Pro gradu -tutkielma
Teoreettisen fysiikan koulutusohjelma

Charged lepton flavour violation

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2015

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Matemaattis-luonnontieteellinen tiedekunta	Fysiikan laitos

Tekijä/Författare – Author
Niko koivunen

Työn nimi / Arbetets titel – Title
Varattujen leptonien maun rikkoutuminen

Oppiaine /Läroämne – Subject
Teoreettinen fysiikka

Työn laji/Arbetets art – Level	Aika/Datum – Month and year	Sivumäärä/ Sidoantal – Number of pages
Pro gradu -tutkielma	Maaliskuu 2015	110

Tiivistelmä/Referat – Abstract

Varatuilla leptoneilla, joita ovat elektroni, myoni ja tau, ei ole koskaan havaittu makua rikkovia reaktioita. Niiden olemassaolo on kuitenkin odotettavissa, sillä maku rikkoutuu kaikilla muilla hiukkasfysiikan standardimallin (SM) fermioneilla. Standardimalli pitää neutriinoja massattomina, mikä kieltää neutriinojen maun sekoittumisen ja siten myös varattujen leptonien maun rikkoutumisen. Tämä on kuitenkin ristiriidassa havaittujen neutriino-oskillaatioiden kanssa. Standardimallia täytyy siis laajentaa sisältämään neutriinojen massat.

Helpoin tapa selittää neutriinojen massa on olettaa, että se syntyy samalla tavalla kuin muidenkin standardimallin fermionien massat: Higgsin mekanismissa. Tämä tapa kuitenkin johtaa ongelmiin neutriinon Yukawa-kytkennän luonnollisuuden kanssa. Eräs suosituimmista tavoista generoida neutriinojen massat luonnollisesti on niin sanottu seesaw-mekanismi (tyyppi-I). Standardimalli, jota on laajennettu neutriinojen massoilla, mahdollistaa varattujen leptonien maun rikkoutumisen. Tämä johtaa kuitenkin havaitsemattoman pieniin todennäköisyyksiin varattujen leptonien makua rikkoville reaktioille. Havainto varattujen leptonien makua rikkovasta reaktiosta olisi selvä todiste standardimallin ja sen triviaalien laajennuksien ulkopuolisesta fysiikasta. Jotta olisi toivoa varattujen leptonien makua rikkovien reaktioiden havaitsemisesta, on oltava olemassa standardimallin laajennus, joka tuottaa havaittavissa olevat, vaikkakin pienet, todennäköisyydet varattujen leptonien makua rikkoville reaktioille.

Eräs suosituimmista standardimallin laajennuksista on niin sanottu minimaalinen supersymmetrinen standardimalli (MSSM). MSSM:ssa neutriinot ovat massattomia, niin kuin standardimallissakin ja näin ollen varattujen leptonien makua rikkovat reaktiot ovat kiellettyjä. Onneksi neutriinojen massat voidaan generoida myös MSSM:ssa seesaw-mekanismilla. MSSM sisältää useampia lähteitä varattujen leptonien maun rikkoutumiselle kuin SM. Ylimääräisiä lähteitä standardimalliin nähden ovat leptonien skalaaripartnerien, sleptonien, pehmeät massatermit.

Supersymmetrisissä malleissa sleptonit kytkeytyvät leptoneihin sleptoni-leptoni-gaugino-verteksien avulla. Nämä aiheuttavat säteilykorjauksissa varattujen leptonien makua rikkovat reaktiot. Yleensä epädiagonaaliset pehmeät massatermit oletetaan MSSM:ssa nolliksi sillä korkealla energiaskaalalla, jossa supersymmetria rikkoutuu. Ihmiset tekevät mittauksia kuitenkin huomattavasti pienemmällä sähköheikolla energiaskaalalla. Pehmeät massaparametrit saavat siis huomattavia radiatiivisia korjauksia, kun ne juoksutetaan supersymmetrian rikkoutumisskaalalta sähköheikolle skaalalle. Tässä seesaw-mekanismi (tyyppi-I) astuu mukaan. Seesaw-mekanismi tuo mukanaan neutriinon Yukawa-kytkentämatriisin, joka sisältää epädiagonaalisia alkioita. Nämä alkiot mahdollistavat sen, että sleptonien pehmeiden massatermien epädiagonaaliset termit kehittyvät nollasta poikkeaviksi, kun niitä ajetaan renormalisaatioryhmällä sähköheikolle skaalalle.

Tässä tutkielmassa käsitellään ensin varattujen leptonien maun rikkoutumista standardimallissa. Sitten varattujen leptonien makua rikkovia reaktioita, $l_i \to l_j\gamma$, $l_i \to l_j l_k l_l$ ja $l_i \leftrightarrow l_j$, tutkitaan yleisimmällä mahdollisella tavalla efektiivisissä teorioissa. Lopuksi varattujen leptonien maun rikkoutumista käsitellään supersymmetrisissä teorioissa yleisesti ja tarkemmin seesaw-mekanismilla laajennetussa minimaalisessa supersymmetrisessä standardimallissa.

Avainsanat – Nyckelord – Keywords
leptoni, supersymmetria, seesaw-mekanismi, standardimalli

Säilytyspaikka – Förvaringställe – Where deposited
Kumpulan tiedekirjasto

Muita tietoja – Övriga uppgifter – Additional information

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HELSINGFORS UNIVERSITET
UNIVERSITY OF HELSINKI

Tiedekunta/Osasto Fakultet/Sektion – Faculty	Laitos/Institution– Department
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Tekijä/Författare – Author
Niko koivunen

Työn nimi / Arbetets titel – Title
Charged lepton flavour violation

Oppiaine /Läroämne – Subject
Theoretical Physics

Työn laji/Arbetets art – Level	Aika/Datum – Month and year	Sivumäärä/ Sidoantal – Number of pages
Master's thesis	March 2015	110

Tiivistelmä/Referat – Abstract

Flavour violating processes have never been observed for charged leptons, electron, muon and tau. The existence of charged lepton flavour violating (CLFV) processes is however expected, since flavour is violated by all the other fermions of the standard model (SM). In the standard model the neutrinos are massless, which forbids the mixing of neutrino flavour and also the violation of lepton flavour. The zero mass of the neutrinos in the SM is in conflict with the experimentally observed neutrino oscillations. The standard model has to be extended to include massive neutrinos.

The easiest way to explain the neutrino mass is to assume that they acquire masses in the same way as the rest of the SM fermions: in the Higgs mechanism. This way however leads to problems with the naturality of the neutrino Yukawa coupling. One of the most popular methods of generating the neutrino mass is the so called seesaw mechanism (type-I). The standard model, extended with the neutrino masses, allows the charged lepton flavour to be violated. This leads to unobservably small transition rates however. Therefore an observation of charged lepton flavour violating process would be a clear evidence of the existence of new physics beyond the standard model and it's trivial extensions. To have hope of ever observing charged lepton flavour violating processes, there must be an extension of the standard model which produces observable, though small, rates for CLFV processes.

One of the most popular extensions of the standard model is the so called minimal supersymmetric standard model (MSSM). The neutrinos are massless in the MSSM, as they are in SM, and therefore CLFV processes are forbidden in the MSSM. Luckily the neutrino masses can be generated via seesaw mechanism in the MSSM as well as in the SM. The MSSM contains more potential sources for CLFV processes than the SM. The extra sources are the soft mass parameters of the sleptons. In supersymmetric models the sleptons couple to the leptons through the slepton-lepton-gaugino-vertices. These generate the CLFV processes at the loop-level.

Often the off-diagonal soft terms are assumed zero in the MSSM at the input scale, where the supersymmetry breaks. Experiments are done at much lower electroweak scale. The soft SUSY-breaking terms acquire large radiative corrections as they are run from the input scale down to the electroweak scale. Here the seesaw mechanism kicks in. The seesaw mechanism brings with it the off-diagonal neutrino Yukawa coupling matrices. This allows the off-diagonal slepton mass terms to evolve non-zero at the electroweak scale.

In this thesis the charged lepton flavour violation is discussed first in the context of the standard model. Then the CLFV processes, $l_i \to l_j \gamma$, $l_i \to l_j l_k l_l$ ja $l_i \leftrightarrow l_j$, are studied in the most general way: in the effective theories. Finally the charged lepton flavour violation is studied in the supersymmetric theories in general and more specifically in the minimal supersymmetric standard model extended with the seesaw mechanism (type-I).

Avainsanat – Nyckelord – Keywords
lepton, supersymmetry, seesaw mechanism, standard model

Säilytyspaikka – Förvaringställe – Where deposited
Kumpula campus library

Muita tietoja – Övriga uppgifter – Additional information

Acknowledgments

I would like to thank my advisor, prof. Katri Huitu for her invaluable advice and useful comments. Especially I want thank Paula who has always supported me.

Contents

1 Introduction

What is *flavour*? It is known that the fermions in the Standard Model appear in three generations. Each generation differs from the others only by the masses of the particles. The particle of a certain generation is said to have a certain flavour. For example the charged leptons in the standard model are electron, muon and tau. These are the flavours of the charged leptons. The particle with certain flavour has a well defined mass. So the flavour is just a quantity that states the particle in question. For each flavour a flavour number can be defined. It counts the number of certain flavour in the system, so that the particle gives a positive contribution and the antiparticle negative. In a flavour violating process that number is violated, i.e. the initial state has different number of a particular flavour than the final state.

Table 1: Flavour quantum numbers of leptons

Particle	Flavour
e^- (e^+)	$+1_e$ (-1_e)
ν_e $(\bar{\nu}_e)$	$+1_e$ (-1_e)
μ^- (μ^+)	$+1_\mu$ (-1_μ)
ν_μ $(\bar{\nu}_\mu)$	$+1_\mu$ (-1_μ)
τ^- (τ^+)	$+1_\tau$ (-1_τ)
ν_τ $(\bar{\nu}_\tau)$	$+1_\tau$ (-1_τ)

Search for charged lepton flavour violation began at early 1940's after the muon was identified as a separate particle [108]. Despite numerous experimental searches, the charged lepton flavour violating processes have never been observed. It is well known that the quark flavour is violated. Also the neutrino oscillations clearly show that also the neutrino flavour is not conserved. Only in the charged leptons, electron e, muon μ, and tau τ, there has been no experimental evidence of flavour violation. Since all other fermions in the standard model of particle physics (SM) have flavour violation, it would be natural that also the charged leptons could mix.

It is known that if the neutrinos were massive and they mixed, the mixing would be transferred to charged leptons as well due to the fact that the neutral and the charged leptons share a coupling (ν_l-e-W). The neutrinos are definitely massive according to the experiment and they mix, so the mixing should exist on the charged leptons as well. The SM, however, considers the neutrinos to be absolutely massless. This clearly contradicts the experiments. The SM has therefore to be extended to include massive neutrinos. The question is how are they introduced? The easiest way is to assume that the neutrino masses are generated the same way as all the other SM fermion masses are: in the Higgs mechanism. This would mean the addition of neutrino Yukawa couplings in to the theory. This method is simple, but it has a serious problem: fine-tuning. The neutrino mass is experimentally known to be smaller than approximately

Table 2: The current experimental bounds of CLFV processes.

Observable	Upper limit
$BR(\mu \to e\gamma)$	5.7×10^{-13} [91]
$BR(\tau \to e\gamma)$	3.3×10^{-8} [92]
$BR(\tau \to \mu\gamma)$	4.4×10^{-8} [92]
$BR(\mu \to eee)$	1.0×10^{-12} [93]
$BR(\tau \to eee)$	3.0×10^{-8} [92]
$BR(\tau \to \mu\mu\mu)$	2.0×10^{-8} [92]
$BR(\tau^- \to \mu^- e^+ e^-)$	1.7×10^{-8} [94]
$BR(\tau^- \to e^- \mu^+ \mu^-)$	2.7×10^{-8} [94]
$BR(\tau^- \to e^+ \mu^- \mu^-)$	1.7×10^{-8} [94]
$BR(\tau^- \to \mu^+ e^- e^-)$	1.5×10^{-8} [94]
$CR(\mu\text{-}e, Ti)$	4.3×10^{-12} [95]
$CR(\mu\text{-}e, Au)$	7.0×10^{-13} [96]

1eV. This forces the neutrino Yukawa coupling to be about 10^{-12}, which clearly is a very precise number. The SM extended with massive Dirac neutrinos, will also predict unobservably small rates for charged lepton flavour violating (CLFV) processes, for example $BR(\mu \to e\gamma) < 10^{-54}$. The small rates are in principle not a problem, they are in an agreement with the experiments, but the need of fine-tuning is considered a serious problem. Some other model for neutrino mass generation is therefore needed.

