Armenian Journal of Physics, 2016, vol. 9, issue 1, pp. 54-59

Cluster Decay Half-Lives of $^{156\text{-}162}$Hf Isotopes Using the Woods-Saxon Potential Model

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Received 12 January 2016

Abstract. Decay half-lives are evaluated in the framework of a Woods-Saxon potential model for various clusters decay. In this model cluster-decay are considered as penetration of the cluster-particle through the potential barrier formed by the nuclear, Coulomb and centrifugal interactions between the cluster - particle and core. The spins and parities of the parent and core nuclei, as well as the quadruple deformations of the core nuclei are taken into account for evaluation of the cluster-decay half-lives. The cluster -decay half-lives for isotopes nuclei$^{156\text{-}162}$Hf coincide well with the experimental data.

Keywords: Alpha decay**,** half-life, decay width, Woods-Saxon potential

Introduction

α- Decay is a very important process in nuclear physics [1]. Experimental information on α- decay half-lives are extensive and are being continually updated. We made the first prediction of the nuclear lifetimes against spontaneous cluster emission by using analytical super asymmetric fission model (ASAFM), prior to the publication of any other model [2].

Cluster radioactivity is the spontaneous emission of particle heavier than alpha particle predicted by Sandulescu et al [3] in 1980, after four years Rose and Jones [4] confirmed that phenomenon in the emission of ^{14}C from ^{223}Ra isotope. After the observation of cluster radioactivity, lots of efforts have been done on both experimental and theoretical fronts for understanding the physics of cluster radioactivity.

Decay Half-Life

The decay half-life of an element is defined as the time taken to half the number of radioactive nuclei in a given sample of the element. It is given by:

$$T_{\frac{1}{2}} = \frac{Ln2}{\lambda} \tag{1}$$

For the break-up of a nucleus into a core and cluster the decay constant λ is defined as the product of the assault frequency *f*, the penetrability *T* of the cluster through the potential barrier and the core-cluster preformation probability *P* in the parent nucleus, that is

$$\lambda = fTP \tag{2}$$

The frequency f is given by [5];

$$f = \frac{\hbar}{2\mu} \frac{1}{\int_{r_1}^{r_2} \frac{dr}{k(r)}} \tag{3}$$

$$f = \frac{\hbar}{4\mu} N \tag{4}$$

Where the wave number *k(r)* is

$$k(r)=\sqrt{\frac{2\mu}{\hbar^2}(Q-V_{Eff}(r))} \tag{5}$$

and the normalization factor *N* is given by [6];

$$N=\frac{2}{\int_{r_1}^{r_2}\frac{dr}{k(r)}} \tag{6}$$

The penetrability *T* is defined as the ratio of the transmitted flux to the incident flux densities. The one dimensional *WKB* connection formulae at the turning points of a reasonably wide potential barrier generalized to three dimensions can further be used to show that the transmission probability reduces to [7];

$$T=\exp(-2\int_{r_1}^{r_2}k(r)dr) \tag{7}$$

Putting Eqns. (7) and (4) in Eqn.(2) gives;

$$\lambda=P\frac{N\hbar}{4\mu}\exp(-2\int_{r_2}^{r_3}k(r)dr) \tag{8}$$

And hence the decay width *Γ*:

$$\Gamma=\hbar\lambda \tag{9}$$

Once all the input parameters are known or optimized as desired, the decay half-life may be obtained from:

$$T_{\frac{1}{2}}=\hbar\frac{Ln2}{\Gamma} \tag{10}$$

Given the energy *Q-value* and the potential $V_{Eff}(r)$ governing the decay, the only quantity that remains to be defined for a complete description of the decay half-life is the probability *P* of having a preformed cluster-core system in the initial state. At its simplest the cluster model assumes that the states of a given band are described by the relative motion of a core and cluster in their respective ground states, so that the probability *P=1*, and this assumption is tested by comparing model derived quantities with their observed values.

Results and Discussion

The drastically expanded use of the Woods-Saxon potential in modern day nuclear physics and the availability of new nuclear data, motivated us to review and optimize the half life time, decay width by this potential to the experimental alpha decay isotopes nuclei$^{156\text{-}162}$ Hf. We demonstrate that the potential provides a good description of the nuclear mean field leading to quality particle decay, prediction of decay width and other properties.

The calculations are done by using Coulomb potential, Woods- Saxon potential and centrifugal potential for the touching configuration and for the separated fragments, the core-cluster interaction in equation (5) includes nuclear and coulomb terms. For the nuclear interaction $V_N(r)$, we use the modified Woods-Saxon potential with different parameter values given by [8],

$$V_N(r) = -\left(\frac{A_1 A_2}{A_1 + A_2}\right) V_0 \frac{F(r,R,a)}{F(0,R,a)} \tag{11}$$

$$F(r,R,a) = \frac{1}{1+\exp\left(\frac{r-R}{a}\right)} \tag{12}$$

In Eq. (11) $V_0 \cong 62.31\ MeV$, and different values of the parameters a and $R = r_0 A_1^{1/3}$ are used and $r_0 \cong 1.20\ fm$, as shown in Table 1.

