DESIGN OF A LABORATORY EXPERIMENT TO TEST NEWTON'S LAW IN THE RANGE 10^{-1}-10^{3} METERS

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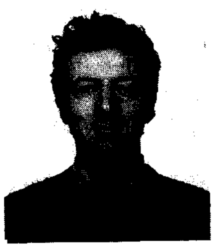

ABSTRACT

The experimental status of the fifth force still needs to be clarified. Almost all the running experiments make use, as source of the gravitational force, of a large amount of matter not controlled by the experimentalist, like mountains, hills or lakes. The use of this kind of sources limits the sensitivity of the experiments and leaves open the disturbing possibility that the observed effects are related to the geographical location of the experiment. In this paper a different experimental approach is discussed, based on the use of a dynamic torsion balance making resonant oscillations in the field of a slowly moving mass of water, located in the laboratory and controlled by the experimentalist. This method is conceptually similar to the one used in the test of the equivalence principle during the 70's and would allow a very high sensitivity over a large interval of interaction ranges.

1. INTRODUCTION

In this paper we discuss a method to test the validity of the newtonian potential in the range 10^{-1}-10^{3} m. This work was originally motivated by the results[1] obtained two years ago in the reanalysis of the Eotvos[2] experiment, indicating a deviation from the 1/r potential in the range ≈ 200 meters. But the interest in this kind of measurement is still high because the data obtained recently in experiments dedicated to this problem are apparently contradictory and the situation is far from being clear [3].

The experiment discussed here[4] makes use of a torsion balance made out of two different materials and having a very long relaxation time (> few years); the balance makes very small oscillations in the periodic gravitational field generated by a liquid moved repeatedly between two containers. "Dynamic" torsion balances have already been used successfully in experiments looking for very small periodic forces, like the test of the equivalence principle [5] where the periodic source of the gravitational field is represented by the sun in its apparent motion around the earth.

This technique has various advantages with respect to most experiments performed or proposed until now. First of all, the balance is sensitive only to the oscillating component of the gravitational field and is then insensitive to any kind of local gravitational anomaly. Second, the gravitational source is under the complete control of the experimentalist who could then modify its nuclear composition: the possibility of modifying the protons/neutrons contents both of the source of the field and of the test masses would allow a systematic exploration of the coupling charge associated with this hypothetical force[6]. Third, with this experiment it is possible to explore distances much smaller (≈10 cm) than the distances accessible to other experiments. Fourth, the technique employed here is possibly the most sensitive to detect small forces: the proposed experiment is not designed to reach the highest sensitivity obtained with this kind of devices[5], but it would be possible, if necessary, to make use of this reserve of sensitivity.

There is also another reason that motivated this study: the sensitivity limit of the resonant torsion balance has not been really explored, and from a theoretical point of view it is possibile to reach sensitivities that are more than ten orders of magnitude better than today state of the art. The improvement in sensitivity of these devices would allow the access to a field of very interesting fundamental measurements ranging from the test of the validity of the equivalence principle for weak forces[8], to the axion search[9] and to "dark matter" searches using macroscopic coherent effects[10]. In fact when designing an earth-based laboratory experiment to test the newtonian potential we did address and solve various problems that are also relevant for the previous searches: a laboratory measurement dedicated to the fifth force would then be a kind of test stand for future developements of this technique.

In the following we refer often to some basic papers: the final paper on the Roll, Krotckov and Dicke[5] experiment (RKD in the following), the paper describing the Braginskii and Panov[5] experiment (BP in the following) and the beautiful book on the measurement of weak forces written by Braginskii[7]. The proposed experiment (PE in the following) has a lot of analogies both with RKD and BP: for that reason we will refer to the previous papers for all the experimental details that are not discussed thoroughly, concentrating ourself mostly on the aspects where the PE is different.

2. THE EXPERIMENT

Our philosophy was to design an experiment having a sensitivity sufficient to explore a region of interaction strength at least one order of magnitude smaller than the effect originally observed by Fischbach et al., without necessarily push the performance of the torsion balance to its limit, in order to semplify the construction. As a matter of fact, in the same experimental conditions, our balance would show a sensitivity ≈ 10 times lower than the BP balance. Various constructive details are taken from RKD or BP, because we concentrated ourself on the original part of the design.The characteristic parameters of the PE with respect to BP are compared in Table 1.

The experiment is schematized in fig. 1, 2 and 3. A torsion balance whose plate is made out of two different materials , for instance Cu ($B/\mu = 1.00112$) and Be ($B/\mu = 0.99865$), oscillates in the periodic acceleration field generated by ≈ 800 liter of water, slowly moving, with the same period of the balance, between two identical containers (400 cm long and having a cross section of 45x45 cm^2), placed symmetrically with respect to the balance. The resulting periodic gravitational acceleration has an amplitude of $\approx 1.10^{-5}$ cm/sec^2.

The total weight of the plate is $\approx$ 44 g: its structure is shown in fig. 2. The plate is suspended to a point higher than its center of gravity through three adjustable wires: this geometry is at the same time very insensitive to the plate construction tolerances and to impulsive multipole gravitational fields generated by objects moving near the laboratory. The plate radius is 5 cm, and all the yoke components are covered with a metallic layer to improve the relaxation time (interaction of the metallic parts with the residual earth magnetic field).

