Zeta Strings

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Abstract

We introduced a nonlinear scalar field model for open strings with spacetime derivatives encoded in the operator valued Riemann zeta function. These new strings we call zeta strings. The corresponding Lagrangian is derived starting from the exact Lagrangians for effective fields of p-adic tachyon strings. Some basic classical properties of the zeta strings are obtained and presented in this paper. In particular, tachyons and free strings are absent in this model. This paper is a short and slightly modified version of arXiv:hep-th/0703008.

1 Introduction

So far p-adic structures have been observed not only in string theory but also in many other models of modern mathematical physics (for a review of the early days developments, see e.g. [1,2]). One of the greatest achievements in p-adic string theory is an effective field description of scalar p-adic strings [3,4]. The corresponding Lagrangians are very simple and exact. They describe not only four-point scattering amplitudes but also all higher ones at the tree-level. This p-adic string theory has been significantly pushed forward when was shown [5] that it describes tachyon condensation and brane descent relations simpler than by ordinary bosonic strings. After that success, many aspects of p-adic string dynamics have been investigated and compared with dynamics of ordinary strings (see, e.g. [6] and references therein). A systematic mathematical study of spatially homogeneous solutions of related nonlinear equations of motion was considered in [7]. Some possible cosmological implications of p-adic string theory have been also investigated [8]. As a result of these developments some nontrivial features of ordinary string theory have been reproduced from the p-adic effective action.

Adelic approach to the string scattering amplitudes is a very useful way to connect p-adic and ordinary counterparts (see [1] as a review). Moreover, it eliminates unwanted prime number parameter p contained in p-adic amplitudes and also cures the problem of p-adic causality violation. Adelic generalization of quantum mechanics was also successfully formulated, and was found a connection between adelic vacuum state of the harmonic oscillator and the Riemann zeta function [9]. Recently, an interesting approach toward foundation of a field theory and cosmology based on the Riemann zeta function was proposed in [10]. The present paper is a result of an attempt to integrate all p-adic effective field actions into one scalar field theory.

2 Open scalar zeta string

The exact tree-level Lagrangian of effective scalar field φ which describes open p-adic string tachyon is

$$\mathcal{L}_p = \frac{1}{g^2}\frac{p^2}{p-1}\Big[-\frac{1}{2}\varphi\, p^{-\frac{\Box}{2}}\varphi + \frac{1}{p+1}\varphi^{p+1}\Big], \tag{1}$$

where p is any prime number, $\Box = -\partial_t^2 + \nabla^2$ is the D-dimensional d'Alembertian and we adopt metric with signature $(- + \dots +)$. The equation of motion for (1) is

$$p^{-\frac{\Box}{2}}\varphi = \varphi^p, \tag{2}$$

and its properties have been studied by many authors (see e.g. [7] and references therein).

It is worth noting that prime number p in (1) and (2) can be replaced by any natural number and such expressions also make sense. Now we want to introduce a model which incorporates all the above n-adic string Lagrangians. To this end, let us take the sum of the corresponding Lagrangians $\mathcal{L}_n$ (1) in the form

$$\begin{aligned} L &= \sum_{n\geq 1} C_n \mathcal{L}_n = \sum_{n\geq 1} \frac{n-1}{n^2}\mathcal{L}_n \\ &= \frac{1}{g^2}\Big[-\frac{1}{2}\phi \sum_{n\geq 1} n^{-\frac{\Box}{2}}\phi + \sum_{n\geq 1}\frac{1}{n+1}\phi^{n+1}\Big], \end{aligned} \tag{3}$$

where coefficients $C_n = \frac{n-1}{n^2}$ are inverses of those within $\mathcal{L}_n$. To emphasize that Lagrangian (3) describes a new field, we introduced notation ϕ instead of φ.

Recall that the Riemann zeta function is defined as

$$\zeta(s) = \sum_{n\geq 1}\frac{1}{n^s} = \prod_p \frac{1}{1-p^{-s}}, \quad s = \sigma + i\tau, \quad \sigma > 1. \tag{4}$$

Employing usual expansion for the logarithmic function and definition (4) we can rewrite (3) in the form

$$L = -\frac{1}{g^2}\Big[\frac{1}{2}\phi\,\zeta\Big(\frac{\Box}{2}\Big)\phi + \phi + \ln(1-\phi)\Big], \tag{5}$$

where $|\phi| < 1$.

