentz invariance. This allows a unified description of the spin-properties in the nuclear systems.

The oldest model of this type is the Walecka model[4]. Nucleons are described by Dirac spinors, which interact through the exchange of effective mesons $\phi_i = (\sigma, \omega, \rho)$ with masses m_i and coupling constants g_i . This interaction generates classical meson fields characterized by the basic quantum numbers of the system, such as spin, parity and isospin. In the isoscalar channel one has scalar fields σ and ω . In symmetric nuclear matter this model contains only two parameters $G_\sigma = (g_\sigma/m_\sigma)^2$ and $G_\omega = (g_\omega/m_\omega)^2$. They are fixed by the binding energy and the saturation density of symmetric nuclear matter. The meson masses or the range of the corresponding forces do not enter explicitly on this level. In fact, the simplest version of such a theory would be a point coupling model of NJL-type

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi - G_\sigma(\bar{\psi}\psi)(\bar{\psi}\psi) - G_\omega(\bar{\psi}\gamma^\mu\psi)(\bar{\psi}\gamma_\mu\psi) \quad (1)$$

It turns out that this model is not able to describe at the same time the binding energies and the radii of finite nuclei. One needs only one additional parameter, which takes care of the fact, that the nuclear fields are not exactly proportional to the corresponding densities. Usually this parameter is the mass of the σ -meson m_σ or, in point coupling models [5, 6], a derivative term of the form

$$D_\sigma(\bar{\psi}\psi)\partial^\mu\partial_\mu(\bar{\psi}\psi). \quad (2)$$

with $D_\sigma = G_\sigma/m_\sigma^2$. With the three parameters G_σ, G_ω and m_σ one is able to describe binding energies and radii of $N = Z$ nuclei. The parameter m_ω does not play a role in this context. In the next step we consider the Coulomb force and systems with $N \neq Z$. In this case one needs also fields carrying isospin $T = 1$. They are usually represented by the ρ -meson and the coupling term

$$G_\rho(\bar{\psi}\gamma^\mu\vec{\tau}\psi)(\bar{\psi}\gamma_\mu\vec{\tau}\psi) \quad (3)$$

with the additional parameter G_ρ , which is adjusted to the symmetry energy of nuclear matter.

At this stage the Walecka model or similar point coupling models have 4 phenomenological parameters $G_\sigma, G_\omega, G_\rho$ and m_σ . They provide a relatively successful description of spherical nuclei all over the periodic table, they miss however essential properties of the equation of state and essential surface properties, in particular the incompressibility [7] and the nuclear deformations [8]. One needs an additional density dependence. There are essentially three possibilities to proceed:

- **The non-linear meson models:** Boguta and Bodmer [7] added non-linear self-couplings for the mesons

$$U(\sigma) = \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}g_2\sigma^3 + \frac{1}{3}g_3\sigma^4 \quad (4)$$

This produces an effective density dependence and adds two additional parameters g_2 and g_3 . Such models have been used with great success. Parameter sets such as NL1 or NL3 are famous examples. However, the density dependence is here restricted to the isoscalar channel. In order to obtain a density dependence in the isovector channel one needs non-linear couplings involving the ρ -meson.

- **The point coupling models:** here the non-linear meson terms are replaced by three- and four-body coupling terms such as $(\bar{\psi}\psi)^3$ and $(\bar{\psi}\psi)^4$ containing again 2 additional parameters.
- **Density dependent couplings:** since renormalizability does not play a role in modern effective field theories density dependent couplings $g_i(\rho)$ or $G_i(\rho)$ are more consistent with density functional theory. ρ is the normal density. In these models one has only linear meson couplings

$$-g_\sigma(\rho)(\bar{\psi}\sigma\psi) - g_\omega(\rho)(\bar{\psi}\gamma^\mu\omega_\mu\psi) - g_\rho(\rho)(\bar{\psi}\gamma^\mu\vec{\tau}\vec{\rho}_\mu\psi) \quad (5)$$

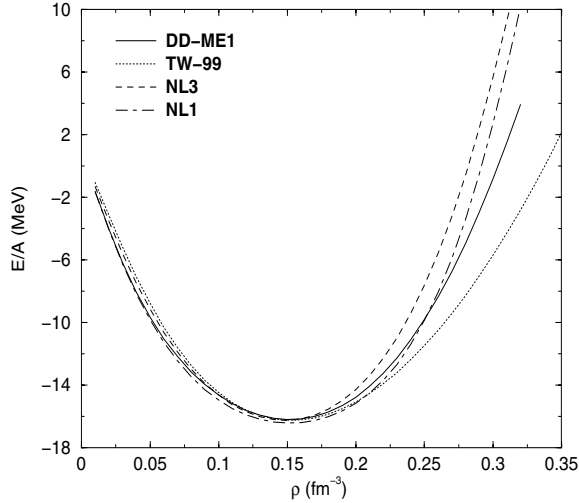


Figure 1. Equation of state of symmetric nuclear matter for various parameter sets.