There is a popular method to generate neutrino masses to SM, which avoids the neutrino Yukawa fine-tuning problem. This model is called the *seesaw mechanism* (type-I). Seesaw-I mechanism one assumes the neutrino to have a Dirac mass and also a right-handed Majorana mass. This allows the left-handed neutrino mass to be tiny ($\sim$eV) and the right-handed mass to be very large (order of the GUT scale $\sim 10^{16}$Gev). This also allows the neutrino Yukawa coupling to be natural (~ 1). Despite its merits, the SM extended with the seesaw-I, still predicts unobservably small ratios for CLFV processes, which is depressing since we would like to find proof of their existence. The observation of charged lepton flavour violating processes would be a clear sign of the existense of the physics beyond the Standard Model (BSM) or its simpler extensions (i.e. just the addition of massive neutrinos).

We have only considered SM extensions that add massive neutrinos and nothing else. They fail to produce observable CLFV rates. The SM can of course be extended more radically. One of the best motivated SM extensions are supersymmetric models. Supersymmetry assumes that for every particle in the theory there is a partner with otherwise the same properties, but with different statistics. So for electron there should be a particle with the same electric charge and mass, but with spin-0. These kind of superpartners have never been observed so the supersymmetry, if it exists, must be a broken symmetry.

The simplest supersymmetric extension of the SM is the so called *Minimal Supersymmetric Standard Model* (MSSM). The MSSM assumes for every SM particle there is a superpartner and it also assumes a second Higgs doublet to avoid anomalies. In the MSSM the supersymmetry breaking mechanism is not spelled out and the breaking is parametrized by the so called *soft supersymmetry-breaking terms*. The soft terms offer new possible sources of flavour violation. These include off-diagonal sfermion mass terms and off-diagonal terms in the trilinear scalar couplings. Since there is no supersymmetry breaking method spesified in the MSSM, the flavour violating off-diagonal terms can be as big as the diagonal ones. This would lead to huge rates for flavour violating processes, which is impossible, since the flavour violating processes are extremely suppressed. The off-diagonal have therefore to be zero or very close to it. It is usually assumed that the off-diagonal soft terms are zero. This means that in the MSSM there can be no charged lepton flavour violation, since also the neutrinos are massless in it just as in the SM. The quark flavour is violated in MSSM as in the SM, since it is due to Yukawa couplings only.

The MSSM has also to be extended to include massive neutrinos. This can also be done by seesaw-I mechanism, as in the SM. Both the SM extended with the seesaw-I and the MSSM produce unobservably small rates of charged lepton flavour violating processes. But when one combines the MSSM and the seesaw-I, interesting things happen. In MSSM the off-diagonal soft terms are (usually) assumed to be zero, but that holds only at the input scale, i.e. the scale where the supersymmetry breaks. The SUSY-breaking scale is assumed to be very high and the quantum corrections can drastically change the values of the soft parameters as they are run from the input scale down to the electroweak scale, where the experiments are done. In MSSM the renormalization group running does not make the off-diagonal terms non-zero at the electroweak scale, but when MSSM is extended with the seesaw-I, the off-diagonal soft terms can deviate from zero at the electroweak scale due to the non-diagonal terms in the neutrino Yukawa coupling. The soft terms generated in this way, produce such rates for CLVF that are observable in the near future.

1.1 Neutrino oscillations

The neutrinos are spin-1/2 particles and have no electric charge. They interact only via the weak interaction. The neutrinos are therefore *leptons*. There are also electrically charged leptons in the Standard Model, called electron, e; muon, μ and tau, τ; of which the electron is the lightest and the tau the heaviest. There are also three neutrinos in the Standard Model, called electron neutrino, ν_e; muon neutrino, ν_μ, and tau neutrino, ν_τ. The neutrinos are named like that, because in the SM when the W^-- boson decays into a charged lepton, it is always accompanied by an antineutrino of same *flavour*:

$$W^- \to l_i + \bar{\nu}_i, \qquad \text{where} \qquad i = e, \mu, \tau. \tag{1}$$

In the Standard Model of particle physics the neutrinos are assumed to be massless. They are also assumed to be Dirac-particles, which means that they have distinct antiparticles. The zero mass forces the neutrino flavour to be absolutely conserved, since the flavour mixing is due to the non-zero off-diagonal terms in the mass matrix. So according to the Standard Model, there can be no neutrino mixing.

However, it is experimentally known that different neutrino flavours can mix to each other [74]. In neutrino mixing a neutrino that started out with a definite flavour (ν_e, ν_μ or ν_τ), can be found with a different flavour after some propagation. This is possible, because the neutrinos taking part in the interactions are not the mass eigenstates, i.e. the states with a definite flavour, but linear combinations of them.

Neutrino mixing has undeniably been verified in "disappearence" experiments with athmospheric and accelerator neutrinos ($\nu_\mu \nrightarrow \nu_\mu$ and $\bar{\nu}_\mu \nrightarrow \bar{\nu}_\mu$), solar neutrinos ($\nu_e \nrightarrow \nu_e$) and reactor neutrinos ($\bar{\nu}_e \nrightarrow \bar{\nu}_e$) and in "appearence" experiments with solar neutrinos ($\nu \to \nu_{\mu,\tau}$)[5]. This kind of mixing is only possible when neutrinos are massive and their masses are different. Since the discovery of the Higgs boson in the LHC [6], the origin of mass of the charged fermions appears to be found. However, the origin of the neutrino masses remains unknown. There are many suggestions concerning the generation of neutrino mass. The structure of the neutrino mixing matrix, Pontecorvo-Maki-Nakagawa-Sakata matrix (PMNS matrix), differs greatly from that of Cabibbo-Kobayashi-Maskawa matrix which describes the mixing of quarks. This suggests that maybe the neutrino masses are not generated in the same way as the masses of quarks and charged leptons, that is when the Higgs field acquires the vacuum expectation value (VEV). One of the suggestions of the neutrino mass generation is the *seesaw mechanism* (there are many of them). It is not even known whether the neutrinos are Majorana or Dirac fermions (Majorana particles are particles that are their own antiparticles).

Anyway, the neutrino oscillations clearly show that the neutrinos have non-zero masses, and therefore that the Standard Model is not the complete description of nature. The SM has to be extended to include the massive neutrinos. The method of their introduction affects the magnitude of charged lepton flavour violation.

In this thesis I will study lepton flavour violation in charged lepton sector. The outlook of this thesis is as follows. In Chapter 2 I will review flavour and flavour violation in the context of the Standard Model. In the Chapters 3 and 4 I will study charged lepton flavour violation in the most general way, using effective theory approach. The Chapter 5 is reserved for the general dicussion about charged lepton flavour violation in supersymmetry in general and specifically in MSSM extended with seesaw-I. In the Chapter 6 I will discuss the following CLFV processes, $l \to l'l''l''$, $l \to l'\gamma$ and e-μ conversion, in the MSSM extended with the seesaw-I. Finally in Chapter 7 I will priefly discuss about the search for the CLFV that is conducted in the LHC.

2 Standard Model and Lepton Flavour

Let us review how the lepton and quark flavours are treated in the Standard Model of particle physics. In this thesis we are of course mainly interested in the flavour violation in the charged leptons, but it is instructive to review also the mixing of quarks, since their mixing is understood a lot better than that of leptons. We will need only a part of the Standard Model in our discussion, and we will only consider topics we need.

The Standard Model gauge group is $SU(3)_C \times SU(2)_L \times U(1)_Y$, where $SU(3)_C$ is the gauge group describing the strong interaction and the $SU(2)_L \times U(1)$ is the gauge group describing the electroweak interaction. At low energies where humans live, the $SU(2)_L \times U(1)$ has been spontaneously broken due to the Higgs field VEV at $\sim$ 246 GeV, resulting into separate weak and electromagnetic interactions. The strong, weak and the electromagnetic interactions are so called gauge interactions, i.e. they are mediated by spin-1 gauge bosons. There is still one interaction remaining in the Standard Model, but it is not a gauge interaction. The remaining interaction is the Higgs-fermion Yukawa interaction. It applies in the energies higher than the electroweak scale, where the Higgs field has not acquired vacuum expectation value.

According to the SM, the neutrinos know only the weak interaction. The charged leptons know the weak interaction as well, and also the electromagnetic interaction due to their electric charge. The quarks know all the interactions: weak, strong and the electromagnetic.

Since only the quarks know the strong interaction and that they don't contribute to the flavour changing processes, we do not discuss it in the rest of this thesis. We are particularly interested in the weak interaction and the Yukawa interaction, since they combined are responsible for the flavour violation in the Standard Model. We will next review the Weinberg-Salam model that treats the topics relevant for us: electroweak and Yukawa interactions.

2.1 Weinberg-Salam model

In the Weinberg-Salam model (named after Nobel laureates Steven Weinberg and Abdus Salam) the weak and electromagnetic interaction are unified under one gauge group, $SU(2)_L \times U(1)_Y$. This is the gauge symmetry exact at the energies larger than the electroweak scale, $\sim$ 100 GeV. At the energies that high all the fermions and gauge bosons are massless. When $SU(2)_L \times U(1)_Y$ gauge symmetry breaks spontaneously, charged leptons, quarks and $W^{\pm}$- and Z^0-bosons acquire masses. The neutrinos and the photon remain massless after the breaking. Let us look into this more closely. First we will describe the general structure of the electroweak interactions and after that we will see how the electroweak $SU(2) \times U(1)$ symmetry is broken and particularly how the fermion masses are created. The fermion mass generation is really important, since it contains the key for understanding the flavour violation.

Before the spontaneous symmetry breaking the Weinberg-Salam Lagrangian divides into three parts:

$$\mathcal{L}_{WS} = \mathcal{L}_G + \mathcal{L}_F + \mathcal{L}_H,$$

where $\mathcal{L}_G$, $\mathcal{L}_F$ and $\mathcal{L}_H$ are gauge boson, fermion and Higgs interaction Lagrangians, respectively. Let us next see what they contain.

2.1.1 Gauge fields

The $\mathcal{L}_G$ describes the kinetic terms of the gauge bosons of the Weinberg-Salam model and also the interactions between the gauge bosons themselves. There are four gauge boson fields $\vec{W}_\mu = (W_\mu^1, W_\mu^2, W_\mu^3)$ and B_μ in this model. The vector fields W_μ^1, W_μ^2 and W_μ^3 correspond to the three generators, τ_1, τ_2 and τ_3[1], of the $SU(2)_L$ gauge group and the vector field B_μ corresponds to the gauge group $U(1)_Y$. These fields are the *gauge eigenstates*. These states are those that appear in the Lagrangian.