A double face mirror of optical quality is mounted on the vertical part of the yoke: the yoke is suspended to a 17 μm, ≈ 100 cm long, tungsten wire. The period of oscillation of this torsion pendulum results $\approx$ 57.2 minutes. The top of the wire is mounted on a mechanism for adjusting the height and the angular position of the torsion balance. This mechanism is mounted inside the vacuum chamber and couples through magnetic interaction with a series of electromagnets placed around the chamber. The mechanism to adjust the angular position of the balance is very accurate: through a series of demultiplying gears the uncertainty on the angular adjustment of the balance is less than 10^{-7} rad. The external electromagnets, controlled by a computer, drive the first gear: the system will allow both small corrections of the equilibrium position of the balance and a very accurate π turn of the balance. This rotation is very important for the PE as will be clear in the following. A conducting ellipsoid is mounted on to the vertical part of the yoke and placed between two condenser plates: electric pulses applied properly to the plates allow to vary the amplitude of the balance oscillations. In this way BP reports they were able to reduce the amplitude of the oscillations to $\approx 1.10^{-5}$ rad; an additional attenuation could be obtained if necessary by moving properly the liquid between the two tanks as described in the following paragraphs.The top of the vacuum chamber is suspended, as shown in fig. 1, to a trapezoidal suspension running parallel to the water tanks: this suspension decouples the balance from the movements of the ceiling, due to the water motion between the two tanks.

An AC driven led could be a reasonable choice for the light source: the beam, after proper focalization, is reflected from the mirror to a charge division silicon device, read by computer: the accuracy of the silicon device is ≈ 2 microns. The silicon device is placed at ≈ 5 meters from the mirror and tilted with respect to the beam direction in order to increase the optical lever by a factor of 5, as suggested by BP. The computer readout allows an online analysis of the behaviour of the experiment, including the Fourier extraction of the relevant informations (see later).

The tungsten wire has to be treated carefully in order to reduce the monotonic drift of its equilibrium position: we plan to follow the prescriptions described in ref.[5,7], in order to obtain less than $<1.\ 10^{-5}$ rad/day monotonic drift.

The balance is mounted in a stainless steel vacuum chamber grounded together with the balance : we plan to reach a stable inner pressure less than ≈ 1.10^{-7} torr, following the same method described by RKD. The vacuum chamber with inside the balance is emptied using a a diffusion pump and backed at 370°: when the pressure reaches ≈ 10^{-6} torr, the chamber is sealed off and an ion pump, mounted on the chamber, started. After pumping few days the pressure stabilizes: RKD report that in this way they obtained a stable 10^{-8} torr during a 15 months period.

The vacuum chamber is carefully shielded with a multilayer mumetal shield, to reduce the earth's magnetic field inside the chamber: we plan to reduce the magnetic field by an order of magnitude. Additional layer of insulating material are placed around the chamber to avoid temperture gradient on the vacuum chamber itself.

One liquid container is shown in fig. 3: each tank is a parellepiped ≈ 400 cm long, having a cross section of ≈45x45 cm^2. The top and bottom sides move symmetrically being mounted on endless gears properly coupled to a phase controlled motor with demultiplying gears. Depending on the versus of rotation of the motor the two sides, moving in opposite directions, pump in or out the liquid: the other tank movement is clearly dephased by 180°. In this way the experiment keeps all the time a symmetry with respect to the z axis (vertical), and the liquid is distributed all the time over ± 200 cm; both facts are important as it is discussed in the following. During a period of oscillation (≈ 1 hour) 1600 liters of liquid should move between the two tanks: that corresponds to the reasonable value of half liter/second.

As discussed in detail by RKD, the full experiment has to be in a room where the temperature is very stable, and the temperature gradients very small. Following RKD, we plan to put the experiment is an underground room, covered by a carefully insulated roof. All the experiment controls and monitoring are done remotely.

It is also important to perform null experiments using a balance made out of only one of the two materials: if the experiment is designed and built correctly we should not observe any periodic torsion in the null experiment.

The measurement is performed observing the variation of the oscillation amplitude of the torsion pendulum with time, when the liquid is moving. If the period of the liquid flow is the same as the period of the torsion balance, the sensitivity of the device is maximized. The data are extracted from the recording of the pendulum oscillations by Fourier analysis (see RKD and BP). The accuracy is inversely proportional to the square root of the number of periods analized. As discussed in the following we should perform a pair of measurements, separated by a rotation of the balance of 180°: the difference of the results obtained in the two measurements is proportional to the fifth force effects.

3. SENSITIVITY OF A RESONANT TORSION BALANCE SUBJECTED TO A PERIODIC FORCE(7)

The minimum detectable acceleration by an oscillator having a mass **m** and a relaxation time, τ_r, longer than the measurement time, τ_m, and subjected to thermal fluctuations results, in the classic approximation,

$$[\Delta a]_{min} = \xi \left(8 k_B T / m \, \tau_m \, \tau_r \right)^{1/2} \qquad \textbf{(1)}$$

where ξ is a coefficient of the order of few units that characterizes the confidence level of the measurement ($\xi \approx 2$ at 95% C.L., $\xi \approx 4$ at 99.99% C.L.), T is the absolute temperature in Kelvin degrees and k_B is the Boltzmann constant.

One can easily show that, if the measurement time is fixed, it is possible to improve the sensitivity of the measurement by increasing the oscillator relaxation time, τ_r. This quantitiy in fact measures the characteristic storage time of the analogic memory rapresented by the torsion balance, and if this time is longer the device can integrate longer the effect due to the external periodic force , thus increasing the sensitivity of the experiment. By substituting in (1) the values relative to BP(5), $\tau_r = 6.\ 10^7$ sec, $\tau_m = 6.\ 10^5$ sec and **m**=4 g, we obtain that the limit in sensitivity imposed by the thermal fluctuations was in this case $\Delta a \approx 10^{-13}$ cm/sec^2.

The sensitivity of the measurement is improved if the period of the torsion balance and the period of the oscillating source are equal. In the case of BP the latter was about 24 hours. It is very difficult to build a torsion balance having such a long oscillating period: in fact

$$\tau_0 = 2 \pi \left(32 \, J_0 \, L / E \, d_0^4 \right)^{1/2} \qquad \textbf{(2)}$$

where L is the wire length, d_0 its diameter, J_0 is the momentum of inertia of the balance and E is the modulus of elasticity of the wire material. This equation shows that, at fixed J_0, to increase the period we have to reduce the wire diameter, because its tensile strength depends on the square of the diameter while its torsional strength decreases like the fourth power of the diameter. In the case of BP, they used a 5. 10^{-4} cm diameter tungsten wire , 280 cm long: the balance plate (m = 4 g) was made out of 8 masses placed at 10 cm from the wire. For tungsten $E=1.5\ 10^{12}$ dyne/cm, it follows that the oscillating period resulted about $2.1\ 10^4$ sec, that is about four times lower than 24 hours.