Table 1: Parameters of Wood-Saxon Potential

Nucleus	Average parameters of Wood- Saxon Potential		
	V_0 (MeV)	*a(fm)*	*R(fm)*
^{156}Hf	63.95	0.68	6.95
^{158}Hf	62.85	0.66	7.12
^{160}Hf	61.23	0.66	7.20
^{162}Hf	61.21	070	7.14

The Coulomb potential $V_C(r)$ works between a uniformly charged spherical core of radius *R*, and a point cluster, then the core cluster interaction can be completely defined. The values of parameters are free parameter in the calculation.

The *Q* values are computed using the experimental atomic mass for fragments of Audi et al [9]. In present work the half- lives are calculated for zero angular momentum transfers.

We have studied [10-12] the radioactive decay of $^{156\text{-}162}$Hf isotopes and are found that the most probable clusters for the decay process from all the selected nuclei are ^{4}He, ^{8}Be, ^{12}C, ^{16}O, ^{20}Ne, $^{24\text{-}26}$Mg, $^{28\text{-}30}$Si and $^{32\text{-}34}$S.

We have applied the Coulomb and Woods-Saxon potential model to calculate the half - lives for various clusters decay of the selected even-even isotopes of the chosen nucleus.

Table 2: The values of $\log_{10}\left(T_{1/2}\right)$ for different isotopes calculated by the CWSM and CPPM, in comparison with the available experimental data.

Parent	Cluster	Daughter	Q-Value (MeV)	$Log_{10}(T_{1/2})$		
				CPPM[11]	CWSM	Exp. [12]
	^{4}He	^{152}Yb	6.0285	-1.620	-0.8594	-1.640
	^{8}Be	^{148}Er	8.6700	21.31	21.311	
	^{12}C	^{144}Dy	18.703	28.32	28.320	
	^{16}O	^{140}Gd	28.652	36.62	36.622	

Parent	Cluster	Daughter	Q-Value (MeV)	$Log_{10}(T_{1/2})$ CPPM[11]	CWSM	Exp. [12]
^{156}Hf	^{20}Ne	^{136}Sm	35.986	46.83	46.833	
	^{24}Mg	^{132}Nd	47.492	52.12	52.119	
	^{26}Mg	^{130}Nd	44.944	56.52	56.522	
	^{28}Si	^{128}Ce	59.159	56.79	56.790	
	^{30}Si	^{126}Ce	57.386	59.99	59.990	
	^{32}S	^{124}Ba	67.238	63.16	63.159	
	^{34}S	^{122}Ba	66.673	65.25	65.247	

Parent	Cluster	Daughter	Q-Value (MeV)	$Log_{10}(T_{1/2})$		
				CPPM[11]	CWSM	Exp. [12]
^{158}Hf	^{4}He	^{154}Yb	5.40470	0.450	0.4499	0.45
	^{8}Be	^{150}Er	10.7871	18.61	18.611	
	^{12}C	^{146}Dy	20.4524	28.33	28.331	
	^{16}O	^{142}Gd	29.5940	38.39	38.392	
	^{20}Ne	^{138}Sm	36.4372	49.67	49.670	
	^{24}Mg	^{134}Nd	47.4775	55.74	55.742	
	^{26}Mg	^{132}Nd	45.5378	59.78	59.780	
	^{28}Si	^{130}Ce	58.8132	60.93	60.932	
	^{30}Si	^{128}Ce	57.8644	63.66	63.663	
	^{32}S	^{126}Ba	66.5829	67.85	67.846	
	^{34}S	^{124}Ba	66.9190	69.41	69.411	

Parent	Cluster	Daughter	Q-Value (MeV)	$Log_{10}(T_{1/2})$		
				CPPM[11]	CWSM	Exp. [12]
^{160}Hf	^{4}He	^{156}Yb	4.90230	1.120	1.1002	1.13
	^{8}Be	^{152}Er	9.62100	20.17	20.170	
	^{12}C	^{148}Dy	21.9219	25.63	25.628	
	^{16}O	^{144}Gd	30.5652	36.24	36.240	
	^{20}Ne	^{140}Sm	36.5666	48.16	48.160	
	^{24}Mg	^{136}Nd	47.2016	54.47	54.471	
	^{26}Mg	^{134}Nd	45.9297	57.86	57.861	
	^{28}Si	^{132}Ce	58.0325	59.92	59.923	
	^{30}Si	^{130}Ce	57.9246	61.98	61.979	
	^{32}S	^{128}Ba	65.4632	66.94	66.941	
	^{34}S	^{126}Ba	66.6703	67.85	67.850	

Parent	Cluster	Daughter	Q-Value (MeV)	$Log_{10}(T_{1/2})$		
				CPPM[11]	CWSM	Exp. [12]
^{162}Hf	^{4}He	^{158}Yb	4.41620	1.580	1.6000	1.59
	^{8}Be	^{154}Er	8.49420	21.45	21.450	
	^{12}C	^{150}Dy	20.1404	26.14	26.140	
	^{16}O	^{146}Gd	31.6535	33.34	33.338	
	^{20}Ne	^{142}Sm	36.8595	45.61	45.609	
	^{24}Mg	^{138}Nd	46.7828	52.34	52.340	
	^{26}Mg	^{136}Nd	46.2448	55.00	54.999	
	^{28}Si	^{134}Ce	57.1566	57.90	57.901	
	^{30}Si	^{132}Ce	57.7349	59.40	59.400	
	^{32}S	^{130}Ba	64.1082	65.03	65.031	
	^{34}S	^{128}Ba	66.6703	64.95	64.950	