The gauge part of the electroweak Lagrangian is

$$\mathcal{L}_G = -\frac{1}{4} F_i^{\mu\nu} F_{\mu\nu}^i - \frac{1}{4} B^{\mu\nu} B_{\mu\nu}, \tag{2}$$

where $F_{\mu\nu}^i$ is the $SU(2)_L$ field strength tensor

$$F_{\mu\nu}^i = \partial_\mu W_\nu^i - \partial_\nu W_\mu^i - g_2 \epsilon^{ijk} W_\mu^j W_\nu^k, \qquad i = 1, 2, 3,$$

g_2 the $SU(2)_L$ gauge coupling and $B_{\mu\nu}$ is the $U(1)_Y$ field strength tensor

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu.$$

When the $SU(2)_L \times U(1)_Y$ gauge symmetry spontaneously breaks, the gauge bosons W_μ^1, W_μ^2, W_μ^3 and B_μ acquire masses. Before the symmetry breaking the gauge bosons are massless (no mass terms in (2)), and the gauge eigenstates W_μ^1, W_μ^2, W_μ^3 and B_μ are also *mass eigenstates*, i.e. states with definite mass. When the gauge fields become massive, the gauge eigenstates no longer are the same thing as the mass eigenstates. After the symmetry breaking one usually wants to use the mass eigenstates instead of gauge eigenstates, since the mass eigenstates are the physical fields that are seen in the experiments. Even though we are going to talk about Higgs mechanism later, we are now going to reveal what the mass eigenstates of the gauge fields are. The later Section concerning Higgs mechanism is reserved for the fermion mass generation, of which we are mainly interested. The physical mass eigenstates are the familiar $W^\pm$-bosons, Z^0-boson, and the photon. These mass eigenstates can be expressed in terms of the gauge eigenstates in the following way:

$$\begin{cases} W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp i W_\mu^2), \\ Z_\mu = cos\theta_W W_\mu^3 - sin\theta_W B_\mu, \\ A_\mu = sin\theta_W W_\mu^3 + cos\theta_W B_\mu. \end{cases} \tag{3}$$

where $W_\mu^\pm$ stands for $W^\pm$-bosons, Z_μ for Z-boson and A_μ stands for the photon. The θ_W is the so called Weinberg angle. In the Higgs mechanism the $W^\pm$- and the Z-bosons acquire masses by absorbing the degrees of freedom from the three Goldstone-bosons. The photon remains massless.

Now that we have dealt with the gauge bosons of the electroweak theory, we are ready to introduce the matter fields: the fermions of the Standard Model.

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[1] The generators of $SU(2)$ group are the Pauli-matrices $\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ and $\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. The Pauli-matrices satisfy the $SU(2)$ algebra: $[\tau_i, \tau_j] = 2i\epsilon_{ijk}\tau_k$.

2.1.2 Fermion fields

The Standard Model fermions can be divided into two categories: quarks and leptons. There are six quarks and six leptons. Three of the quarks has the electric charge $+2/3$ and they are called: up, u; charm, c and top, t. The rest three quarks have the electric charge $-1/3$ and they are called: down, d; strange, s and bottom, b. Three of the leptons have the electric charge -1 and they are called: electron, e; muon, μ and tau, τ. The rest three leptons are electrically neutral and they are called: electron neutrino, ν_e; muon neutrino, ν_μ and tau neutrino, ν_τ. Every fermion of certain electric charge are said to exist in three generations. The fermion fields appearing in the electroweak Lagrangian are in the *gauge basis*. Let us denote gauge eigenstates by prime:

$$\begin{aligned} e'_i &= (e', \mu', \tau') \\ \nu'_i &= (\nu'_e, \nu'_\mu, \nu'_\tau) \\ u'_i &= (u', c', t') \\ d'_i &= (d', s', b'), \end{aligned}$$

where i is the generation index or equivalently *flavour index*, which run from 1 to 3. We have chosen to use prime in the names of the fields, because the gauge eigenstates are in general different from the mass eigenstates. Even though the fermions are massless when the $SU(2)_L \times U(1)_Y$ gauge symmetry is exact, the fermions will become massive in the electroweak symmetry breaking. The unprimed symbols are reserved for more fundamental, physically observable mass eigenstates.

Let us now see how the fermions couple to the electroweak gauge bosons. The electroweak interactions treat fermions of different handedness differently. The $SU(2)_L$ gauge bosons, W^1_μ, W^2_μ and W^3_μ, couple only to the left-handed fermions and the $U(1)_Y$ gauge boson B_μ couples to both left- and right-handed fermions.

Left-handed fermions reside in the weak isospinors and right-handed fermions reside in weak isosinglets. The weak isodoublets are

$$l'_{i,L} = \begin{pmatrix} \nu'_i \\ e'_i \end{pmatrix}_L \qquad \text{and} \qquad q'_{i,L} = \begin{pmatrix} u'_i \\ d'_i \end{pmatrix}_L \tag{4}$$

and weak isosinglets are $e_{i,R}$, $u_{i,R}$ and $d_{i,R}$. Two quantum numbers are now defined for the fermions: the weak isospin third component, I_3, and the weak hypercharge, Y. The upper field, in the weak isodoublets in (4) have the weak isospin third component $1/2$ and the lower ones have $-1/2$. So the left-handed fermions have weak isospin third component either $1/2$ or $-1/2$. The right-handed fermions are defined to have isospin 0. We define the weak hypercharge of a particle as[2]:

$$Y = 2(Q - I_3), \tag{5}$$

where Q is the electric charge of the particle in question. The weak isospins and weak hypercharges of the Standard Model fermions are given in the Table 3. Note that there is no right-handed neutrino in the Standard Model, since the Standard Model treats the neutrino as an absolutely massless particle, even after the electroweak symmetry breaking.

[2]Sometimes the weak hypercharge is defined as $Y = Q - I_3$.

Table 3: Hypercharges and isospins

Particle	I_3	Y
$e_{i,L}$	-1/2	-1
$e_{i,R}$	0	-2
$\nu_{i,L}$	1/2	-1
$u_{i,L}$	1/2	1/3
$u_{i,R}$	0	4/3
$d_{i,L}$	-1/2	1/3
$d_{i,R}$	0	-2/3

The fermion part of the Lagrangian is:

$$\mathcal{L}_F = \sum_{\psi_L} \bar{\psi}_L i \not{D} \psi_L + \sum_{\psi_R} \bar{\psi}_R i \not{D} \psi_R, \tag{6}$$

where the first sum sums over all the left-handed weak isodoublets and the second sum sums over all the right-handed weak isosinglets. The $\not{D}$ represents the covariant derivative. As usual, the covariant derivative contains the possible gauge interactions the fermions have. The right-handed fermions don not couple to weak isospin so their covariant derivative is simpler than that of the left-handed fermions. The covariant derivative of the right-handed fermions is

$$D_\mu \psi_R = (\partial_\mu + i\frac{g_1}{2} Y_W B_\mu)\psi_R, \tag{7}$$

where g_1 is the $U(1)$ coupling constant. As we see from (7), the right-handed fermions couple only to the B_μ-boson.

The covariant derivative of the left-handed fermions is

$$D_\mu \psi_L = \left(\mathbf{I}(\partial_\mu + i\frac{g_1}{2} Y_W B_\mu) + i g_2 \frac{\vec{\tau}}{2} \cdot \vec{W}_\mu \right) \psi_L, \tag{8}$$

where $\mathbf{I}$ and the 2×2 matrices $\vec{\tau}$ are the Pauli matrices. As is evident from (8), the left-handed fermions couple both to the gauge bosons W_μ^1, W_μ^2 and W_μ^3, as well as to the B_μ-boson. Now that we know all the possible electroweak gauge interactions, we are fit to deal with the only non-gauge interaction in the: the interaction with the spin-0 Higgs boson.

2.1.3 The Higgs sector

We have already discussed about the fermions and the vector bosons of the electroweak theory. There is however two complex-scalar fields in the Standard Model, electrically charged and electrically neutral Higgses: ϕ^+ and ϕ^0. The Higgs fields are assigned to the weak isospinor:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}. \tag{9}$$

So the charged Higgs has $I_3 = 1/2$ and the neutral Higgs has $I_3 = -1/2$. Therefore they have the weak hypercharge $Y = 1$.

The Higgs field couples both to the fermions and the gauge bosons. Therefore the Higgs sector Lagrangian divides into two parts, the Higgs-gauge part and the Higgs-fermion part:

$$\mathcal{L}_H = \mathcal{L}_{HG} + \mathcal{L}_{HF}.$$

The Higgs-gauge part can be written as

$$\mathcal{L}_{HG} = (D^\mu \Phi)^\dagger D_\mu \Phi - V(\Phi), \tag{10}$$

where the D^μ is the covariant derivative for Higgs fields and the $V(\Phi)$ is the Higgs self-interaction potential

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2, \tag{11}$$

with positive μ^2 and λ parameters. This Higgs scalar potential is the most important ingredient, when one considers the electroweak symmetry breaking. We will come to this in the next section.

The covariant derivative can be written as:

$$D_\mu \Phi = (\mathbf{I}(\partial_\mu + i\frac{g_1}{2} B_\mu) + ig_2 \frac{\vec{\tau}}{2} \cdot \vec{W}_\mu)\Phi. \tag{12}$$

The Higgs doublet couples to all of the electroweak gauge bosons.

The Higgs-fermion part describes the Yukawa-type interaction between the Higgs fields and the Standard Model fermions. In general Yukawa couplings couple one scalar field to two fermion fields. The Higgs-fermion part of the electroweak Lagrangian is given by:

$$-\mathcal{L}_{HF} = \sum_{i=1}^{3} \sum_{j=1}^{3} (y_u^{ij} \bar{q}'_{i,L} \widetilde{\Phi} u'_{j,R} + y_d^{ij} \bar{q}'_{i,L} \Phi d'_{j,R} + y_e^{ij} \bar{l}'_{i,L} \Phi e'_{j,R}) + h.c., \tag{13}$$

where y_u^{ij}, y_d^{ij} and y_e^{ij} are arbitrary, dimensionless, Yukawa-coupling constants. Primes denote gauge eigenstates. We have also employed charge conjugation to Φ:

$$\widetilde{\Phi} = i\tau_2 \Phi^*,$$

where τ_2 is the second Pauli matrix.

Note that there is no term

$$y_\nu^{ij} \bar{l}'_{i,l} \widetilde{\Phi} \nu'_{j,R} \tag{14}$$

in (13). It is so, because neutrinos are assumed massless in the Standard Model and there can not therefore be any right-handed neutrinos. It is, however, experimentally known that the neutrinos are massive. The Standard Model could be trivially extended to include massive neutrinos by adding the term (14) to the Standard Model. This however leads ot unwanted properties. We talk more about this in the next section, where we dive into the Higgs mechanism.

2.2 The Higgs mechanism

The Weinberg-Salam lagrangian is invariant under $SU(2)_L \times U(1)_Y$ gauge transformation. However, the ground state of the Higgs field (9) is not invariant under $SU(2)_L \times U(1)_Y$ gauge transformation. More specifically the groundstate of the Higgs field is not invariant under $SU(2)_L$ transformation. It still must be invariant under $U(1)$ electromagnetic gauge transformation in order to ensure zero photon mass and the conservation of the electric charge [3]. Because the groundstate is not invariant under $SU(2)_L \times U(1)_Y$ gauge transformation the system is subject to *spontaneous symmetry breaking*. Let us look into this more closely.

The minimum of the Higgs field corresponds to the minimum of its scalar potential (11):

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2. \tag{15}$$

One easily finds out that the minimum of the potential is:

$$\Phi^\dagger \Phi = |\phi^+|^2 + |\phi^0|^2 = \frac{\mu^2}{2}.$$

To ensure that only neutral component breaks we demand that the Higgs field vacuum expectation value $\langle \Phi \rangle$ is:

$$\langle \Phi \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix},$$

where v is:

$$v = \sqrt{\frac{\mu^2}{\lambda}}.$$

So the vacuum expectation value of the charged Higgs is zero. If it differed from zero, the Yukawa terms would then contain terms that would break the conservation of the electric charge, which clearly contradicts the experimental evidence.