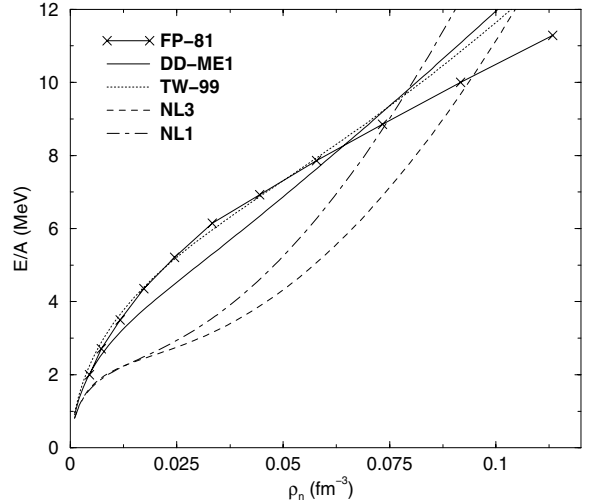


Figure 2. Equation of state for neutron matter for the same parameter sets.

The essential problem is the functional form of the coupling constants $g_i(\rho)$ or $G_i(\rho)$. In the past one has used the polynomial forms or the Typel-Wolter ansatz [9]: $g_i(\rho) = g_i f_i(x)$ where $x = \rho/\rho_{\text{sat}}$ is the density in units of the saturation density of symmetric nuclear matter and

$$f_i(x) = a_i \frac{1 + b_i(x + d_i)^2}{1 + c_i(x + d_i)^2} \quad \text{for } i = \sigma, \omega \quad \text{and} \quad f_\rho(x) = e^{-a_\rho(x-1)} \quad (6)$$

This form has been chosen in order to reproduce the shape of the corresponding effective coupling constants obtained from relativistic Brueckner calculations in nuclear matter. In order to reduce the number of fit parameters additional constraints have been introduced. At the end one has 3 additional parameters for the σ - and ω -couplings and one parameter for the ρ -mesons

The total number of parameters is determined by the form of the density dependence. Parameter sets with non-linear σ -coupling in isoscalar channel have therefore ≈ 6 phenomenological parameters. If one introduces also a density dependence in the isovector channel, one needs at least 7 parameters. The Typel-Wolter ansatz has 8 parameters. If they are carefully adjusted to experimental data, as for instance in the case of DD-ME1 or DD-ME2 (Density-Dependent-Meson-Exchange) one obtains an excellent description of nuclear properties all over the periodic table [10].

3. Density dependence in the isovector channel

In the following we discuss results obtained by the density dependent meson coupling models DD-ME1 [11] and DD-ME2 [12] within relativistic Hartree-Bogoliubov (RHB) theory [13]. Here pairing correlations are described by a density functional

$$E_{RHB}[\hat{\rho}, \hat{\kappa}] = E_{RMF}[\hat{\rho}] + E_{pair}[\hat{\kappa}] \quad (7)$$

with the RMF density functional $E_{RMF}[\hat{\rho}]$ depending on the relativistic single particle density matrix $\hat{\rho}$ and the pairing energy $E_{pair}[\hat{\kappa}]$ depending on the pairing-density $\hat{\kappa}$. Since pairing

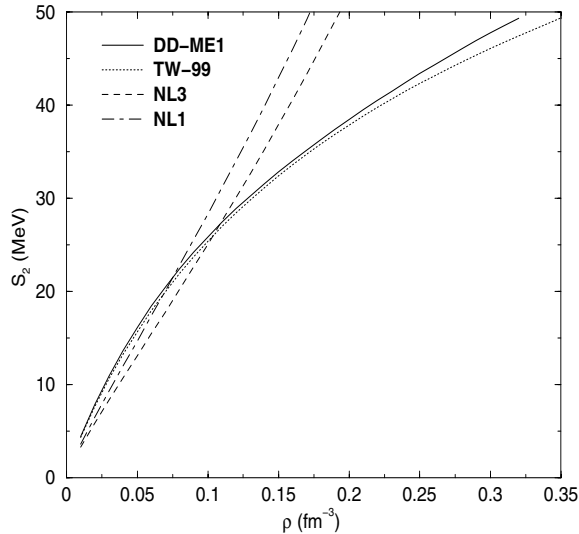


Figure 3. Symmetry energy S in symmetric nuclear matter as a function of the density

