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Photons as solitons

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Abstract. The field model of space-time film is considered. New class of exact solutions is discussed. These solutions are solitons moving with the speed of light or light-like solitons. Subclass of these solutions with approximate helical symmetry or having twist is considered. Determinate conformity of twisted light-like solitons and photons is shown. In particular, the obtained expression of the soliton energy coincides with the appropriate expression for photon in high-frequency approximation. Planck formula is obtained (in some approximation) for the equilibrium energy spectral density of the ideal gaze of the twisted solitons. An experimental check of the modified formula for the energy of photon (in low-frequency region) as the twisted light-like soliton is proposed.

1. Introduction

The concept of general relativity in physics is closely coupled with one of unified field. According to this concept the material world can be described as a solution of equations of the unified field. Also the elementary particles must be represented by localized solutions of the unified field model. The model equations must be nonlinear for the natural description of the interaction between the particles. As well known the creator of the relativity theory A. Einstein adheres to just this position [1].

The term “soliton” in wide sense means a localized wave solution of a nonlinear field model. Thus in the framework of the concept under consideration the elementary particles are solitons of the unified field model.

In particular, Born - Infeld electrodynamics was considered as a possible unified field model. In the framework of this nonlinear electrodynamics the electromagnetic and gravitational interactions appear naturally [2-4].

But the appropriate means for the description of the interactions used in nonlinear electrodynamics are model independent. That are integral conservation law of energy-momentum and characteristic equation for electromagnetic and gravitational interactions accordingly. These means used for the model of space-time film give the similar results [5-6]. Thus the model of space-time film can be considered as possible unified field model.

Here we discuss the obtained exact solutions for the model of space-time film which can be correlated to real photons [7].

2. Extremal space-time film

We start from the following action integral:

$$\mathrm{A} = \int\limits_V \sqrt{|M|}(dx)^4, \qquad M = \det\left(M_{\mu\nu}\right) \ , \qquad M_{\mu\nu} = m_{\mu\nu} + \chi^2 \frac{\partial\Phi}{\partial x^\mu}\frac{\partial\Phi}{\partial x^\nu}, \tag{1}$$

where Φ is the scalar real field function, V is a space-time volume, Greek indices take the values $\{0,1,2,3\}$, $x^0 = ct$, c is the speed of light, t is time, $m_{\mu\nu}$ are components of the metric tensor for flat four-dimensional space-time, χ is a dimensional constant.

The action (1) is a generalization of one for minimal two-dimensional thin film in three-dimensional space to four-dimensional space-time film in a five-dimensional space.

The model equation has the following form in Cartesian coordinates:

$$\left(\bar{m}^{\mu\nu}\left(1 + \chi^2 \frac{\partial\Phi}{\partial x^\alpha}\frac{\partial\Phi}{\partial x_\alpha}\right) - \chi^2 \frac{\partial\Phi}{\partial x_\mu}\frac{\partial\Phi}{\partial x_\nu}\right)\frac{\partial^2\Phi}{\partial x^\mu \partial x^\nu} = 0, \tag{2}$$

where $\bar{m}_{\mu\nu}$ are components of the diagonal metric constant tensor with the signatures $\{+,-,-,-\}$ or $\{-,+,+,+\}$.

3. Lightlike solitons

Let us consider solutions of the equation (2) depending on three variables:

$$\Phi = \Phi\left(\theta, x^1, x^2\right) \ , \qquad \theta = \omega\, x^0 - k\, x^3 \,, \qquad \omega > 0 \ , \qquad \omega^2 = k^2. \tag{3}$$

This function is a wave packet propagating with the speed of lighjt along x^3 axis.

The substitution (3) to the equation (2) gives the following one:

$$\left(1 \mp \chi^2\left(\frac{\partial\Phi}{\partial x^2}\right)^2\right)\frac{\partial^2\Phi}{(\partial x^1)^2} \pm 2\chi^2 \frac{\partial\Phi}{\partial x^1}\frac{\partial\Phi}{\partial x^2}\frac{\partial^2\Phi}{\partial x^1 \partial x^2} + \left(1 \mp \chi^2\left(\frac{\partial\Phi}{\partial x^1}\right)^2\right)\frac{\partial^2\Phi}{(\partial x^2)^2} = 0, \tag{4}$$

where top and bottom signs correspond to two possible signatures of metric in (2).

As we see the obtained equation (4) does not include derivatives on phase θ of wave (3).

The exact solution class of equation (4) with two arbitrary functions was obtained by means of a nontrivial transformation of variables [7].

In particular, we consider the solutions which called twisted lightlike solitons. These solutions have the following asymptotic form:

$$\Phi \sim \frac{Q}{\rho^m}\ \cos\left(m\,\varphi\ \mp\ \theta\right) \ \text{ for } \ \rho \to \infty \tag{5}$$

where $\{\rho, \varphi\}$ are the polar coordinates in the $\{x^1, x^2\}$ plane, $Q = Q(\theta)$ is an arbitrary scaling function of phase θ, m is twist parameter (natural number), top $(-)$ and bottom $(+)$ signs correspond to right and left twist of the soliton propagating along the positive direction of x^3 axis that is for $k > 0$ in (3).