The masses of all the fermions and the gauge bosons are determined by substituting the Higgs field by its vacuum expectation value in the Weinberg-Salam Lagrangian. Gauge boson masses are obtained from the Higgs-gauge part of the Lagrangian (10). Fermion masses are obtained from the Higgs-fermion Yukawa coupling (13). Thus the fermion mass Lagrangian is

$$\begin{aligned} &-\mathcal{L}_{F,\text{mass}} = \langle -\mathcal{L}_{HF} \rangle \\ &= \tfrac{v}{\sqrt{2}} y_u^{ij} \bar{u}'_{i,L} u'_{j,R} + \tfrac{v}{\sqrt{2}} y_d^{ij} \bar{d}'_{i,L} d'_{j,R} + \tfrac{v}{\sqrt{2}} y_e^{ij} \bar{e}'_{i,L} e'_{j,R} + h.c. \\ &= \bar{u}'_{i,L} (\mathbf{m}'_u)^{ij} u'_{j,R} + \bar{d}'_{i,L} (\mathbf{m}'_d)^{ij} d'_{j,R} + \bar{e}'_{i,L} (\mathbf{m}'_e)^{ij} e'_{j,R} + h.c., \end{aligned} \tag{16}$$

where we have identified the fermion mass matrices:

$$\mathbf{m}'_i = \frac{v}{\sqrt{2}} \mathbf{y}_i \qquad i = u, d, e. \tag{17}$$

The neutrino masses are not generated like this in the Standard Model since the neutrinos are assumed to be massless in Standard Model. Their masses could however be generated like this if the neutrinos have distinct antiparticles i.e. they are *Dirac particles*. We would only have to add term

$$y_\nu^{ij}\bar{l}'_{i,L}\widetilde{\Phi}\nu'_{j,R},$$

to the Higgs-fermion part (13) of the Weinberg-Salam Lagrangian. When Higgs field acquires VEV, this term would become the neutrino Dirac-mass term:

$$\frac{v}{\sqrt{2}}y_\nu^{ij}\bar{\nu}'_{i,L}\nu'_{j,R} = \bar{\nu}'_{i,L}(\mathbf{m}'_\nu)^{ij}\nu'_{j,R} \tag{18}$$

This seems unnatural, however. According to measurements the neutrino masses less than 1eV [12]. This would need unsatisfactory fine-tuning:

$$g_\nu \lesssim 10^{-12} \lll g_e.$$

It would be more natural if the coupling would be closer to the order of one. This need of fine tuning of neutrino Yukawa coupling could suggest, that the neutrino masses are not generated in this way. It is not even known whether the neutrinos are their own antiparticles or not. The particles, that are their own antiparticles, are called *Majorana particles.* The Standard Model treats the neutrinos as Dirac particles. The Standard Model has to be extended to have massive neutrinos. One of the popular methods to give the mass to the neutrino is the *seesaw-I mechanism.* This allows the neutrino to have a natural Yukawa coupling. The seesaw-I mechanism is discussed in the Appendix C. Next we see how the fermions mix with each other in the Standard Model.

2.3 Fermion mixing

When electroweak symmetry breaks the Higgs-fermion interaction terms (13) become the fermion mass terms:

$$-\mathcal{L}_{F,\text{mass}} = \bar{u}'_L\mathbf{m}'_u u'_R + \bar{d}'_L\mathbf{m}'_d d'_R + \bar{e}'_L\mathbf{m}'_e e'_R + h.c.$$

There is no reason why fermion mass matrices, $\mathbf{m}'_u$, $\mathbf{m}'_e$ and $\mathbf{m}'_e$, should be diagonal in gauge basis (denoted by prime). We want to make the mass matrices diagonal, since the physical particles we observe have well defined masses. The mass matrices can be made diagonal in mass basis (unprimed). The diagonalization can done by introducing the following 3×3 unitary matrices:

$$\mathbf{S}_i^\alpha \qquad \alpha = u, d, e \text{ and } i = L, R,$$

and insert them next to appropriate fermion fields. The mass terms, that were originally in gauge basis (16) can now be written in mass basis:

$$\begin{aligned}
-\mathcal{L}_{F\text{mass}} &= \bar{u}'_L\mathbf{S}_L^u\mathbf{S}_L^{u\dagger}\mathbf{m}'_u\mathbf{S}_R^u\mathbf{S}_R^{u\dagger}u'_R + \bar{d}'_L\mathbf{S}_L^d\mathbf{S}_L^{d\dagger}\mathbf{m}'_d\mathbf{S}_R^d\mathbf{S}_R^{d\dagger}d'_R \\
&\quad + \bar{e}'_L\mathbf{S}_L^e\mathbf{S}_L^{e\dagger}\mathbf{m}'_e\mathbf{S}_R^e\mathbf{S}_R^{e\dagger}e'_R + h.c. \\
&= \bar{u}_L\mathbf{m}_u u_R + \bar{d}_L\mathbf{m}_d d_R + \bar{e}_L\mathbf{m}_e e_R + h.c. \\
&= \bar{u}\mathbf{m}_u u + \bar{d}\mathbf{m}_d d + \bar{e}\mathbf{m}_e e.
\end{aligned}$$

The matrices $S_{L,R}^\alpha$ are absorbed into the definitions of mass eigenstates:

$$\begin{array}{lll} u_L = \mathbf{S}_L^{u\dagger}u'_L, & d_L = \mathbf{S}_L^{d\dagger}d'_L, & e_L = \mathbf{S}_L^{e\dagger}e'_L \\ u_R = \mathbf{S}_R^{u\dagger}u'_R, & d_R = \mathbf{S}_R^{d\dagger}d'_R, & e_R = \mathbf{S}_R^{e\dagger}e'_R. \end{array} \tag{19}$$

Here we use notation

$$u_{i,L} = \sum_{j=1,2,3} (\mathbf{S}_L^{u\dagger})_{ij} u'_{j,L} \qquad \bar{u}_{i,L} = \sum_{j=1,2,3} \bar{u}'_{j,L} (\mathbf{S}_L^u)_{ji} \tag{20}$$

for matrix multiplication in (19). Here we see that the different generations are mixed into each other. According to (20), an up-type quark $u_{i,L}$ is a linear combination of gauge eigenstates of every generation. This mixing will be visible in the interaction Lagrangian, and allow flavour violating processes to take place.

The diagonalizations of mass matrices $\mathbf{m}'_u$, $\mathbf{m}'_e$ and $\mathbf{m}'_e$ therefore are:

$$\mathbf{m}'_u = \mathbf{S}_L^u \mathbf{m}_u \mathbf{S}_R^{u\dagger}, \qquad \mathbf{m}'_d = \mathbf{S}_L^d \mathbf{m}_d \mathbf{S}_R^{d\dagger}, \qquad \mathbf{m}'_e = \mathbf{S}_L^u \mathbf{m}_e \mathbf{S}_R^{u\dagger}.$$

The diagonal mass matrices then are:

$$\mathbf{m}_u = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}, \qquad \mathbf{m}_d = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}, \qquad \mathbf{m}_e = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}.$$

The electroweak theory was originally expressed in terms of gauge eigenstates. In order to be able to compare calculations to observations, we should express theory in terms of observed fields i.e. the mass eigenstates. This can be done by using equations (19). The introduction of mass eigenstates leads to modifications in electroweak currents. Also for a general theory, the currents can be divided into neutral and charged currents. Neutral currents couple to the neutral gauge bosons and the charged currents couple to the electrically charged gauge bosons.

We are interested in the flavour violation. In the Standard Model the flavour violation can only occur after the electroweak symmetry breaking, when the fermion masses are generated. The off-diagonal mass terms act as sources for the flavour violation. We will also express the neutral and charged current Lagrangians in terms of physical gauge bosons: $W^{\pm}_\mu, Z_\mu$ and A_μ.

2.3.1 Electroweak neutral current

The structure of electroweak neutral current does not change when we write the current in terms of mass eigenstates. The neutral weak current Lagrangian is [1]:

$$\begin{aligned}\mathcal{L}_{NC} &= g_2 {J'}^3_\mu W^{3\mu} + \frac{1}{2} g_1 J'^Y_\mu B'^\mu \\ &= e J'^{em}_\mu A^\mu + \frac{g_2}{cos\theta_W} J'^0_\mu Z^\mu,\end{aligned}$$

where J'^{em}_μ is the *electromagnetic current*, which contains both left- and right-handed fields [3]:

$$J'^{em}_\mu = \sum_{i=1}^{3} q_i \bar{e}'_i \gamma^\mu e'_i + \sum_{i=1}^{3} q_i \bar{u}'_i \gamma^\mu u'_i + \sum_{i=1}^{3} q_i \bar{d}'_i \gamma^\mu d'_i, \tag{21}$$

[3] $P_L + P_R = \frac{1}{2}(1-\gamma_5) + \frac{1}{2}(1+\gamma_5) = 1$

and J'^0_μ is the current which couples to the Z^0-boson:

$$J'^0_\mu = J'^3_\mu - sin^2\theta_W J'^{em}_\mu .$$

The prime in the names of the currents signifies, that they are written in terms of the gauge eigenstate fermions.
Neutral weak current J'^3_μ is:

$$J'^3_\mu = \frac{1}{2}\sum_{i=1}^{3} \bar{l}'_{i,L}\gamma_\mu\tau^3 l'_{i,L} + \frac{1}{2}\sum_{i=1}^{3} \bar{q}'_{i,L}\gamma_\mu\tau^3 q'_{i,L},$$

$$= \sum_{i=1}^{3} \bar{\nu}'_{i,L}\gamma_\mu\nu'_{i,L} - \sum_{i=1}^{3} \bar{e}'_{i,L}\gamma_\mu e'_{i,L} + \sum_{i=1}^{3} \bar{u}'_{i,L}\gamma_\mu u'_{i,L} - \sum_{i=1}^{3} \bar{d}'_{i,L}\gamma_\mu d'_{i,L}.$$

Neutral currents J'^{em}_μ and J'^0_μ retain their form when we express them in terms of mass eigenstates (19), since unitary matrices $\mathbf{S}^\alpha_{L,R}$ cancel. To show this, let us write leptonic part of the electromagnetic current (21) in terms of mass eigenstates (19):

$$J'^{em}_{\mu,lepton} = \sum_{i=1}^{3} q_i \bar{e}'_i\gamma_\mu e'_i = \sum_{i=1}^{3} q_i\bar{e}'_{i,L}\gamma_\mu e'_{i,L} + \sum_{i=1}^{3} q_i\bar{e}'_{i,R}\gamma_\mu e'_{i,R}$$

$$= \sum_{i=1}^{3} q_i(\bar{e}_L\mathbf{S}^{e\dagger}_L)_i\gamma_\mu(\mathbf{S}^e_L e_L)_i + \sum_{i=1}^{3} q_i(\bar{e}_R\mathbf{S}^{e\dagger}_R)_i\gamma_\mu(\mathbf{S}^e_R e_R)_i.$$

We can commute $\mathbf{S}^\alpha_{L,R}$ and γ_μ because they operate in different spaces: $\mathbf{S}^\alpha_{L,R}$ operates in flavour space and γ_μ operates in spin space. Therefore due to unitarity of $\mathbf{S}^\alpha_{L,R}$:

$$J'^{em}_{\mu,lepton} = \sum_{i=1}^{3} q_i\bar{e}_{i,L}\gamma_\mu e_{i,L} + \sum_{i=1}^{3} q_i\bar{e}_{i,R}\gamma_\mu e_{i,R}$$

$$= \sum_{i=1}^{3} q_i\bar{e}_i\gamma_\mu e_i \equiv J^{em}_{\mu,lepton}.$$

So indeed electromagnetic current retains its structure when we express gauge eigenstates in terms of mass eigenstates. That is because each term in the electromagnetic current contains both a field and the adjoint of the same field. Also neutral weak current has this property. However that is not the case with the charged current as we shall see.