Table 2 represents the comparison of computed logarithmic half-life time for suitable cluster emissions from $^{156\text{-}162}$Hf parents for the ground state. We have found that these parents are stable against light clusters (except alpha particle) and instable against heavy cluster emissions. For e.g. in the case of ^{16}O, ^{20}Ne, ^{24}Mg, ^{28}Si and ^{32}S emission from ^{156}Hf, $T_{1/2} = 4.19\times10^{+36}, 6.81\times10^{+46}, 1.32\times10^{+52}, 6.17\times10^{+56}\, and\, 1.44\times10^{+63}\, s$ respectively, which are above the present experimental limit for measurements $T_{1/2} \le 10^{+30}\, s$.

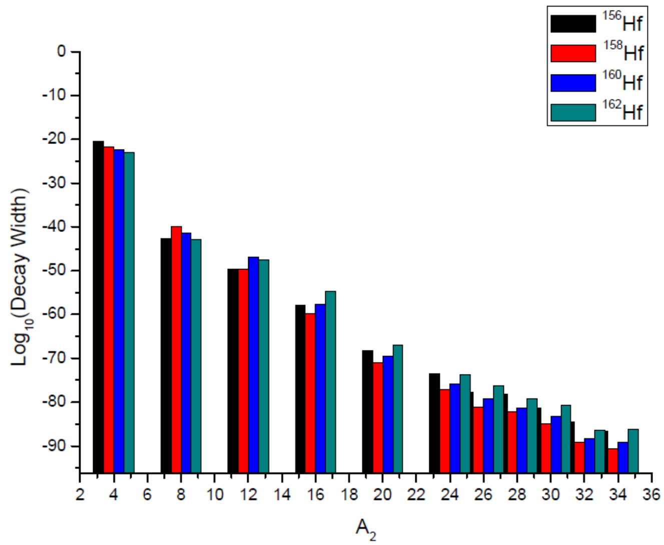

Fig. 1: The values of $log_{10}(\Gamma)$ versus the mass number (A_2) of the clusters are ^{4}He, ^{8}Be, ^{12}C, ^{16}O, ^{20}Ne, $^{24\text{-}26}$Mg, $^{28\text{-}30}$Si and $^{32\text{-}34}$S for $^{156\text{-}162}$Hf isotopes.

The calculated values of $log_{10}(\Gamma)$ using the CWSM are plotted as a function of the mass number of the clusters for all the parents presented in Fig. 1, where the value of $log_{10}(\Gamma)$ is plotted against the mass number of the clusters with different isotopes. As the atomic number is fixed at $Z=72$ for $^{156\text{-}162}$Hf, from this figure, it is clear that decay width of alpha cluster for the chosen isotopes are decrease with increase A_2, while decay width of ^{16}O, ^{20}Ne, $^{24\text{-}26}$Mg, $^{28\text{-}30}$Si and $^{32\text{-}34}$S clusters are volatile, and decay width of ^{8}Be, ^{12}C are less volatility. We conclude that this deviation is due to the difference in the mass numbers of the clusters. Also it is obvious that the upper column is for the ^{4}He cluster, while the lower one is for ^{34}S. Therefore, $log_{10}(\Gamma)$ decrease with increasing mass number of the clusters. This means that the width decay of the decay process from the parent nuclei is shorter as the mass number of the clusters is larger.

Within the Coulomb and Woods-Saxon potential model (CWSM) effects of barrier penetrability in cluster decay half- life are studied, using one dimensional WKB approximation, the barrier penetrability T is given as equation (7), here the reduces mass $\mu = m\frac{A_1A_2}{A_1+A_2}$, where m is nucleon mass and A_1,A_2 are the mass number of core and emitted cluster respectively. The turning points r_2,r_3 are determined from the equation $V_{Eff}(r_2) = V_{Eff}(r_3) = Q$. The integral can be evaluated numerically or analytically. In the present work, numerical method has been adopted for calculating the penetrability.

It is to be noted that half-lives decrease due to the reduction of the height and width of the barrier (increases the barrier penetrability, this enables us to use our model for the half- life calculations for clusters with mass $A_2 < 36$, including alpha particle.

Conclusion

Stability of $^{156\text{-}162}$Hf nuclei against alpha and cluster emission is studied within the Coulomb and Woods- Saxon potential model (CWSM). It is found that these nuclei are stable against light clusters (except alpha particle) and instable against heavy cluster $A_2 < 36$, emissions. For heavy cluster emissions the core nuclei lead to doubly magic cores or neighboring one. The effect of parameters of potential on half -lives is also studied. The computed cluster decays half-life values are in close agreement with experimental data. Inclusion width of the barrier increases the barrier penetrability, and hence the half- life decreases.

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