$\zeta\big(\frac{\Box}{2}\big)$ acts as pseudodifferential operator in the following way (see also [10]):

$$\zeta\Big(\frac{\Box}{2}\Big)\phi(x) = \frac{1}{(2\pi)^D}\int e^{ixk}\zeta\Big(-\frac{k^2}{2}\Big)\tilde{\phi}(k)\,dk, \quad -k^2 = k_0^2 - \vec{k}^2 > 2+\varepsilon, \tag{6}$$

where $\tilde{\phi}(k) = \int e^{(-ikx)}\phi(x)\,dx$ is the Fourier transform of $\phi(x)$. The region of integration in (6) is $-k^2 = k_0^2 - \vec{k}^2 > 2+\varepsilon$, where ε is an arbitrary small positive number, and it follows from the definition of zeta function (4). Here and in the sequel, it is understood that this zeta function depends also on ε. The usual tachyon with mass $m^2 = -k^2 = -2$ is absent in this theory and the energy of this new string is bounded from below by $k_0^2 = 2$ in the string mass scale.

Dynamics of this field ϕ is encoded in the (pseudo)differential form of the Riemann zeta function. When the d'Alembertian is an argument of the Riemann zeta function we call such string a zeta string. The equation of motion for the zeta string ϕ is

$$\zeta\Big(\frac{\Box}{2}\Big)\,\phi = \frac{1}{(2\pi)^D}\int_{k_0^2-\vec{k}^2>2+\varepsilon} e^{ixk}\,\zeta\Big(-\frac{k^2}{2}\Big)\,\tilde\phi(k)\,dk = \frac{\phi}{1-\phi} \tag{7}$$

which has an evident solution $\phi = 0$.

The above zeta string potential is given by

$$V(\phi) = \frac{1}{g^2}\,[\phi + \ln(1-\phi)] = -\frac{1}{g^2}\sum_{n\geq 2}\frac{\phi^n}{n}\,, \tag{8}$$

where $V(\phi) \leq 0$ for $-1 < \phi < 1$: it increases from $V(\phi \to -1) = -\frac{0{,}31}{g^2}$ to the maximum $V(\phi = 0) = 0$ and then $V(\phi)$ decreases so that $V(\phi) \to -\infty$ as $\phi \to +1$.

For the case of time dependent spatially homogeneous solutions one has to consider the equation of motion

$$\zeta\Big(\frac{-\partial_t^2}{2}\Big)\,\phi(t) = \frac{1}{(2\pi)}\int_{|k_0|>\sqrt{2}+\varepsilon} e^{-ik_0t}\,\zeta\Big(\frac{k_0^2}{2}\Big)\,\tilde\phi(k_0)\,dk_0 = \frac{\phi(t)}{1-\phi(t)}\,. \tag{9}$$

In the weak field approximation ($|\phi(t)| \ll 1$) the above expression $\phi/(1-\phi) \approx \phi$ and (9) becomes a linear equation which can be written in the form

$$\int_{\mathbb{R}} e^{-ik_0t}\Big[\zeta\Big(\frac{k_0^2}{2}\Big)\,\theta(|k_0| - \sqrt{2} - \varepsilon) - 1\Big]\,\tilde\phi(k_0)\,dk_0 = 0\,, \tag{10}$$

where θ is the Heaviside function. Since $\zeta\Big(\frac{k_0^2}{2}\Big) > 1$ when $|k_0| > \sqrt{2}$ the equation (10) has solution only for $\tilde\phi(k_0) = 0$. This also means the absence of free scalars with mass.

3 Concluding remarks

We derived an effective field Lagrangian for open scalar string, which contains all n-adic Lagrangians. An infinite number of spacetime derivatives and related nonlocality are governed by the Riemann zeta function. Potential is nonlinear. The tachyon is absent in this theory, although it is contained in the constitutive p-adic Lagrangians. In this model there are no mass and free strings. Energy is bounded from below. A similar procedure can be extended to the open-closed scalar strings [11].

There are still many classical aspects which need to be investigated. One of them is a systematic study of the equation of motion (7). In the quantum sector it is very desirable to investigate some basic properties.

In this short paper we have restricted our attention to the case when field satisfies $|\phi| < 1$ and the Riemann zeta function $\zeta(s)$ is defined for $\mathcal{R}e\,s > 1$. Analytic continuation of the potential and the zeta function should provide more complete insight.

This paper can be regarded as a first step towards more realistic model of zeta strings and effective Lagrangian for adelic strings [12].

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