The relaxation period τ_r, is determined by the residual pressure in the vacuum chamber,

$$\tau_{r gas} = (m / 4S)(\mu k_B T)^{-1/2}(k^*)^{-1} \quad (3)$$

and by the internal dissipation of the wire,

$$\tau_{r fibra} = E\ \tau_0 2 / \eta\pi^2 , \quad (4)$$

where m is the oscillator mass, S is its surface area, μ is the mass of the gas molecules, k^* is the molecules density and η is the dissipation coefficient of the wire material. Putting in equations (3) and (4) the parameters of BP(m=4 g, S=2 cm^2, $\mu = 5.5\ 10^{-23}$ g, $k^* \leq 3.\ 10^{-8}\ cm^{-3}$ (residual pressure $p \leq 10^{-8}$ torr), and $\eta = 10^{10}$ poise) we obtain

$$\tau_{r gas} \geq 3.\ 10^9 \text{ sec}, \qquad \tau_{r fiber} = 8.\ 10^9 \text{ sec}.$$

In addition a significant contribution to the dissipation could be given by the passive currents induced by the earth's magnetic field on the moving pendulum's metallic parts. This contribution to the relaxation time depends on the construction geometry of the balance: for BP it was (without the additional permalloy screen used in the experiment)

$$\tau_{r mag} \leq 3.\ 10^8 \text{ sec} \quad (5).$$

In all the three cases the relaxation time is about three order of magnitude longer than the measurement time: it follows that the experimental sensitivity is properly described by equation (1) . It results then $\Delta a \approx 10^{-13}\ cm/sec^2$ at 95% of CL.

In reality the minimum detectable acceleration was limited in BP by the minimum angle detectable in the experiment, $\Delta\phi$:

$$\Delta\phi = g_{sun}\Delta(Al,Pt) / 3.07\ R\ \omega_0^2 \quad (6)$$

where $g_{sun} = 0.62\ cm/sec^2$, R is the balance radius, ω_0 is its oscillating period, $\Delta(Al,Pt) = (M/m)_{Al} - (M/m)_{Pt}$, where m and M are the inertial and gravitational masses of the weights. The factor 3.07 is due to BP geometry and latitude:this factor is equal to one in the PE. The minimum detectable angle in PE will be similar to BP, that is $\Delta\phi \approx 1.8\ 10^{-7}$ rad.

4. RESONANT TORSION BALANCE: LABORATORY VS FREE FALL EXPERIMENTS

There are some important difference between PE and RKD or BP:

1) the forces/torques acting on the balance are not the same;

2) the distance between the source and the balance differs by many order of magnitude.

Let us analyse the first point. In the case of RKD or BP the equation of the forces acting on the balance results (fig.4a):

$$\underline{\mathbf{F}} = \Sigma_i M_i \underline{\mathbf{a}}_i^G = m_{TOT} \underline{\mathbf{a}} \qquad (7)$$

(M is the gravitational mass while m is the inertial mass and the index i run on all the elements of the balance). The two vectors represent respectively the sun acceleration on the earth surface, $\underline{\mathbf{a}}_i^G$, and the corresponding inertial acceleration, $\underline{\mathbf{a}}$; the earth gravitational acceleration and the earth centrifugal acceleration are not present in eq. (7) because they are not periodic and then are undetectable with the technique discussed in the paper.

The sun's field at the earth's surface is very uniform on the region occupied by the balance, excluding negligible corrections $O(\varnothing_{balance}/ d_{earth\text{-}sun}) = O(10^{-12})$. If we rewrite the previous equation putting $\underline{\mathbf{a}}_i^G = \underline{\mathbf{a}}^G$ we obtain:

$$\underline{\mathbf{F}} = \Sigma_i M_i \underline{\mathbf{a}}^G = M_{TOT} \underline{\mathbf{a}}^G = m_{TOT} \underline{\mathbf{a}} \qquad (8).$$

In the same way we can write the equation of the periodic torque acting on the balance that is equal to the sum of all the torques, including the terms due to inertial force:

$$\underline{\mathbf{T}} = \Sigma_i m_i \underline{\mathbf{r}}_i \wedge \{\underline{\mathbf{F}}_i^G + \underline{\mathbf{F}}_i^I\} = \Sigma_i m_i \underline{\mathbf{r}}_i \wedge \{(M/m)_i \underline{\mathbf{a}}^G + \underline{\mathbf{a}}^I\}, \qquad (9)$$

where $\underline{\mathbf{r}}_i$ is the position of the i-th mass with respect to the center of gravity of the balance. The resulting torque $\underline{\mathbf{T}}$ is null if the quantity M/m is the same for all the balance elements; in fact, if the equivalence principle holds for each element the quantity included in the parenthesis is exactly zero.

Let us now study the corresponding equations in the PE case (fig. 4b) where the balance is not in free fall towards the source of the field.

In this case the balance is constrained to stay at a fixed distance from the oscillating source: in fact the acceleration acting on the center of mass of the balance due to the gravitational source is $\approx 10^8$ times smaller than the earth's gravitational field and the swing in the vertical plane has a negligible amplitude. That means that the inertial acceleration due to the presence of the oscillating field is essentially zero and the equation for the forces acting on the various pieces of the balance results :

$$\underline{\mathbf{F}} = \Sigma_i M_i \underline{\mathbf{a}}_i^G + \underline{\mathbf{R}} = 0 \qquad (10)$$

where $\underline{\mathbf{R}}$ is the constraint reaction. This force is applied on the balance axis and it does not contribute to the (periodic) torque that then results :

$$\underline{\mathbf{T}} = \Sigma_i m_i \underline{\mathbf{r}}_i \wedge \underline{\mathbf{a}}_i^G \qquad (11).$$

If there is a force coupling to the baryonic number of the different pieces of the balance, the newtonian gravitational acceleration, $\underline{\mathbf{a}}_i^G$,is then multiplied by a function, $f(B_i/\mu_i,R_i)$, close to unity and depending both on the ratio between the baryonic number and the atomic weight of the i-th piece and on its position with respect to the source. In eq. (11) we assumed the equivalence between the gravitational and the inertial mass in the free fall motion of the balance towards the sun (and the moon). In addition we neglected all the torques due to masses other than the the oscillating liquid; in fact the resonant torsion balance is sensitive only to slowly oscillating fields and not to static contributions to the total torque.