2.3.2 Electroweak charged current and quark mixing

The charged weak currents are [1]:

$$J^+_\mu = \sum_{i=1}^{3} \bar{\nu}'_{i,L}\gamma_\mu e'_{i,L} + \sum_{i=1}^{3} \bar{u}'_{i,L}\gamma_\mu d'_{i,L}$$

$$J^-_\mu = \sum_{i=1}^{3} \bar{e}'_{i,L}\gamma_\mu \nu'_{i,L} + \sum_{i=1}^{3} \bar{d}'_{i,L}\gamma_\mu u'_{i,L}.$$

The neutral currents hold their form, whether one represents them in the gauge or the mass basis, so they can not be responsible for flavour violation in the Standard Model. In fact, the charged currents are those responsible. Let us show that for the quarks. The charged current J^+_μ for quarks can be written in terms of mass eigenstates (19) as follows:

$$J^+_{\mu,quark} = \sum_{i=1}^{3} \bar{u}'_{i,L}\gamma_\mu d'_{i,L} = \bar{u}_L \mathbf{S}_L^{u\dagger}\gamma_\mu \mathbf{S}_L^d d_L$$

$$= \bar{u}_L \gamma_\mu \mathbf{S}_L^{u\dagger}\mathbf{S}_L^d d_L = \sum_{i=1}^{3} \bar{u}_{i,L}\gamma_\mu d''_{i,L},$$

where we have denoted

$$d''_{i,L} \equiv \sum_{j=1}^{3} \mathbf{V}_{ij} d_{j,L}, \tag{22}$$

and

$$\mathbf{V} \equiv \mathbf{S}_L^{u\dagger}\mathbf{S}_L^d.$$

The matrix $\mathbf{V}$ is called *Cabibbo-Kobayashi-Maskawa matrix*, abbreviated as *CKM matrix.* It describes the mixing of quarks. Because the quark gauge eigenstates differ from mass eigenstates, which is experimentally known, the physically observed quarks (i.e. the mass eigenstates) are linear combinations of gauge eigenstates. Traditionally the mixing matrix $\mathbf{V}$ is added to the lower generations d_a, but it can of course be added to the upper generations u_A instead of the lower ones.

The double primed state in equation (22) is called *weak eigenstate*, since it is the field that directly couples to the $W^\pm$-bosons. So the physical mass eigenstates do not directly couple to the charged current, but they couple to it as a linear combination (22).

Since the electroweak neutral currents retained their form when we switched from the gauge eigenstates to the mass eigenstates, the flavour is not violated by them. It is said that there is no *flavour changing neutral currents* (FCNC) at the Lagrangian level. This means that the flavour cannot change in any process, which has only neutral gauge boson (photon and Z-boson) exchanges. The flavour can only change in processes where charged gauge bosons ($W^\pm$) are exchanged. There can be flavour changing neutral currents in Standard Model beyond Lagrangian level, i.e. in diagrams containing loops. The flavour changing neutral current are however highly suppressed in the Standard Model due to the GIM-mechanism.

2.3.3 Possibility of charged lepton mixing?

In the Standard Model charged leptons cannot mix, because the neutrinos are massless in it. It is, however, known that the neutrinos have mass and that they mix. Could the charged leptons mix? It is known that quark generations mix to each other. The mixing is often associated to the lower fields in the $SU(2)_L$ doublet, but the mixing could as well be associated to the upper fields in the doublet. Based on the quark mixing case, we then expect that the neutrino mixing matrix could be associated to the charged lepton fields instead of neutrino fields.

If neutrinos were Dirac fermions, their masses could be generated through Higgs mechanism. Then the neutrinos would have a mass term (18). The neutrino mass matrix in (18) could be diagonalized in the same fashion as in the equation (19), by introducing a unitary matrices $\mathbf{S}^{\nu}_{L,R}$. With these we could represent the neutrino mass eigenstates in terms of gauge eigenstates:

$$\nu_L = \mathbf{S}^{\nu\dagger}_L \nu'_L \qquad \nu_R = \mathbf{S}^{\nu\dagger}_R \nu'_R.$$

We could now introduce lepton mixing in a same way as in the previous section, by writing charged weak current in terms of mass eigenstate leptons instead of gauge eigenstates. The lepton mixing would not be possible without neutrino mass: mixing matrix could always be taken as unit matrix.

$$\begin{aligned} J^{+}_{\mu,lepton} &= \sum_{i=1}^{3} \bar{\nu}'_{i,L}\gamma_\mu e'_{i,L} = \bar{\nu}_L \mathbf{S}^{\nu\dagger}_L \gamma_\mu \mathbf{S}^{e}_L e_L \\ &= \bar{\nu}_L \gamma_\mu \mathbf{S}^{\nu\dagger}_L \mathbf{S}^{e}_L e_L = \bar{\nu}_L \mathbf{S}^{\nu\dagger}_L \mathbf{S}^{e}_L \gamma_\mu e_L. \end{aligned}$$

So the mixing of neutrinos could suggest that also the charged leptons e, μ and τ could mix. As we have already noted the Dirac-neutrinos would require unsatisfactory fine-tuning of neutrino Yukawa-coupling. So the Standard Model is preferably not extended in this way. But as a conclusion to our Standard Model section, we can say that there are no charged lepton flavour violating processes in the Standard Model. So any observation of charged lepton flavour violating process would prove the existence of physics beyond the Standard Model.

3 CLFV in effective theories

3.1 Effective theories

The Standard Model Lagrangian consists of renormalizable terms only, i.e. the constant coefficients of every term in the Lagrangian have non-negative mass dimensions. Also all the terms are invariant under the Standard Model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. If one abandons the requirement of renormalizability, the Standard Model can be extended by including gauge invariant effective operators with mass dimensions larger than four. These operators are remnants of the higher theory in the low energy limit. The propagators of the heavy particles reduce into points in the low momentum limit leaving behind only the contact interactions. The vertex function of this new vertex is called an *effective operator*. Let us now briefly study an example of this kind of effective operator.

Let us study the decay of a muon, $\mu \to \nu_\mu e \bar{\nu}_e$, in the standard theory of weak interactions. At the tree level there is only one diagram contributing to this process and it is displayed at Figure 1 (a). The Feynman amplitude for this process at tree level is therefore [3]:

$$\mathcal{M} = -g_W^2 [\bar{e}\gamma^\alpha(1-\gamma_5)\nu_e] \frac{i(-g_{\alpha\beta} + \frac{k_\alpha k_\beta}{m_W^2})}{k^2 - m_W^2 + i\epsilon} [\bar{\nu}_\mu \gamma^\beta (1-\gamma_5)\mu], \tag{23}$$

where g_W is related to weak coupling constant g_2 as $g_W = \frac{g_2}{2\sqrt{2}}$.

However, when the energy is small compared to the mass of the W-boson, the W boson propagator reduces to a much simpler form:

$$\lim_{m_W \to \infty} \frac{i(-g_{\alpha\beta} + \frac{k_\alpha k_\beta}{m_W^2})}{k^2 - m_W^2 + i\epsilon} = \frac{i}{m_W^2} g_{\alpha\beta}.$$

At this low energy limit the Feynman amplitude (23) becomes:

$$\mathcal{M} = -i\frac{g_W^2}{m_W^2}[\bar{e}\gamma^\alpha(1-\gamma_5)\nu_e][\bar{\nu}_\mu \gamma_\alpha (1-\gamma_5)\mu]. \tag{24}$$

This corresponds to a contact interaction shown in figure 2 (b).

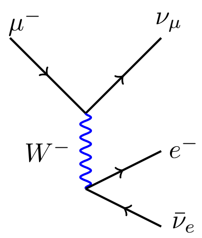

Figure 1: (a)

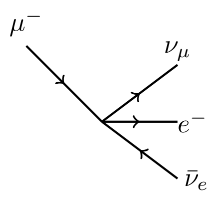

Figure 2: (b)

So when the energy scale is much lower than the scale of the weak interactions, m_W, the propagator of the heavy high energy scale particle (W-boson) contracts to a suppression factor $\sim 1/m_W^2$. It is said that the heavy degrees of freedom have been "integrated out" (the term comes from the path integral formalism where the heavy fields are got rid of by performing a functional integration over the heavy fields) . This is the fundamental idea of effective field theories: at energies much lower than the energy scale of the higher theory, the propagators of the heavy particles become factors which are suppressed by the energy scale of the higher theory.

At lower energies than the scale of the higher energy theory the heavy high energy theory particles cannot be produced in collisions or in decays, so they are absent in the external states. The heavy fields are, however, still present as virtual particles. Therefore the effects of the heavy fields are still there even though the heavy fields themselves are not detected. This is how the effects of the electroweak theory were first detected, as virtual effects beyond QED, which was the Standard Model at the time. The nuclear beta decays were detected and they seemed to happen as the contact interaction presented in our example. The heavy field mediating the process, W boson, was detected later.

At low energies compared to the scale of the higher theory the heavy particles effectively vanish. As a result two vertices of the original high energy theory merge into a one vertex. This usually ruins the renormalizability of the effective theory. As the propagator disappears more fields accumulate to that one vertex making mass dimension of the effective coupling constant negative. This is exactly what happens in our example: once the W-boson propagator vanishes four fermion fields get connected in the same vertex. The mass dimension of a fermion field is $3/2$ so the total mass dimension of the fermion fields is 6, making the mass dimension of the effective coupling constant, g_W^2/m_W^2, -2.

We would have obtained the expression (24) if we had used the interaction Lagrangian

$$\mathcal{L}_{int}^F = -\left(\frac{g_W}{m_W}\right)^2 [\bar{e}\gamma^\alpha(1-\gamma_5)\nu_e][\bar{\nu}_\mu\gamma^\beta(1-\gamma_5)\mu].$$

This is the contact interaction which Fermi proposed in 1934 to describe nuclear β-decay process $n \to p + e^- + \bar{\nu}_e$ [3][41].

The effective theories (effective QFT) can be used in two ways. If the Lagrangian of the full theory is known, the low energy situations of that theory can be expressed in a much simpler form by integrating out the heavy degrees of freedom. When the full high energy theory is not known, which is the case now when one tries to study the physics beyond SM, the effective field theory can be used to give the phenomenology of the high energy physics. It is straightforward to use an effective field theory at tree level, but at loop level complications arise, since the higher dimensional operators are not renormalizable. In general these non-renormalizable interactions give divergent results. However using appropriate techniques one can scrape together a finite result [**?**].

In the most *New Physics* theories the Standard Model is recovered by decoupling the heavy degrees of freedom whose mass scale is $\Lambda \gg M_Z$. This kind of decoupling is possible because of the *Appelquist-Carazzone theorem* which states that heavy fields decouple at low momenta except for their contribution to renormalization effects [28]. When low momentum limit is taken from a new physics model the propagators in diagrams containing heavy new physics particles wither, leaving only contact interactions. This gives rise to so called higher dimensional operators i.e. operators whose mass dimension is greater than four. The new physics effective Lagrangian at low energy limit can be written as a sum of the Standard Model Lagrangian and the effective higher dimensional operators:

$$\mathcal{L}_{newphys} = \mathcal{L}_{SM} + \frac{1}{\Lambda}\sum_k C_k^{(5)} Q_k^{(5)} + \frac{1}{\Lambda^2}\sum_k C_k^{(6)} Q_k^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right),$$

where C_k^n's are dimensionless coupling constants known as *Wilson* coefficients and Q^n's are the n-dimensional effective operators.

New physics can be studied in a model independent way by studying the Standard Model extended with gauge invariant effective higher dimensional operators. When we have chosen our New Physics model, we can calculate the Wilson coefficients. However, in the following three Sections we want to study lepton flavour violation in charged lepton sector without specifying the model. This can then be done using higher dimensional effective operators. We assume so called *"Minimal Flavour Violation hypothesis"* (MFV) [27]. It assumes that the Standard Model Yukawa couplings are the only sources of lepton flavour symmetry breaking. This means that effective higher dimensional operators describing lepton flavour violation contain only Standard Model fields.

In general two different high energy theories will both produce the same effective higher dimensional operators. The two models differ in the Wilson coefficients.