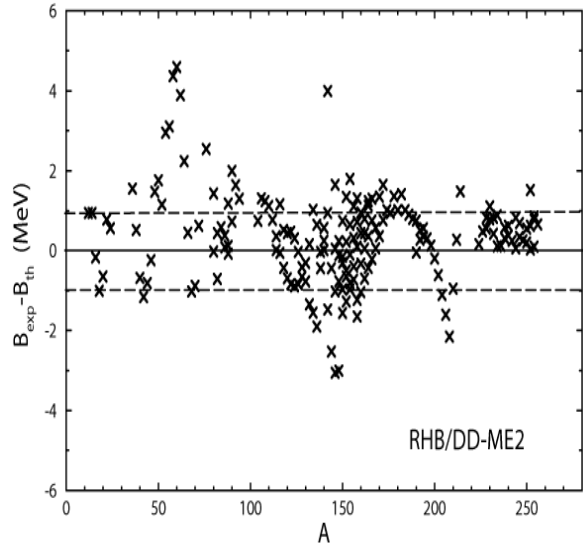


Figure 4. Nuclear binding energies B : difference between experiment and theory.

properties are completely non-relativistic we use here the pairing energy of Gogny with the parameter set D1S [14].

The eight independent parameters: three coupling constants g_σ , g_ω , g_ρ , the mass of the σ -meson m_σ and four additional parameters describing the density dependence of the Typel-Wolter ansatz are simultaneously adjusted to reproduce the properties of symmetric and asymmetric nuclear matter, and to ground state properties of twelve spherical nuclei, such as binding energies, charge radii and neutron radii. In Figs. 1 we show the equation of state (EOS) for symmetric nuclear matter for the relativistic parameter sets NL1 [15], NL3 [16], TW-99 [9] and DD-ME1 [11]. We notice that, for the symmetric case, all these sets show rather similar equations of state in the neighborhood of the saturation point. The principal difference between the older sets NL1, NL3 and the modern sets with density dependence in the isovector channel are the properties of asymmetric nuclear matter. The equation of state of pure neutron matter is shown in Fig. 2. The new parameter sets DD-ME1 and TW-99 are in much better agreement with the equation of state calculated by Freedman and Pandharipande [17], because they contain a density dependence in the isovector channel (6). In DD-ME1 the corresponding parameter a_ρ has been adjusted to the experimental size of the neutron skin in several spherical nuclei.

In Fig. 3 we show the symmetry energy $S_2(\rho)$ for symmetric nuclear matter as a function of the density. It is proportional to the second derivative of the binding energy per particle $\epsilon(\rho, \alpha)$ with respect to the asymmetry parameter $\alpha = (N - Z)/(N + Z)$

$$\epsilon(\rho, \alpha) = \epsilon(\rho, \alpha) + S_2(\rho)\alpha^2 + \dots \quad (8)$$

The Bethe-Weizsäcker formula contains the empirical value $a_4 = S_2(\rho_{\text{sat}}) \approx 30 \pm 4$ MeV. The older sets NL1 and NL3 have no density dependence in the isovector channel. Therefore the corresponding symmetry energies $S_2(\rho)$ are nearly linear functions of the density with rather large values for a_4 . The density dependence in the isovector channel induces a bending of the curve $S_2(\rho)$ as it is seen also in extended non-relativistic Brueckner calculations of nuclear matter. This leads to smaller values for the parameter a_4 at the saturation density and to larger values of S_2 for smaller values of the density in the surface region. In this way the modern parameter sets yield much better values for the energy of the IVGDR, which is a surface mode

and therefore influenced by the symmetry energy at smaller densities. There is a close connection between the value of a_4 and the nuclear incompressibility K_{nm} . A careful analysis [18] of the interplay between the symmetry energy and the energies of the ISGMR and the IVGDR in many nuclei shows that the present experimental data on these giant resonances yield in connection with relativistic models based on the Typel-Wolter ansatz the following limits

$$32 \leq a_4 \leq 34 \text{ MeV} \quad \text{and} \quad 250 \leq K_{\text{nm}} \leq 270 \text{ MeV} \quad (9)$$

This is close to the values obtained by the analysis of non-relativistic Skyrme models with a power-law for the density dependence [19].