The total torque in eq. (11) depends explicitly on positions and masses of the various balance elements: that is for a not free falling experiment only the sum of all contributions can cancel. Which are the implications of this fact on the constructive tolerances and on the sensitivity limit? It turns out that if the balance plate is suspended at a point higher than its center of gravity , like it happens to a bell, it is very insensitive to the construction tolerances! In fact the equilibrium position of the plate will be by definition such that the c.o.g. is located on the projection of the suspension wire, that is

$$\Sigma_i M_i \mathbf{r}_i = 0 \qquad (12).$$

One could argue that if there is a short range composition dependent force then eq. (12) is not exact because the equilibrium position of the balance will be determined also by this additional term. It is easy to show that eq.(12) is valid up to corrections of order 10^{-11} or smaller, the reason being that the mass of all the earth contributes to vertical acceleration while the effect of the short range force is much smaller, $O(10^{-8})$, and, in addition, the difference of this contribution for the two parts of the plate is at most $O(10^{-3})$. In conclusion, thanks to earth gravity a "bell-shaped" balance is accurately aligned following eq. (12) and it then results quite insensitive to construction tolerances.

Let us discuss now the second point.There are at least two aspects to consider when exploring a range 10^{-9} times smaller than the distance earth-sun;

2a) first of all, to generate a given acceleration field the mass needed is much smaller, being inversely proportional to square of the distance: it follows that the experiment will make use of much smaller masses when exploring smaller distances;

2b) second, the non uniformity of the acceleration field in the region of space occupied by the balance is not negligible and its effect is bigger at smaller distances.

These two points (the use of small source masses and the uniformity of the generated field) represent conflicting requests on the optimal distance between the source and the balance; in the following we discuss the criteria used to design the proposed laboratory experiment trying to optimize the sensitivity vs the difficulty to build the apparatus.

Let discuss the first point. The quantity $g\Delta$ in eq. (6) represents, for a given $\Delta\phi_{min}$, the minimum detectable periodic acceleration: this quantity is the product of a gravitational acceleration times an adimensional parameter that represents the effect we are looking for. As we will see, this quantity is $\approx 3.\ 10^{-12}$ cm/sec^2 in the PE and $\approx 5.\ 10^{-13}$ cm/sec^2 in BP. For a given sensitivity, if the acceleration decreases (increases), Δ increases (decreases)

When looking for a fifth force effect, it is interesting to study a range of interaction length (λ) much smaller ($O(10^{-1}$-$10^{3})$ m) than the distance earth-sun ($O(10^{11})$ m) and in addition we expect a value for Δ much bigger ($O(10^{-6})$) than the limit obtained by BP ($O(10^{-12})$). It follows that it is sufficent to use a torsion balance one order of magnitude less sensitive than BP (in the PE $g\Delta \approx 3.\ 10^{-12}$ cm/sec^2) placed in a oscillating field of$\approx 1.10^{-5}$ cm/sec^2 obtained for example using a small quantity of water ($\approx$ 800 liters) placed very near to the balance.In other words, it would be sufficent to perform an experiment at **small** distances and using a **small** source .

The limit that can be obtained with such an experiment is given , in the λ versus α plot, by the following equation :

$$|\kappa\, \beta_{sune} \{ \beta_1 - \beta_2 \}| \, (1 + r/\lambda) \exp(-r/\lambda) < \Delta_{12} \qquad \textbf{(12)}$$

where $\kappa\beta_1\beta_2 = \alpha$, $\beta_i = (N_i \cos\theta + Z_i \sin\theta)/(m_i / 1 \text{ amu})$ and θ is a mixing angle between protons and neutrons in the nucleus(3). In fig.5(11) the limit given by eq. (12) is plotted both for RKD (line labeled DICKE) and for BP (line labeled PANOV): for $\lambda > r$ the limit reaches the maximum sensitivity and it stays flat, while for $\lambda < r$ the sensitivity becomes rapidly much worse. This behaviour is a consequence of the fact that the presence of a non newtonian, short range term modifies the value of G_N at distances $O(\lambda)$ and smaller; it follows that, given a distance r where the experiment is performed, all the values λ bigger than $\approx$ r are automatically measured.

It is important to stress that eq. (12) implies that , for a given amplitude of the periodic acceleration, a **smaller** distance between the balance and the source of the field correspond to a **wider** λ range explored! This observation strongly favours experiments performed at the smallest distance compatible with the other experimental needs.

Let us suppose to use a torsion balance having a sensitivity $g\Delta \approx 3.10^{-12}$ cm/sec^2: g is the amplitude of the oscillating acceleration generated by the moving liquid, while, for a barion number coupling (θ=45°), $\Delta \approx 2\alpha \{ \beta_1 - \beta_2 \}$. If we take the result of the reanalysis of the Eotvos experiment that suggests $2\alpha \approx 10^{-2}$, and then $\Delta \approx 10^{-5}$ (in fact $\beta_1 - \beta_2 \approx 10^{-3}$), it follows that an acceleration $g_{min} \approx 3.\ 10^{-7}$ cm/sec^2 is sufficent to detect such an effect. This value is very small and corresponds to the acceleration due to a mass of about 500 g of water placed at $\approx$ 10 cm! A more detailed calculation shows that if we take in to account the extension of the source and if we want to place a limit 25 times more sensitive, we need $\approx$ 800 liters moving between two tanks placed at $\approx$10 cm from the torsion balance (see fig.1). We recall that we plan to use a balance that is purposedly $\approx$10 times less sensititive than BP, in order to simplify the construction of the experiment: this fact illustrates well how large is the sensitivity of this kind of device!