3.2 Effective operators

In the following sections we will take a closer look into three kinds of processes which violate charged lepton flavour. These processes are $l_i \to l_j$, $l_i \to l_j\gamma$ and $l_i \to l_j l_k l_l$. We will study properties of these reactions by examining the effective Lagrangians for given processes. The effective Lagrangians we use to generate charged lepton flavour violation consist of dimension-six effective operators at electroweak scale. There is only one dimension five operator, the *Weinberg operator* [30], and it only contributes to the neutrino masses. The mixing of neutrino masses gives a source of flavour violation in charged lepton sector, due to the fact that the neutrinos couple to the charged leptons. The effect is unobservably small however. We don't need to include the Weinberg operator in our studies then. The dimension-seven operators are order $\mathcal{O}\left(1/\Lambda^3\right)$ and are therefore contributing very little compared to the dimension-six operators which are suppressed only by $\mathcal{O}\left(1/\Lambda^2\right)$. The dimension-six operators are then the only ones we need.

We will first study $\mu \to e$. It is in some sense more general than closely related $\mu \to e\gamma$. Vertex (or vertices) that contain photon exists in both reactions. In $\mu \to e\gamma$ the photon is on mass shell but $\mu \to e$ can contain a photon that is virtual.

Then we will concentrate on $l_i \to l_j l_k l_l$ which has many different properties compared to previous ones. But before we can go to examine specific reactions, we have to review some of the effective operators that are relevant to our discussion. The complete list of independent dimension-5 and dimension-6 operators which are constructed from the SM fields and which also are invariant under SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ was first derived by W. Buchmuller and D. Wyler in 1985 [42]. That list was not irreducible however. The classical equations of motion can be used to get rid of redundant operators [46]. In this thesis minimal reducible set of dimension-six operators is used [43].

Let us now list the effective dimension-six operators that contribute to the charged lepton flavour violating processes at tree-level or at the 1-loop level (after the heavy fields are integrated out). Operators listed here are the ones that give the main contribution to lepton flavour violating processes. We have omitted operators which give only the contribution to the flavour violating Higgs boson coupling since they are suppressed by higher orders of m_l/m_{h^0} [14].

Table 4: Effective dimension-six operators with leptons [14]

$llll$	$llX\phi$	$ll\phi\phi D$
$Q_{ll} = (\bar{l}_i\gamma_\mu l_j)(\bar{l}_k\gamma^\mu l_l)$	$Q_{eW} = (\bar{l}_i\sigma^{\mu\nu}e_j)\tau^I\phi W^I_{\mu\nu}$	$Q^{(1)}_{\phi l} = (\phi^\dagger i \overleftrightarrow{D}_\mu \phi)(\bar{l}_i\gamma^\mu l_j)$
$Q_{ee} = (\bar{e}_i\gamma_\mu e_j)(\bar{e}_k\gamma^\mu e_l)$	$Q_{eB} = (\bar{l}_i\sigma^{\mu\nu}e_j)\phi B_{\mu\nu}$	$Q^{(3)}_{\phi l} = (\phi^\dagger i \overleftrightarrow{D}^I_\mu \phi)(\bar{l}_i\tau^I\gamma^\mu l_j)$
$Q_{le} = (\bar{l}_i\gamma_\mu l_j)(\bar{e}_k\gamma^\mu e_l)$		$Q_{\phi e} = (\phi^\dagger i \overleftrightarrow{D}_\mu \phi)(\bar{e}_i\gamma^\mu e_j)$

Table 5: Effective dimension-six operators with leptons and quarks [14]

$llqq$		
$Q^{(1)}_{lq} = (\bar{l}_i\gamma_\mu l_j)(\bar{q}_k\gamma^\mu q_l)$	$Q_{ld} = (\bar{l}_i\gamma_\mu l_j)(\bar{d}_k\gamma^\mu d_l)$	$Q_{lu} = (\bar{l}_i\gamma_\mu l_j)(\bar{u}_k\gamma^\mu u_l)$
$Q^{(3)}_{lq} = (\bar{l}_i\gamma_\mu\tau^I l_j)(\bar{q}_k\gamma^\mu\tau^I q_l)$	$Q_{ed} = (\bar{e}_i\gamma_\mu e_j)(\bar{d}_k\gamma^\mu d_l)$	$Q_{eu} = (\bar{e}_i\gamma_\mu e_j)(\bar{u}_k\gamma^\mu u_l)$
$Q_{eq} = (\bar{e}_i\gamma_\mu e_j)(\bar{q}_k\gamma^\mu q_l)$	$Q_{ledq} = (\bar{l}^a_i e_j)(\bar{d}_k q^a_l)$	$Q^{(1)}_{lequ} = (\bar{l}^a_i e_j)\epsilon_{ab}(\bar{q}^b_k u_l)$
		$Q^{(3)}_{lequ} = (\bar{l}^a_i\sigma_{\mu\nu}e_j)\epsilon_{ab}(\bar{q}^b_k\sigma^{\mu\nu}u_l)$

Indices i, j, k and l are flavour indices. These operators are the basis from which the effective Lagrangians are constructed. In the case of operators, Q_{eW} and Q_{eB}, which contain gauge eigenstates $\vec{W}_\mu = (W^1_\mu, W^2_\mu, W^3_\mu)$ and B_μ it is customary to use the mass eigenstates W^+_μ, W^-_μ, Z_μ and A_μ instead. The relation between mass eigenstates and gauge eigenstates is given by (3).

The operators listed in Tables 4 and 5 give rise to lepton flavour violating effective vertices. The effective vertices are represented in Figures 5-16. The operators which contribute to a given vertex are written in to the captions of the figures. In the Appendix E we have written the effective operators explicitly in terms of the fields. The Feynman rules associated to these vertices can be found in [14]. The reason why we are assuming so small energy, that the Higgs can be neglected, is that the processes $l_i \to l_j$, $l_i \to l_j\gamma$ and $l_i \to l_j l_k l_l$ are predominantly searched in the low-energy experiments [75]-[86].

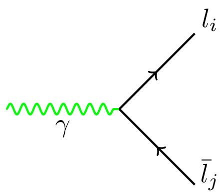

Figure 3: Q_{eW}, Q_{eB}

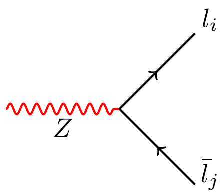

Figure 4: $Q_{eW}, Q_{eB}, Q_{\phi l}^{(1)}, Q_{\phi l}^{(3)}, Q_{\phi e}$

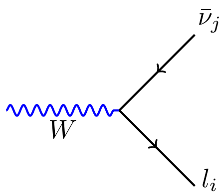

Figure 5: $Q_{eW}, Q_{\phi l}^{(3)}$

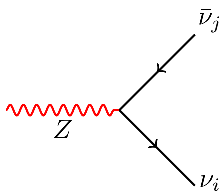

Figure 6: $Q_{\phi l}^{(1)}, Q_{\phi l}^{(3)}$

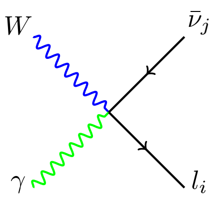

Figure 7: Q_{eW}

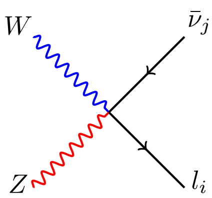

Figure 8: Q_{eW}

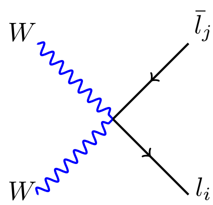

Figure 9: Q_{eW}

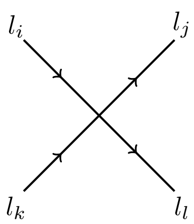

Figure 10: Q_{ll}, Q_{ee}, Q_{le}

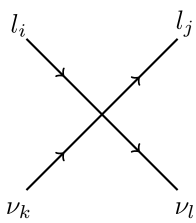

Figure 11: Q_{ll}, Q_{le}

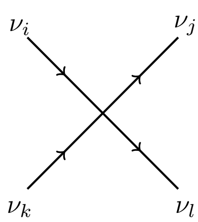

Figure 12: Q_{ll}

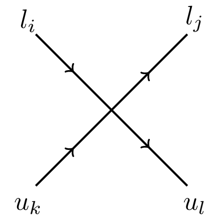

Figure 13: $Q_{lq}^{(1)}, Q_{lq}^{(3)}, Q_{eq}, Q_{lu}, Q_{eu}, Q_{lequ}^{(1)}, Q_{lequ}^{(3)}$

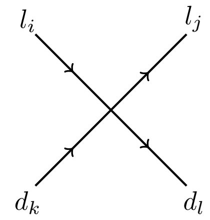

Figure 14: $Q_{lq}^{(1)}, Q_{lq}^{(3)}, Q_{eq}, Q_{ld}, Q_{ed}, Q_{ledq}$

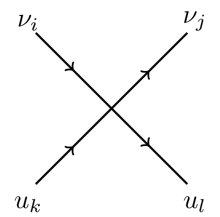

Figure 15: $Q_{lq}^{(1)}, Q_{lq}^{(3)}, Q_{lu}$

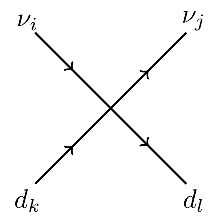

Figure 16: $Q_{lq}^{(1)}, Q_{lq}^{(3)}, Q_{ld}$

By taking into account all the operators in the Tables 4 and 5 we can write the most general charged lepton flavour violating Lagrangian to one loop order:

$$\begin{aligned}
\mathcal{L}_{CLFV} &= \mathcal{L}_{llX\phi} + \mathcal{L}_{llll} + \mathcal{L}_{llqq} + \mathcal{L}_{ll\phi^2 D} \\
&= \frac{1}{\Lambda^2}\sum_{ij}\left(C_{eW}^{ij}Q_{eW}^{ij} + C_{eB}^{ij}Q_{eB}^{ij}\right) + \frac{1}{\Lambda^2}\sum_{ij}\left(C_{\phi l}^{ij}Q_{\phi l}^{(1)ij} + C_{\phi l}^{ij}Q_{\phi l}^{(3)ij} + C_{\phi e}^{ij}Q_{\phi e}^{ij}\right) \\
&+ \frac{1}{\Lambda^2}\sum_{ijkl, i\geq k, j\geq l}\left(C_{ll}^{ijkl}Q_{ll}^{ijkl} + C_{ee}^{ijkl}Q_{ee}^{ijkl}\right) + \frac{1}{\Lambda^2}\sum_{ijkl}\left(C_{le}^{ijkl}Q_{le}^{ijkl}\right) \\
&+ \frac{1}{\Lambda^2}\sum_{ijkkl}\left(C_{lq}^{ijkl}Q_{lq}^{(1)ijkl} + C_{lq}^{ijkl}Q_{lq}^{(3)ijkl} + C_{eq}^{ijkl}Q_{eq}^{ijkl} + C_{ld}^{ijkl}Q_{ld}^{ijkl} + C_{ed}^{ijkl}Q_{ed}^{ijkl}\right. \\
&\left. + C_{ledq}^{ijkl}Q_{ledq}^{ijkl} + C_{lu}^{ijkl}Q_{lu}^{ijkl} + C_{eu}^{ijkl}Q_{eu}^{ijkl} + C_{lequ}^{ijkl}Q_{lequ}^{(1)ijkl} + C_{lequ}^{ijkl}Q_{lequ}^{(3)ijkl}\right), \quad (25)
\end{aligned}$$

where the C's are the Wilson coefficients and Λ is the some energy scale where the new physics enters. In the four-lepton part of the Lagrangian we have required that $i \geq j$ and $j \geq l$ to avoid counting the same operator multiple times [14].