The twisted lightlike soliton can have a cavity near its symmetry axis for $\rho = 0$. The field function Φ on the plan $\{x^1, x^3\}$ of the twisted soliton for $m = 1$ and Gaussian scaling function $Q(\theta)$ is shown on Figure 1.

Let us consider the case for slowly varying scaling function $Q(\theta)$ and sufficiently high frequency ω. In this case we have the following notable relations between energy E, momentum P, and angular momentum J of the twisted lightlike soliton:

$$E = P = \frac{\omega}{m} J. \tag{6}$$

That is for the case $m = 1$ and $J = \hbar$ we have the relations which are characteristic for photon.

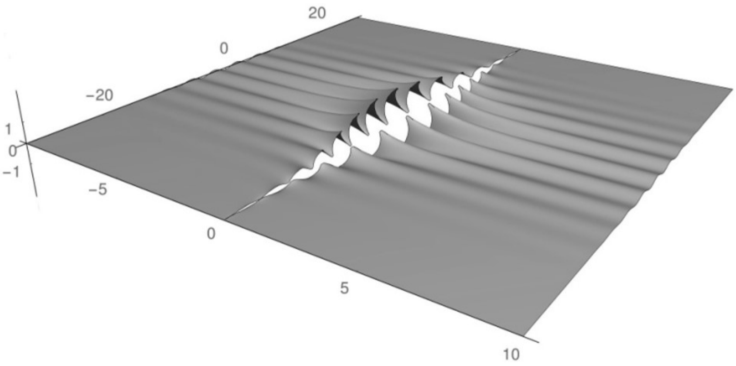

Figure 1. The field function of a Gaussian twisted soliton on the horizontal axial plane.

4. Lightlike twisted solitons as photons

Let us consider an ideal gas of the twisted lightlike solitons with $m = 1$ in bounded three-dimensional volume. Let us suppose also that absorptive and emissive capacities of the walls are provided by soliton-particles having the following constant absolute value of angular momentum:

$$J_e = \frac{\hbar}{2}. \tag{7}$$

Then, because of the angular momentum conservation in absorption and emission events each lightlike solitons in the volume must have the angular momentum equals to $\hbar$:

$$J_p = \hbar. \tag{8}$$

See also [7] for details.

Based on these assumptions and using obtained energy expression for the soliton we derive the equilibrium distribution function by soliton frequencies which coincide with Planck distribution in high-frequency approximation [7].

We have the following expression for the soliton energy in high-frequency approximation:

$$E_p = \hbar\omega. \tag{9}$$

But this dependence can be broken for low frequencies. The appropriate dependence is shown on Figure 2 obtained for the explicit values of parameters. Of course, it must be considered mainly for a qualitative analysis. As we see on Figure 2, the distinction of soliton energy function from the linear one (8) (dashed line) can be noticeable in a low-frequency region.

The appropriate experimental check may be possible with the help of the extrinsic photoeffect. If the photon energy not exactly equals to $\hbar\omega$, then the frequency dependence of photoelectron energy may have a weak nonlinearity near photoemission threshold. The substances with low photoemission threshold is preferable for such experiments.

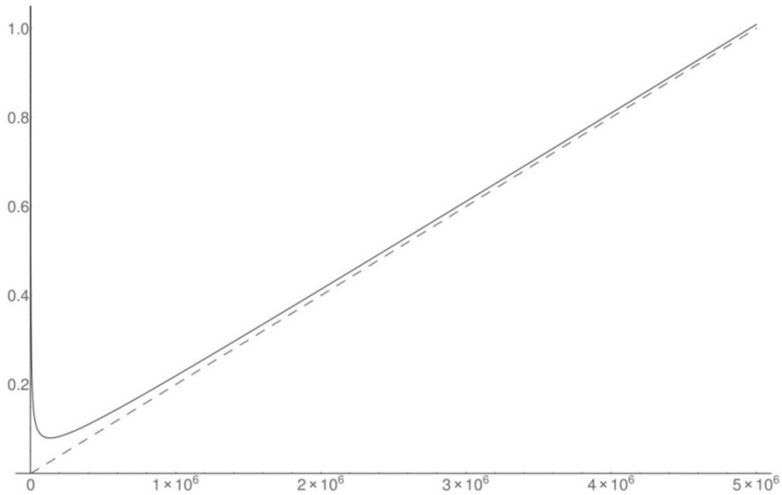

Figure 2. Dependence of the twisted soliton energy in a volume from its frequency.

5. Conclusions

Thus we have considered the field model of extremal space-time film.

We have discussed the new class of exact solutions for this model that is the class of lightlike solitons. We have considered the significant subclass of these solutions that are twisted lightlike solitons. It is notable that the energy of these solitons is proportional to its angular momentum in high-frequency approximation.

For the case of the minimal value of the twist parameter $m = 1$ we have the asymptotic relation between soliton energy, momentum, and angular momentum, which is characteristic for photon.

Then we have investigated relations of the twisted lightlike solitons with $m = 1$ to photons. The model of ideal gas of the twisted lightlike solitons in a bounded volume has considered for this purpose. Planck formula for the soliton energy spectral density in the volume has obtained with explicit assumptions in some approximation.

An experimental check for a validity of the obtained soliton energy true formula for real photon is proposed.

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