In conclusion, the limit on α that we could reach with the proposed balance is α less than $\approx 10^{-4}$ in the range $\lambda >$ 10 cm: such a limit would fill completely the interesting region of the plot α versus λ.

Let us discuss now the second point, 2b. If the gravitational field is not uniform over the space occupied by the balance, the different elements will undergo different accelerations and a periodic torque acting on the balance balance could appear. In other words, eq. (11) describing the total periodic torque becomes:

$$\underline{\mathbf{T}} = \Sigma_i m_i \underline{\mathbf{r}}_i \wedge f(B_i/\mu_i, \underline{\mathbf{R}}_i)\, \underline{\mathbf{a}}_i^G(\underline{\mathbf{R}}_i) \quad \textbf{(13).}$$

Let us discuss this problem in the simpler case of a two arms balance in the geometry shown in fig. 6: we can distinguish two effects, one due to the gradient of the acceleration ($\mathbf{a}_x$) in the direction parallel to the arms ($\partial_y \mathbf{a}_x$) and one due to the gradient in the direction of the field ($\partial_x \mathbf{a}_x$).

The first gradient $\partial_y \mathbf{a}_x$ can simulate a spurious periodic torque in two ways. First of all there is a term :

$$[2(\partial_y \mathbf{a}_x)\ \sigma_r/\mathbf{a}_x] \quad \textbf{(14)}$$

due to the mechanical tolerances on the balance arms (the corresponding term for the masses is much smaller and it is neglected), and in addition there is a term:

$$[2(\partial_y \mathbf{a}_x)\ \sigma_b/\mathbf{a}_x] \quad \textbf{(15)}$$

due to the tolerance on the position of the wire (it should be placed at y=0).

Let we study the effect of the gradient $\partial_x \mathbf{a}_x$ analysing the work done on the balance by the periodic force as a function of time.

The work performed by a periodic force with frequence ω_1 on a two arms balance oscillating with frequence ω_0 and making small oscillations with respect to its equilibrium position, results, in the notation of fig. 7;

$$\mathbf{L}(t) = \Sigma_{i=1,2}\ {}_0\!\int^t m_i a_x(x_i(t'), y_i(t'))(dx_i/dt')dt' \quad \textbf{(16).}$$

In the case of very small oscillations we can write:

$$x_1(t) = r_1[\Theta_0(t)\cos(\omega_0 t+\phi)+\delta\theta], \quad x_2(t) = -r_2[\Theta_0(t)\cos(\omega_0 t+\phi)+\delta\theta],$$

$$y_1(t) = r_1\ ,\ y_2(t) = -r_2,$$

$$dx_1/dt = -r_1\Theta_0(t)\omega_0\sin(\omega_0 t+\phi), \quad dx_2/dt = r_2\Theta_0(t)\omega_0\sin(\omega_0 t+\phi) \quad \textbf{(17)}$$

where ϕ represents the phase between the balance and the source oscillation and $\delta\theta$ is a quantity describing by how much the equilibrium position of the balance is displaced with respect to the axis of symmetry of the experiment **y**. If the period of oscillation of the balance and of the source is the same and $\phi=0$ then it is impossible to transfer energy between the two systems. In eq.s (17) Θ_0 is a function of the time, because the amplitude of the balance oscillations will change if there is a transfer of energy: but if the energy exchanged during a period of oscillation is small with respect to the balance total energy it is possible to approximate $\Theta_0(t)$ con

Θ_0=k, as we do in the following.

Having already taken in to account the effect of the $\partial_y \mathbf{a}_x$ gradient when there is a difference between the two balance arms, in the following we assume $\mathbf{r}_1 = \mathbf{r}_2 = \mathbf{r}$, neglecting higher order corrections proportional to the product of the two gradients:

$$x_1(t)=r[\Theta_0\cos(\omega_0 t+\phi)+\delta\theta]=-x_2(t)$$
$$y_1(t)=-y_2(t)= r,$$
$$dx_1/dt= -r\Theta_0\omega_0\sin(\omega_0 t+\phi)=-dx_2/dt \qquad (18)$$

If we develop with respect to **x**=0 the space dependence of the acceleration $\mathbf{a_x}(\mathbf{x_i}(\mathbf{t}),\pm\mathbf{r})= \mathbf{a_x}(\mathbf{x_i}(\mathbf{t}),\mathbf{r})$ we obtain :

$$a_x(x(t),r)=A\cos(\omega_1 t)f(x),$$
$$f(x)=f(0)+df/dx|_{x=0}x+O(x^2) \qquad (19)$$

Given that the balance oscillations are very small, $<10^{-5}$ rad, we keep only the first order. Using eq.s (17)-(19) we rewrite eq. (16) and, putting $r\Theta_0=X_0$, $r\delta\theta=\varepsilon$, we obtain:

$$L(t)=AmX_0\omega_0\{$$
$$-\int_0^t[f(0)+df/dx|_{x=0}[X_0\cos(\omega_0 t'+\phi)+\varepsilon]]\sin(\omega_0 t'+\phi)\cos(\omega_1 t')dt'+$$
$$\int_0^t[f(0)+df/dx|_{x=0}[-X_0\cos(\omega_0 t'+\phi)-\varepsilon]]\sin(\omega_0 t'+\phi)\cos(\omega_1 t')dt'\}=$$
$$2AmX_0\omega_0 df/dx|_{x=0}\{X_0\int_0^t\cos(\omega_0 t'+\phi)\sin(\omega_0 t'+\phi)\cos(\omega_1 t')dt'-$$
$$\varepsilon\int_0^t\sin(\omega_0 t'+\phi)\cos(\omega_1 t')dt'\}= L_1(t)+ L_2(t) \qquad (20).$$

Solving the integrals assuming $\omega_1=\omega_0=\omega$, i.e. the best experimental situation, only the second gives a contribution that over a period is different from zero:

$$L_2(t)= -2AmX_0\omega\varepsilon df/dx|_{x=0}\sin\phi[\sin(2\omega t)/2\omega+t] \qquad (21).$$

It contains a linear term in **t** that lets the balance energy increase or decrease depending on the sign of $\varepsilon\sin\phi$. If $\sin\phi=0$ the transfer of energy is zero but this particular case is not interesting because also the sensitivity of the device to the presence of the fifth force vanishes.