Depending on the relative size of the Wilson coefficients, different operators can dominate the others. The operators in Tables 4 and 5 are the leftovers of the higher energy diagrams where the heavy particles have been integrated out. Some of the higher energy diagrams contained loops, some didn't. If the effective operator is a leftover from a tree level diagram, it is called *tree-generated*

(TG) and if it is a leftover from a loop diagram it is called *loop-generated* (LG) [33]. But since we don't know what is the theory beyond Standard Model, we don't really know if a operator is tree-generated or not. Therefore one uses usually term *potential tree-generated operator* (PTG) instead of tree-generated operator. The Wilson coefficients of PTG operators are generically larger than the Wilson coefficients of the LG operators. Therefore one might expect that LG operators give smaller contribution compared to the PTG operators. In the next section we will study the CLFV processes $l_i \to l_j \gamma$, $l_i \to l_j l_k l_l$ and the l_i-l_j conversion in the effective theory approach.

4 CLFV processes in effective theories

In the this section we will study CLFV processes, $l_i \to l_j\gamma$, $l_i \to l_j l_k l_l$ and the l_i-l_j conversion, in terms of effective operators. We will consider the contribution of all the effective operators in the first order they appear. If the operator enters already at tree level, the same operator is not taken into the account at the loop level, since the tree level should always dominate the loop level diagrams. All the operators enter not later than at the one loop level. When we begin our study of each process we assume that the energy scale $\sim M_Z$, so that the electroweak symmetry has been broken and draw all the relevant Feynman diagrams to the given process.

4.1 $l_i \to l_j$ effectively

We will now study charged lepton conversion process, $l_i \to l_j$, that is a process where a charged lepton l_i changes to a different charged lepton l_j. This conversion process however can't happen without involving other external particles due to four-momentum conservation. So actually we are considering processes where also other particles appear in the initial and final states. The term "lepton - lepton" conversion is therefore somewhat misleading.

We must now decide what other particles than the converting leptons we include to our process. Lepton decay to lepton and photon, $l_i \to l_j\gamma$, is studied in the next section, so we can't include a photon. We can't include other charged leptons either since in the section 4.3 we are considering $l_i \to l_j l_k l_l$ which already contains leptons.

The particles we are left with are quarks (or composite structures formed by them: hadrons). Experimentally interesting processes are the expected lepton-lepton conversions near the nucleus of an atom. In those a heavy charged lepton (muon or tau) orbiting the nucleus converts into another charged lepton by interacting with the quarks in the nucleus. Therefore we wish to study processes

$$l_i + q \to l_j + q,$$

where q is a quark. The quark is the same in the initial and the final states. The lepton conversion near the nucleus, where the nucleus does not change is called *coherent conversion*. Let us next study coherent conversion at the tree-level of the effective theory.

4.1.1 $l_i + q \to l_j + q$ at the tree level

The coherent conversion $l_i + q \to l_j + q$ can happen already at tree level. The Feynman diagrams contributing to this in the effective theory, are constructed by using the vertices listed in the Figures 5-16. At the tree-level the possible diagrams are:

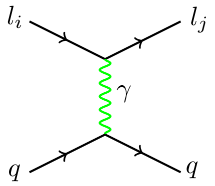

Figure 17: Lepton conversion with quarks mediated by a photon (Q_{eW}, Q_{eB})

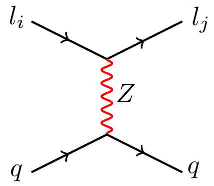

Figure 18: Lepton conversion with quarks mediated by Z boson (Q_{eW}, Q_{eB})

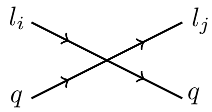

Figure 19: Lepton conversion with quarks as a contact interaction (*llll* class)

Process in Figure 17 contains a lepton flavour violating $l_i l_j \gamma$-vertex. It is generated by LG operators Q_{eW} and Q_{eB}. Processes in the Figure 18 contain lepton flavour violating vertex ($l_i l_j Z$) which is generated by LG operators Q_{eW} and Q_{eB} but also by PTG operators $Q_{\phi l}^{(1)}$, $Q_{\phi l}^{(3)}$ and $Q_{\phi e}$. The contact interaction in Figure 19 is generated by PTG operators in *llqq*-class.

The processes mediated by gauge bosons are generated when heavy new physics particles are integrated out. The diagram in Figure 17 could be generated for example from the following penguin-diagram:

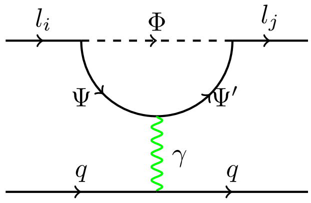

Figure 20: Penguin diagram containing New Physics particles

The diagram in Figure 20 contains New Physics particles: Ψ (fermion), Ψ' (fermion) and Φ (scalar). In Supersymmetric models the Ψ and Ψ' could be *gauginos* (the fermionic partners of the gauge bosons) and the Φ could be the scalar partner of some neutrino.

The four-lepton contact interaction in the Figure 19 could be generated from the following box diagram:

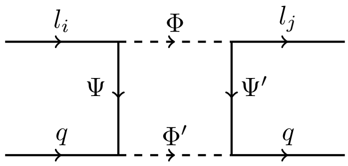

Figure 21: New Physics box diagram

In a supersymmetric model the Ψ and Ψ' could be some gauginos, Φ the scalar partner of some lepton and the Φ' could be the scalar partner of some quark.

We have not used all of the effective CLFV operators at the tree-level, so we must also study the one-loop level to get all the possible CLFV contributions.

4.1.2 $l_i + q \to l_j + q$ at one-loop level

The other operators also contribute to process $l_i + q \to l_j + q$, but at loop order. We are interested in the leading order contribution to the process. It would be tempting to stick with the tree level, discarding the loops as higher order contributions. This is not acceptable however. We do not know what is the New Physics theory which allows CLFV. We don't therefore know which CLFV operators are generated and which aren't. In order to be consistent one must include contributions from all CLFV operators at the lowest order they enter. If some operator enters already at tree level we ignore the same operator at one loop level, since the tree level contribution should always dominate.

The operators in classes $llX\phi$, $ll\phi\phi D$ and $llqq$ (in the Table 4) may enter at tree level. So the only remaining operators that enter at one-loop order are the operators of the $llll$ class (Table 4). The only one-loop diagrams that introduce new operators are the diagrams in the Figures 22 and 23 involving lepton self energy contributions and the diagrams in the Figures 25 and 26.

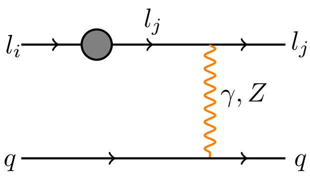

Figure 22: $l_i \to l_j$ with lepton self energy (a)

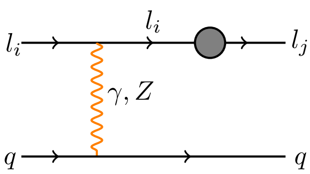

Figure 23: $l_i \to l_j$ with lepton self energy (b)

The gray blob is the following charged lepton flavour violating self energy diagram:

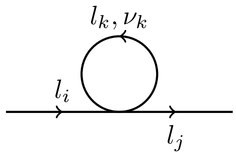

Figure 24: Self energy generated by CLFV operators in *llll* class

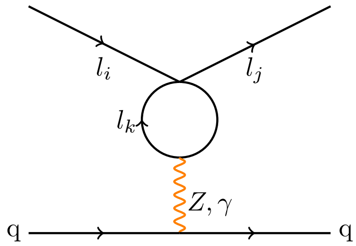

Figure 25: *llll* operator contribution

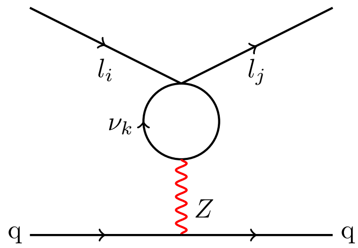

Figure 26: Q_{ll}, Q_{le} contribution

At a first glance the gray blobs in the diagrams in the Figures 22 and 23 look like corrections to external leg, making the diagram not *one particle irreducible* (1PI). However it is the gray blob that contains the flavour changing effective vertex (Figure 24); cutting off the gray blob does not reduce to the same process, making the diagrams in the Figures 22 and 23 1PI.

All the effective diagrams describing CLFV process $l_i + q \to l_j + q$ in this section contain only one effective CLFV vertex. The CLFV processes are experimentally known to be extremely rare: they have never been observed. Even one effective CLFV vertex comes from a loop constructed from heavy new physics particles or only from the situation where heavy virtual particle has been integrated out. Latter case gives suppression by mass (squared) in case on boson (fermion) and the former gives loop suppression, $(16\pi^2)^{-1}$, on top of that. This is why we can safely take only one effective CLFV vertex, two is negligible. In the following sections we will continue allow only one CLFV vertex in the effective diagrams.

4.1.3 $l_i + q \to l_j + q$ at low energy limit

The experimentally interesting $l_i \to l_j$ -conversion is the μ-e conversion in a muonic atom. In the muonic atom muon has replaced an electron in an ordinary atom. Tau has so short life time that the similar structure formed by it and the nucleus is not experimentally significant. We will therefore focus on

$\mu \to e$ conversion in a muonic atom. Normally the muon decays on orbit, $\mu^- \to e^- \nu_\mu \bar{\nu}_e$, or is captured by the nucleus

$$\mu^- + (A, Z) \to \nu_\mu + (A, Z-1),$$

where A is the mass number and Z is the atomic number of the nucleus [31]. In the context of charged lepton flavour violation the conversion,

$$\mu^- + (A, Z) \to e^- + (A, Z),$$

is expected. This kind of conversion where the nucleus remains unchanged is coherent conversion. In process $l_i + q \to l_j + q$ the quark remains the same and therefore also the nucleus.

Conversions of muons to electrons (or taus to muons and electrons) are low energy processes (in order the atom to stay together), so the heavy Z-boson can be ignored as mediator of the conversion process. Then only the diagrams with virtual photon and contact interaction remain. The diagrams in Figures 22, 23 and 25 contain lepton loops. Since we are at low energy we consider them as corrections to $l_i l_j \gamma$ vertex. So there are only two diagrams: photon mediated and contact interaction.

So at the low energies the conversion process $l_i \to l_j$ can be described by two interactions: lepton flavour violating virtual photon exchange with the quarks in the nucleus as in Figure 17 and the $l_i l_j qq$-contact interaction as in Figure 19. The CLFV process $\mu + q \to e + q$ that can happen through a photon mediation and a contact interaction is described by the following Lagrangian [31]

$$\mathcal{L}_{\mu\to e} = \mathcal{L}_{\mu\to e\gamma} + \mathcal{L}_{\mu\to e}^{non-photo}$$
$$= A\bar{\mu}_R\sigma^{\mu\nu}e_L F_{\mu\nu} + B\bar{\mu}_L\sigma^{\mu\nu}e_R F_{\mu\nu} + \sum_{q=u,d,s\ldots}\Big[(g_{LS}\bar{e}_L\mu_R) + (g_{RS}\bar{e}_R\mu_L)\bar{q}q$$
$$+(g_{LP}\bar{e}_L\mu_R + g_{RP}\bar{e}_R\mu_L)\bar{q}\gamma_5 q + (g_{LV}\bar{e}_L\gamma^\mu\mu_L + g_{RV}\bar{e}_R\gamma^\mu\mu_R)\bar{q}\gamma_\mu q$$
$$(g_{LA}\bar{e}_L\gamma\mu_L + g_{RA}\bar{e}_R\gamma^\mu\mu_R)\bar{q}\gamma_\mu\gamma_5 q + \frac{1}{2}(g_{LT}\bar{e}_L\sigma^{\mu\nu}\mu_R + g_{RT}\bar{e}_R\sigma^{\mu\nu}\mu_L)\bar{q}\sigma_{\mu\nu}q + h.c.\Big],$$

where the A, B, g_{LS}, g_{RS}, g_{LP}, g_{RP}, g_{LV}, g_{RV}, g_{LA}, g_{RA}, g_{LT} and g_{RT} are dimensionful coupling constants.