The theoretical binding energies of approximately 200 nuclei calculated in the RHB model with the DD-ME2 plus Gogny D1S interactions, are compared with experimental values in Fig. 4. Except for a few Ni isotopes with $N \approx Z$ that are notoriously difficult to describe in a pure mean-field approach, and several transitional medium-heavy nuclei, the calculated binding energies are generally in very good agreement with experimental data. Although this illustrative calculation cannot be compared with microscopic mass tables that include more than 9000 nuclei [20], we emphasize that the rms error including all the masses shown in Fig. 4 is less than 900 keV. Moreover, since a finite-range pairing interaction is used, the results are not sensitive to un-physical parameters like, for instance, the momentum cut-off in the pairing channel. When compared with data on absolute charge radii and charge isotope shifts from Ref. [21], the calculated charge radii exhibit an rms error of only 0.017 fm.

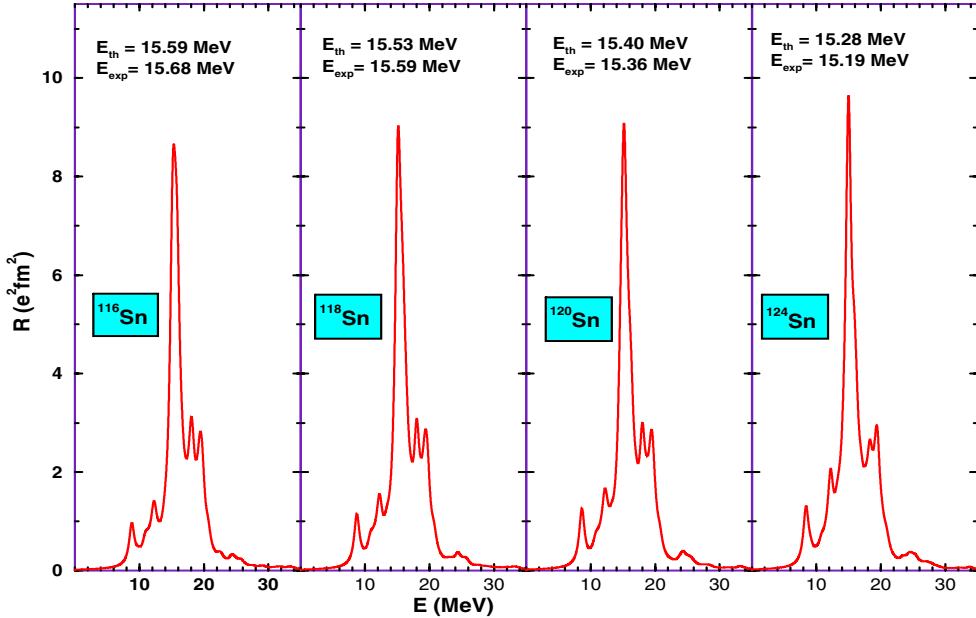


Figure 5. The Giant Monopole Resonance (ISGMR) in four Sn-isotopes. Experimental centroid energies are compared with theoretical values obtained with the parameter set DD-ME2/D1S

In Fig. 5 we compare relativistic quasi-particle RPA (RQRPA) results for the Sn isotopes with experimental data on IVGDR excitation energies [22]. In contrast to the case of ^{208}Pb , the strength distributions in the region of giant resonances exhibit fragmentation and the energy of the resonance E_{GDR} is defined as the centroid energy $\bar{E} = m_1/m_0$, calculated in the same

energy window as the one used in the experimental analysis (13–18 MeV). The RHB+RQRPA calculation with the DD-ME2 interaction reproduces in detail the experimental excitation energies and the isotopic dependence of the IVGDR.

4. Conclusions

Effective nuclear interactions with density-dependent meson-nucleon vertex functions represent a significant improvement in the relativistic self-consistent mean-field description of the nuclear many body problem. In a number of recent studies it has been shown that, in comparison with standard non-linear meson-exchange models, this class of effective interactions provides a more realistic description of asymmetric nuclear matter, neutron matter and finite nuclei. In particular, these interactions allow for a softer equation of state of nuclear matter (i.e. lower incompressibility) and a lower value of the symmetry energy at saturation.

Of course there are specific properties, which cannot be described in a satisfying way on the mean field level, as for instance the structure of single particle levels in the vicinity of the Fermi surface, where low-lying collective surface vibrations cause additional shifts and fragmentation [23], or the decay width of giant resonances, where $2p$ - $2h$ -configurations have to be taken into account. Other examples are transitional nuclei, which have to be treated by superposition of many configurations with different deformation within the framework of the Generator Coordinate Method (GCM) [24].

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