The fraction of the total balance energy exchanged during a period is then proportional to $\sin\phi$:

$$\Delta E/E\approx 2AmX_0\omega\varepsilon\sin\phi\, df/dx|_{x=0}T/(.5\ kX_0^2/r^2) \qquad (22)$$

where **k** is the torsion constant of the balance. By substituting in the equation the parameters of the PE, we obtain:

$$\Delta E_\varepsilon/E\approx -1.02\sin\phi\,\delta\theta/\Theta_0 \qquad (23).$$

If we repeat the same analysis in the case where it is present a fifth force effect, $\Delta = \delta m/m$, we obtain a similar result, except for the field gradient:

$$L_\Delta(t) = -AmX_0\Delta\omega f(0)\sin\phi[\sin(2\omega t)/2\omega + t] \qquad (24),$$

and by substituting the parameters of the PE, we obtain:

$$\Delta E_\Delta/E \approx -1.1 \; 10^2 \sin\phi\Delta/\Theta_0 \qquad (25).$$

Eq.s (23) and (25) hold only if $\Delta E/E$ <<1 during a period.

It clearly turns out from this analysis that in the PE a displacement of the equilibrium position from the symmetry axis could simulate a fifth force effect. To separate the two contributions we observe that if we rotate by exactly 180° degrees the balance the contribution due to the gravitational anomaly changes sign while the contribution due to $\delta\theta$ does not: that means that if we perform a first measurement of $\Delta E/E$ over a reasonable number of periods, and then we rotate the balance by exactly 180° and repeat the measurement , the sum of the two results gives a term proportional to $2\delta\theta$ while the difference between the two results gives a term proportional to 2Δ. If necessary by repeating iteratively the double measurement it is possible to reduce at pleasure the effect due the gradient $\partial_x a_x$, correcting the equilibrium position of the balance by reducing the offset $\delta\theta$: in any case if $\delta\theta$ is different from zero the presence of the gradient $\partial_x a_x$ could also be used to reduce the amplitude of the balance oscillations before starting a set of measurements.

The above method to eliminate the gradient of the field in the x direction is essential for the feasibility of the PE: in fact , when using small masses placed at small distances from the balance, the term $\partial_x a_x$ is large with respect to the sensitivity of the experiment, for almost any shape of the the oscillating source.

We conclude this paragraph writing the equation for the spurious periodic torque due to the only effect contributing to it, the y gradient of the field:

$$(\sigma_{T^*_2}/T_1)^2 = 4\{[(\partial_y a_x)\sigma_r/a_x]^2 + [(\partial_y a_x)\sigma_b/a_x]^2\} \qquad (26).$$

where σ_{T_2} is the total spurious periodic torque acting on the balance while T_1 is the torque acting on a single arm. The square root of this quantity has to be << 4 10^{-7}, that is less than our goal sensitivity. If we assume the following construction parameters:

a) $\sigma_r/r \approx 2\;10^{-4}$, $r = 5$ cm => $\sigma_r \approx 10\;\mu m$

b) $\sigma_m/m \approx 10^{-5}$, $m = 23$ g => $\sigma_m \approx 230\;\mu g$

c) $\partial_y a_x/a_x \approx 1.5\;10^{-5}$ => $\sigma_b \approx 50\;\mu m$

then the spurious torque of eq. (26) results smaller than the proposed sensitivity. The tolerances a)-b) are relatively easy to obtain and the condition c) is also fulfilled by a proper design of the water containers as discussed in the next paragraph.

5. DESIGN OF THE SOURCE OF THE OSCILLATING GRAVITATIONAL FIELD

We need to shape our field source in such a way the quantity $\partial_y \mathbf{a}_x$ is small over the region occupied by the balance. It is then reasonable to start from a geometry where the liquid-container has a shape that extends itself along the y coordinate.

A simple calculation shows that for a mass uniformly distributed along a straight segment $-s < y < +s$, the two gradients $\partial_x \mathbf{a}_x$ and $\partial_y \mathbf{a}_x$, computed at the extremity y=r of the arm of a balance placed at a distance $\mathbf{x_0}$ from the source, result, if $r \ll s$, $x_0 \ll s$:

$$\partial_x \mathbf{a}_x / \mathbf{a}_x \big|_{y=\pm r} \approx 1/s, \qquad \partial_y \mathbf{a}_x / \mathbf{a}_x \big|_{y=\pm r} \approx X^2_0/s^3 \qquad (27).$$

From a more detailed calculation we obtained that a square cross-section tank (45x45 cm^2) extending over $\approx \pm$ 200 cm generates a y gradient sufficiently small if the maximum distance from the balance is $\approx$10 cm. The volume of water oscillating between the two symmetric containers is in this case $\approx$ 800 liters, and the maximum intensity of the acceleration turns out what we need, that is$\approx 1.\ 10^{-5}$ cm/sec^{-2} .

6. CONCLUSIONS

In this paper we discussed a laboratory experiment, based on the technique of the resonant torsion balance, to test the existence of a short range gravitational anomaly with interaction range bigger than $\approx$ 10 cm. This technique is well known and has been used until now only in non-laboratory experiments, to set very stringent limits on the equivalence priciple, for interaction ranges of the order of the earth-sun distance.

