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Momentum anisotropy at freeze out

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Abstract. The transition from a hydrodynamical modeling to a particle-based approach is a crucial element of the description of high-energy heavy-ion collisions. Assuming this "freeze out" happens instantaneously at each point of the expanding medium, we show that the local phase-space distribution of the emitted particles is asymmetric in momentum space. This suggests the use of anisotropic hydrodynamics for the last stages of the fluid evolution. We discuss how observables depend on the amount of momentum-space anisotropy at freeze out and how smaller or larger anisotropies allow for different values of the freeze-out temperature.

1. The need for a kinetic freeze out

Hydrodynamics has emerged as being very successful at describing a large amount of the evolution of the fireball created in a ultrarelativistic heavy ion collision.

However, hydrodynamics is an effective field theory which assumes the medium to be continuous. An appropriate measure for this continuity is the Knudsen number Kn, defined as the ratio between the mean free path and some macroscopic length scale like the system size. For hydrodynamics to be applicable, this number has to be small. Even if at the beginning of the evolution the condition $\mathrm{Kn} \ll 1$ is fulfilled, yet as the fireball expands, it becomes more dilute and the Knudsen number grows. Obviously, at the very end detectors detect individual particles, which have long ceased to interact with each other. Going backwards in time to the point where interactions became rare, this amounts to a very large mean free path (and Kn), in which case kinetic theory provides an appropriate modeling of the particles evolution.

Thus, the description of the fireball evolution needs to include a prescription for switching from the hydrodynamic approach at $\mathrm{Kn} \ll 1$ to the transport models used at $\mathrm{Kn} \gg 1$. This prescription, which for consistency should conserve all relevant quantum numbers of the fireball as well as its local energy and momentum densities, is the kinetic freeze out. Note that in the simplified application we shall present hereafter, we consider the direct conversion from a fluid to free particles, omitting the stage consisting of interacting particles described by a kinetic theory.

2. The Cooper–Frye formula

The basic idea of the Cooper–Frye freeze out is that the medium description changes suddenly from a fluid-dynamical to a kinetic one as the medium is passing through a hypersurface Σ [1]. From this point on, the particles decouple from the fluid and, assuming the emitted particles are then free, their momenta are frozen. Σ is a closed three-dimensional surface in space-time, since eventually the whole fluid has to freeze out.

Introducing the on-shell phase-space distribution $f(x^\mu, \vec{p})$, the Lorentz-invariant particle spectrum, which basically expresses the number of particles per momentum interval, is given by the Cooper–Frye integral

$$E_p \frac{d^3N}{d^3\vec{p}} = \frac{1}{(2\pi)^3} \int_\Sigma f(x^\mu, \vec{p})\, p^\mu d^3\sigma_\mu(x^\mu), \tag{1}$$

where $d\sigma_\mu(x^\mu)$ is a unit vector standing perpendicular to the hypersurface Σ. For the particle distribution function $f(x^\mu, \vec{p})$ at freeze out entering the Cooper–Frye formula, popular choices are the Maxwell–Jüttner distribution, or its quantum generalizations to fermions and bosons, possibly with terms accounting for dissipative effects in the fluid.

An important issue is now to characterize the hypersurface Σ. It is often assumed to correspond to the space-time points where some property of the fluid (e.g. its temperature T, or its particle-number or energy density) reaches some "critical" value, or when a given proper time is reached.

One problem here is that there are regions of Σ where $d\sigma_\mu d\sigma^\mu < 0$, which likely leads to negative contributions to the particle spectrum (1). However the idea of the particle spectrum is just to count particles, not quantum numbers, so that negative contributions are unexpected (for a proposal on how to deal with this problem, see e.g. Ref. [2]).

A second issue with the Cooper–Frye freeze out is the dependence of the computed observables on the initially chosen freeze out parameter, which characterizes Σ. Since the hypersurface is not associated with a phase transition, there is no reason why an observable should depend on the point where a theoretical physicist changes her/his "tool" to describe the medium. Accordingly, one should rather expect that nature performs a smooth transition between the two asymptotic models valid at small and for large Knudsen numbers.

Because of its simplicity, the Cooper–Frye recipe is however much too attractive to be discarded. Our idea is, instead of just gluing together the two models (hydrodynamic and kinetic theory), to try to first tune them, so that they fit together in a better way. The desired result should be a much weaker (or almost no) dependence of observables on the point where we glued together the two models.

3. Anisotropic Cooper-Frye freeze out

3.1. Motivation

In our opinion, one way of achieving this "improved gluing" is to resort to an explicitly anisotropic distribution at freeze out [3]. A first hint towards that direction comes from a series of studies, starting with Ref. [4], of non-relativistic hypersonic flows which expand through a nozzle into vacuum. The conclusion of these studies is that the frozen-out particle distribution can be characterized by two effective temperatures, which stand on the one hand for the "thermal" motion perpendicular to the streamlines ($T_\perp$) and on the other hand for the motion parallel to the streamlines ($T_\parallel$). We are currently working on a generalization of these studies to the relativistic heavy ion collision case.

A second motivation for invoking a momentum-asymmetric phase-space distribution is that there is already a transition between models in the description of heavy ion collisions, namely from the pre-equilibrium model(s) to hydrodynamics, which can be improved by the use of anisotropic hydrodynamics, see e.g. Ref. [5].

3.2. Implementation

The basic idea of the anisotropic freeze out [3] is to implement an anisotropy in the phase-space distribution along the radial direction. As ansatz for the functional form of $f(x^\mu, \vec{p})$, we used

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the Romatschke–Strickland distribution [6], which reads in the local rest frame:

$$f_{aniso}^{LRF}(x^\mu, \vec{p}, \Lambda, \xi) \propto exp\left(-\frac{\sqrt{(p\cdot u)^2 + \xi(x^\mu)(p^r)^2}}{\Lambda(x^\mu)}\right), \quad (2)$$

where $\xi(x^\mu)$ is the anisotropy parameter and $\Lambda(x^\mu)$ the "anisotropic temperature", which is going to be the temperature in the case of isotropy. To implement this distribution function into the Cooper–Frye formula, one must boost it into the laboratory frame.

As a consequence of using the Romatschke–Strickland distribution, we get an additional parameter, which gives us the possibility to tune the anisotropy in the radial direction. In principle this parameter has only the lower boundary -1, since we need the distribution function to be real valued. In order to generate a higher effective temperature/pressure in the radial direction, we need to choose ξ to be less than 0.

Because we are up to now just interested in how the technique of the anisotropic freeze out works, we inserted a blast-wave like fluid velocity profile of the form $u^\phi = u^\eta = 0$, $u^r = \bar{u}_{max}\,\frac{r}{R}\,(1 + 2\sum_n V_n \cos(n\phi))$, where we chose the parameters to be $R = 10$ fm, $\tau_{\rm fo} = 7.5$ fm/c, $V_2 = V_3 = 0.05$ and we investigated the emission of pions. Because of the lack of a microscopical model we ignored a possible position-space dependence of the anisotropic temperature Λ and the anisotropy parameter ξ. Now that we have all ingredients (1) and (2) together, we are able to compute observables for the anisotropic freeze out.

4. Observables

4.1. Particle spectra

The first observable which we present is the transverse particle spectrum. We plotted this for various values for the anisotropy parameter in Figure 1. As one can see, the smaller the anisotropy parameter is chosen, the more particle one will detect with higher momenta. This behavior is to be expected, since in anisotropic hydrodynamics one obtains a higher pressure in a direction if one chooses the corresponding anisotropy parameter to be negative. Such a higher pressure will then push the particles to higher momenta in the out-direction, which results in the deviations of the ideal momentum spectrum. As a cross-check we plotted the spectrum for positive ξ-values as well.

In Figure 2 we now "optimized" (by eye) the pairs of anisotropy parameter ξ and anisotropic temperature Λ. As one can see there, we are able to reproduce the same particle spectrum while varying the "temperature" over a range of 30 MeV. This behavior is a first hint that the additional parameter ξ works the way it should, relaxing the need for a well-defined value of the freeze-out temperature

4.2. Anisotropic flow coefficients v_n

The second observables which we now present are the anisotropic flow coefficients v_n. These coefficients arise through a Fourier series of the particle spectrum in the momentum angle ϕ_p. We computed the coefficients up to v_5. They all follow the same trend, so here we are only showing the elliptic flow coefficient v_2.

In Figure 3 we plotted v_2 as a function of momentum for different values of the anisotropy. As one can see, v_2 decreases when the anisotropy decrease. This trend reflects the fact that the negative ξ induces a higher effective temperature in the out-direction. This higher effective temperature results in a higher thermal motion along this direction, which then overcomes the effect of collective motion that is measured by the flow coefficient v_2.