The importance of the role of laboratory experiments in the test of the existence of a composition dependent force is out of question as it has been stressed at this conference(12): all the (positive) evidences and most of the negative results until now come from non-laboratory experiments and it appears troublesome to avoid inconsistencies between the different limits obtained.

We recalled the huge sensitivity of the resonant torsion balance, and we have presented the design of a relatively simple but very precise experiment that is under the complete control of the experimenter. We have shown that in a laboratory search for short range forces it is convenient to use small masses making a periodic motion and placed very near to the resonant torsion balance. In this conditions the sensitivity of the balance is such we could place very stringent limits on the strength of a composition dependent force for all the interaction ranges bigger than the distance between the balance and the source.

We discussed the influence of the construction tolerances and of the field gradients on the experiment sensitivity: we have shown that it is quite easy to keep both aspects under control by choosing the right geometry for the balance plate and the liquid tanks, and the right measurement strategy.

Most of the above arguments apply to the more general problem of the search for weak (periodic) forces: the technique presented here could be applied and developed for this kind of experiment, making also use of the huge reservoir of sensitivity avaliable when using resonant torsion balances but still unused.

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	Braginskii & Panov	Proposed experiment	Units
a_{min}	$2\ 10^{-13}$	$2\ 10^{-13}$	cm/sec^2
Δ	$9\ 10^{-13}$	$4\ 10^{-7}$	
g	$6\ 10^{-1}$	$1\ 10^{-5}$	cm/sec^2
$g\Delta$	$5\ 10^{-13}$	$4\ 10^{-12}$	cm/sec^2
balance :			
r	10	5	cm
L	280	100	cm
m_{TOT}	4	46	g
∅ vire	5	17	μm
ω_0	350	57.2	min
P	$1\ 10^{-8}$	$1\ 10^{-7}$	torr
$\Delta\phi_{min}$	$2\ 10^{-7}$	$2\ 10^{-7}$	rad

TABLE 1

Comparison between the parameters of the Braginskii&Panov experiment and of the proposed experiment

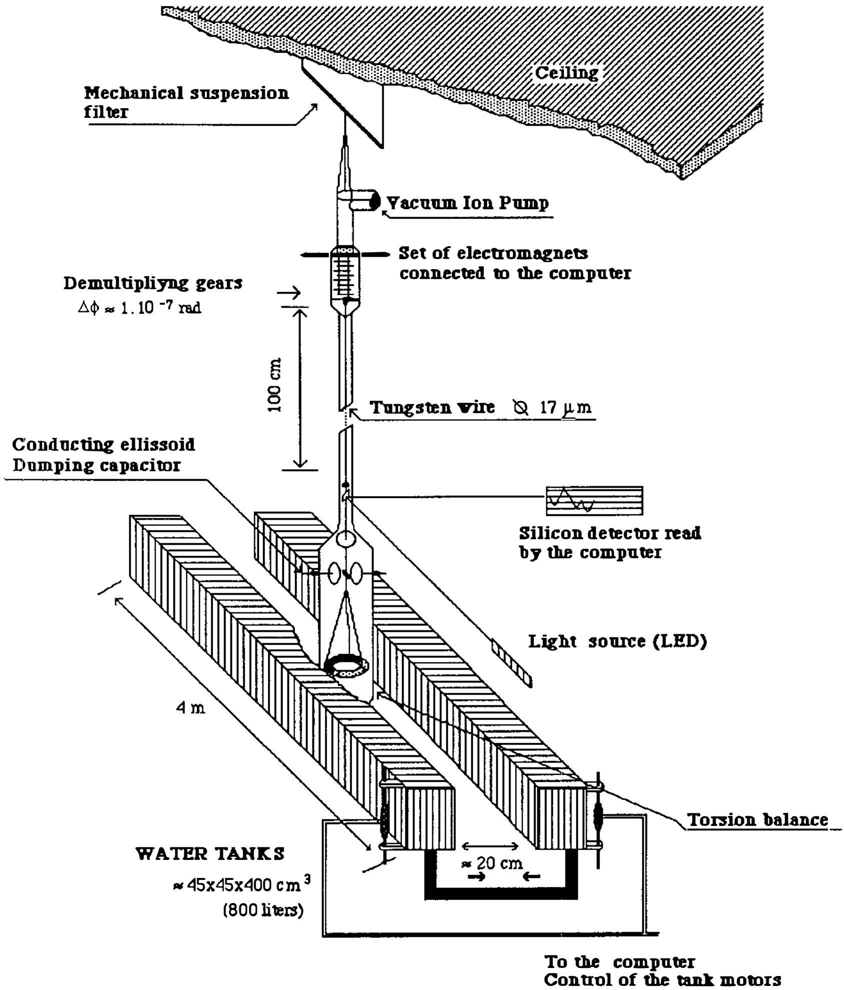


FIG. 1

SCHEMATIC VIEW OF THE DETECTOR

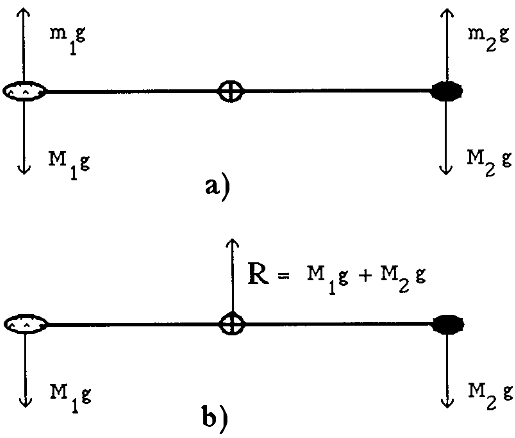


Fig. 4

Forces acting on a torsion balance (seen from above)
a) free falling;
b) constrained.