Now we followed the same procedure as before and optimized the pairs of anisotropic temperature Λ and anisotropy parameter ξ, so that we are able to reproduce the same observable although we vary the value of the temperature at freeze out. The results are plotted in Figure 4. Again we are able to reproduce nearly the same v_2 for an freeze out temperature interval of nearly 30 MeV.

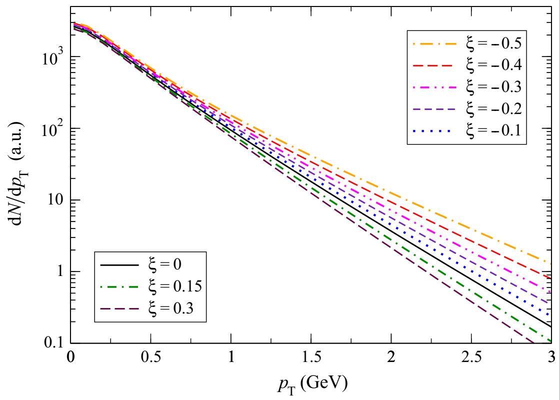

Figure 1. Transverse particle spectrum for various anisotropies ξ.

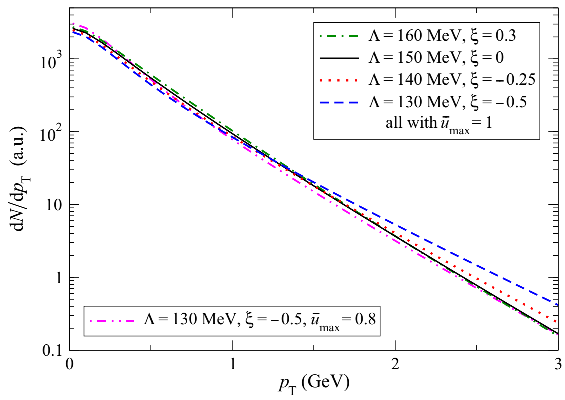

Figure 2. Transverse particle spectrum for optimized ξ, Λ pairs.

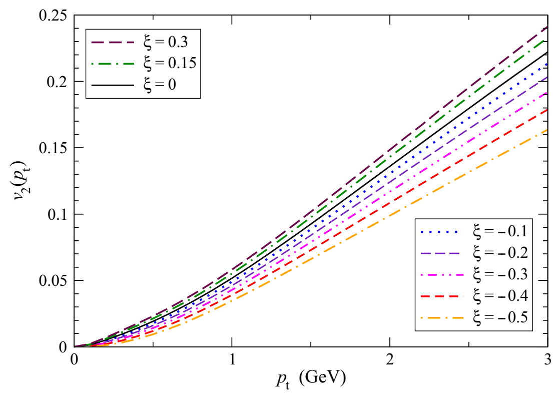

Figure 3. Elliptic flow coefficient v_2 for various ξ.

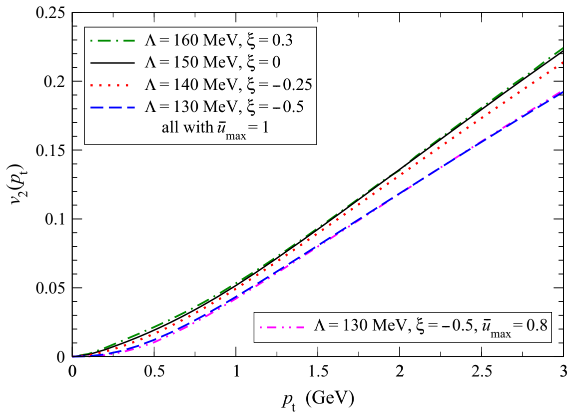

Figure 4. Elliptic flow coefficient v_2 for various pairs of ξ and Λ.

5. Conclusion

We showed that at the cost of an additional parameter, namely the anisotropy parameter ξ we are able to establish a interval of freeze out temperatures, which all give the same observables. So we get rid of the strong dependence of observables on the parameter where we glued together the two asymptotic models, hydrodynamics and kinetic theory. As we showed for decreasing freeze out temperatures Λ we have to choose more negative anisotropy parameters ξ. This reflects the fact that we are able to tune the late times hydrodynamics in such a way, that it crosses over to the (trivial) kinetic theory in a smoother manner.

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