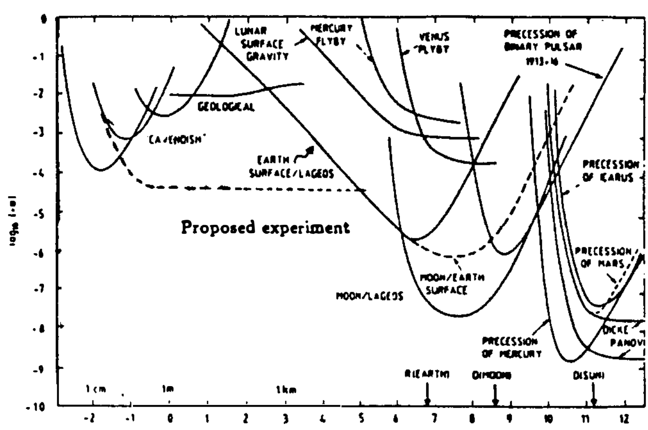


Fig. 5

The sensitivity of the proposed experiment in the plot α vs λ (dashed line) . The data from the various fifth force experiments are not reported.

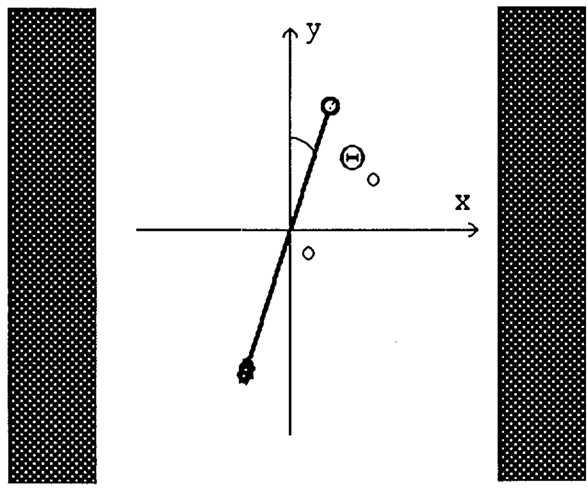


Fig. 6

Torsion balance oscillating with amplitude Θ_0 around the equilibrium point $\theta = 0$

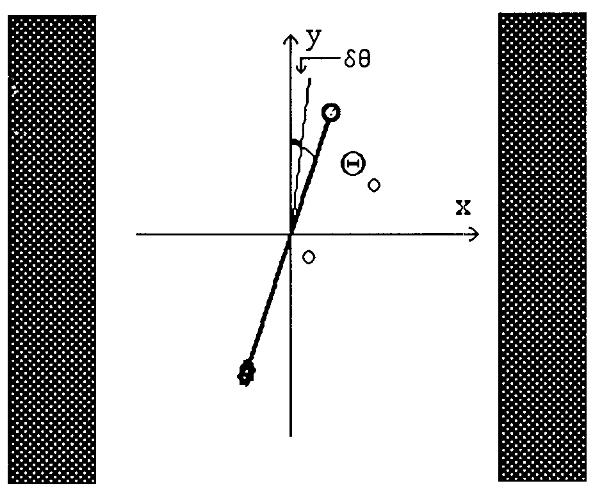
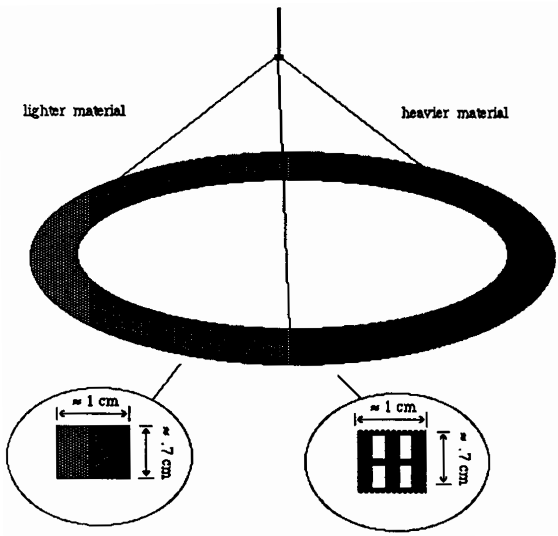


Fig. 7

Torsion balance oscillating with amplitude Θ_0 around the equilibrium point $\theta = \delta\theta$



Enlarged sections of the two plate halves
The heavier material is shaped properly and the two surfaces are covered in order to show the same surface to the residual gas in the vacuum chamber.

Fig. 2
The balance plate

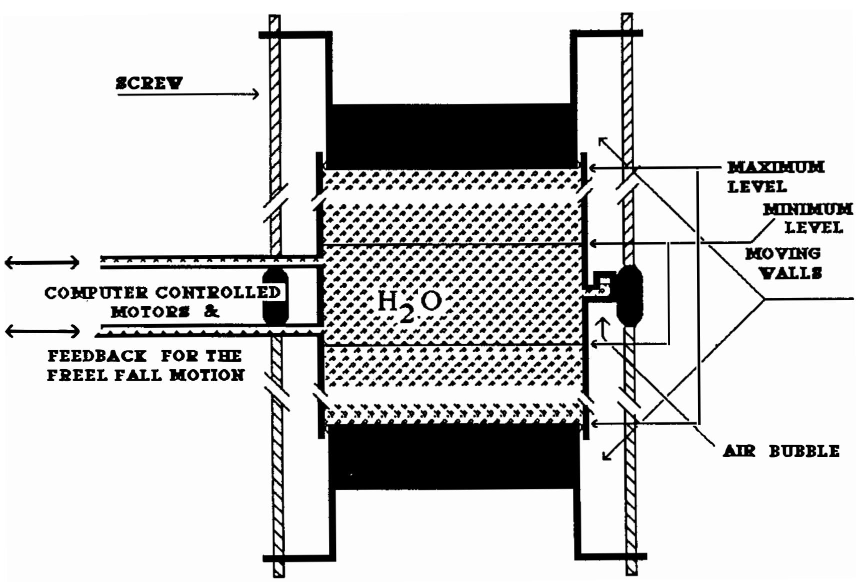


Fig. 3
Layout of a water tank