It depends on the New Physics model which dominates: photonic or the non-photonic interaction. Non-photonic mechanism is important when process $l_i + q \to l_j + q$ can happen at tree level in the New Physics model. In the models with extended gauge groups the lepton conversion can happen through exchange of new gauge boson Z' [37], which is not flavour diagonal. Similarly the charged lepton conversion can happen at the tree level in models with non-diagonal Higgs coupling [38]. In supersymmetric models with broken R-parity the lepton conversion can also happen at tree level [39].

We have now discussed enough about charged lepton conversion. It is time to study the charged lepton decay into a lighter charged lepton and a photon: $l_i \to l_j\gamma$.

4.2 $l_i \to l_j\gamma$ effectively

Let us now consider process $l_i \to l_j\gamma$ where a heavier charged lepton l_i decays into a lighter charged lepton l_j by emitting an on-shell photon. There are three different decays: $\tau \to \mu\gamma$, $\tau \to e\gamma$ and $\mu \to e\gamma$. In one-loop order the effective Lagrangian (25) describes the process $l_i \to l_j\gamma$. The process can happen already at tree level in the effective theory due to operators Q_{eW} and Q_{eB}:

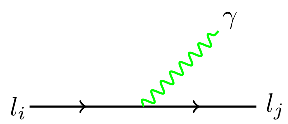

Figure 27: Tree level contribution to $l_i \to l_j\gamma$ generated by operators Q_{eW} and Q_{eB}

Again it might feel good to stay at the tree level to avoid nasty loops and difficult integrals associated with them, but it would be cheating. We again can't know for sure what effective operators are actually generated when the New Physics contributions are integrated out. We must consider all the possible CLFV operators in order to be consistent. We consider those loop diagrams where the CLFV operators first appear, and reject diagrams containing operators that have already appeared at the tree level.

The other CLFV operators contribute at one-loop level through the following diagrams:

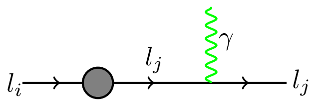

Figure 28: $l_i \to l_j\gamma$ generated by lepton self energy $(Q_{\phi l}^{(1)}, Q_{\phi l}^{(3)}, Q_{\phi e}, llll, llqq)$

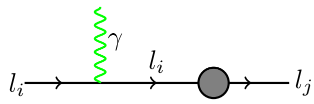

Figure 29: $l_i \to l_j\gamma$ generated by lepton self energy $(Q_{\phi l}^{(1)}, Q_{\phi l}^{(3)}, Q_{\phi e}, llll, llqq)$

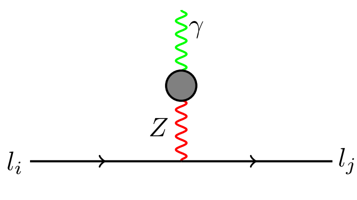

Figure 30: $l_i \to l_j\gamma$ generated by gauge boson self energy $(Q_{\phi l}^{(1)}, Q_{\phi l}^{(3)}, Q_{\phi e})$

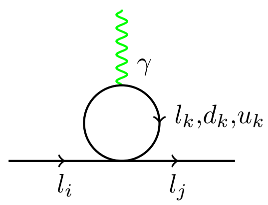

Figure 31: $l_i \to l_j\gamma$ generated by vertex correction $(llll, llqq)$

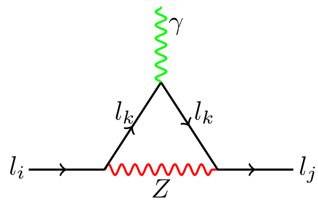

Figure 32: $l_i \to l_j\gamma$ generated by vertex correction ($Q_{\phi l}^{(1)}, Q_{\phi l}^{(3)}, Q_{\phi e}$)

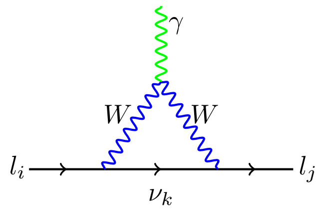

Figure 33: $l_i \to l_j\gamma$ generated by vertex correction ($Q_{\phi l}^{(3)}$)

The diagrams in Figures 28-33 divide into two categories. The Figures 28-30 are corrections to particle propagators and the Figures 31-33 are corrections to the photon-lepton vertex. In the next two subsections we look into these more closely.

4.2.1 Self-energy contributions to $l_i \to l_j\gamma$

The gray blob in diagrams in the Figures 28 and 29 represents the fermion self energy given by the diagrams in the Figures 34, 35 and 36. These fermion self energy diagrams each contain one CLFV vertex. The diagram in Figure 34 contains a CLFV l_il_jZ-vertex which is generated by operators $Q_{\phi l}^{(1)}$, $Q_{\phi l}^{(3)}$ and $Q_{\phi e}$. The l_il_jZ-vertex could also be generated by operators Q_{eW} and Q_{eB}, but since they appear already at the tree level, they are omitted at loop level.

In the diagram in the Figure 35 the violation of charged lepton flavour is introduced by the $l_i\nu_jW$-vertex, which is generated by the operator $Q_{\phi l}^{(3)}$. This vertex could also be generated by the operator Q_{eW}, but this is again neglected since it appears at the tree level already.

The self energy diagram in the Figure 36 contains a charged lepton flavour violating four-lepton vertex, which is generated by the operators in the classes $llll$ and $llqq$.

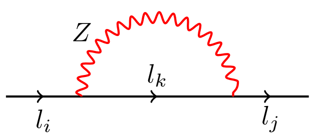

Figure 34: Lepton Z-self-energy ($Q_{\phi l}^{(1)}, Q_{\phi l}^{(3)}, Q_{\phi e}$)

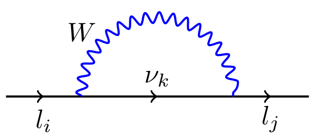

Figure 35: Lepton W-self-energy ($Q_{\phi l}^{(3)}$)

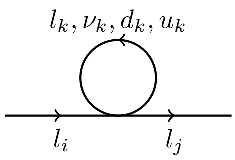

Figure 36: Lepton fermion loop self-energy ($llll, llqq$-classes)

The gray blob in the Figure 30 contains Standard Model Z-photon self-energies where virtual particles contain W-bosons, charged ghosts and charged fermions. The violation of the charged lepton flavour is introduced by the Zl_il_j-

vertex which is generated by the operators $Q_{\phi l}^{(1)}$, $Q_{\phi l}^{(3)}$ and $Q_{\phi e}$. We are again omitting operators Q_{eW} and Q_{eB}.

4.2.2 Vertex correction contributions to $l_i \to l_j\gamma$

Let us now examine the corrections to the $l_i l_j \gamma$-vertex. The vertex corrections are represented in Figures 31, 32 and 33. The diagram in the Figure 31 contains charged fermion loop and it violates charged lepton flavour through four-fermion vertex, which is generated by the diagrams in the $llll$ and $llqq$ classes.

The diagram 32 contains one CLFV vertex $l_i l_j Z$ generated by the operators $Q_{\phi l}^{(1)}$, $Q_{\phi l}^{(3)}$ and $Q_{\phi e}$. The photon vertex $l_k l_k \gamma$ is SM-like; operators Q_{eW} and Q_{eB} have already appeared at tree level.

Finally the diagram in the Figure 33 contains CLFV vertex $l_i \nu_j W$ which is generated by operator $Q_{\phi l}^{(3)}$. The same vertex could also be generated by the operator Q_{eW} but as said it appears already at the tree level.

4.2.3 $l_i \to l_j\gamma$ in a low energy limit

When we study $l_i \to l_j\gamma$ at energies lower than weak scale, the fermion self energies (Figures 34, 35 and 36) become insignificant (the gauge boson masses suppress in the Figures 34 and 35 and the corrections to the $l_i l_j \gamma$ vertex (Figures 31, 32 and 33) merge into one vertex. So at the low energy where all the heavy new physics particles and the usual Standard Model gauge bosons have been integrated out, the process $l_i \to l_j\gamma$ can be studied as a simple tree level process as in Figure 27.

In general the Feynman amplitude of the process $l_i \to l_j\gamma$ can be written in a form :

$$\mathcal{M} = \bar{l}_f V_{ll\gamma}^{fi\mu} l_i \varepsilon_\mu,$$

where the photon-lepton vertex $V_{ll\gamma}^{fi\mu}$ is [14]:

$$\begin{aligned} V_{ll\gamma}^{fi\mu} = \tfrac{i}{\Lambda^2} \Big[& \gamma^\mu (F_{VL}^{fi} P_L + F_{VR}^{fi} P_R) \\ & + (F_{SL}^{fi} P_L + F_{SR}^{fi} P_R) q^\mu + (F_{TL}^{fi} i\sigma^{\mu\nu} P_L + F_{TR}^{fi} i\sigma^{\mu\nu} P_R) q_\nu \Big]. \end{aligned} \tag{26}$$

The factors F_{ab}^{ij} are form factors and q^μ is the momentum of the photon. We are studying process $l_i \to l_j\gamma$ where the photon is real. This gives restrictions to the form of the vertex. The gauge invariance of the electromagnetism states that $q^\mu J_\mu^{em} = 0$, which means that the form factors F_{VL}^{fi} and F_{VR}^{fi} must vanish. For an on-shell photon $\varepsilon^\mu q_\mu = 0$, so the term proportional to q^μ must vanish as well. This means that we are only left with magnetic transition term proportional to $\sigma^{\mu\nu}$:

$$\mathcal{M} = \bar{l}_f \frac{i}{\Lambda^2} (F_{TL}^{fi} i\sigma^{\mu\nu} P_L + F_{TR}^{fi} i\sigma^{\mu\nu} P_R) q_\nu l_i \varepsilon_\mu.$$

The branching ratio of the $l_i \to l_j\gamma$ can now be expressed in terms of the form factors F_{TL}^{fi} and F_{TR}^{fi} [14]:

$$Br[l_i \to l_j\gamma] = \frac{m_{l_i}^3}{16\pi\Lambda^4\Gamma_{l_i}} \left(|F_{TR}^{fi}|^2 + |F_{TL}^{fi}|^2\right),$$

where Γ_{l_i} is the decay width of the muon or the tau. The contributions to the form factors F_{TL}^{fi} and F_{TR}^{fi} from the different diagrams are given in the Appendix F.

4.3 $l_i \to l_j l_k l_l$ effectively

Finally we study processes $l_i \to l_j l_k l_l$, that is processes where a charged lepton decays into three leptons so that individual lepton flavour is violated.

This process can happen already at tree level of the effective theory through following diagrams:

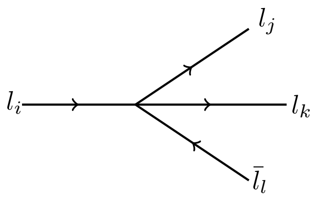

Figure 37: $l_i \to l_j l_k l_l$ with 4-lepton vertex generated by operators in $llll$-class

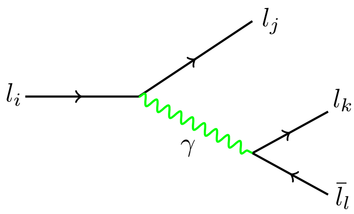

Figure 38: $l_i \to l_j l_k l_l$ mediated by a photon (Q_{eW}, Q_{eB})

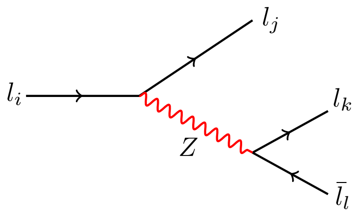

Figure 39: $l_i \to l_j l_k l_l$ mediated by a Z ($Q_{eW}, Q_{eB}, Q_{\phi l}^{(1)}, Q_{\phi l}^{(3)}, Q_{\phi e}$)

The diagram in the Figure 37 contains one CLFV vertex which is generated by PTG operators in the $llll$-class. The two other tree level contributions are mediated by neutral gauge bosons γ and Z through charged lepton flavour violating $l_i l_j \gamma$ and $l_i l_j Z$ couplings, which are generated by LG operators Q_{eW} and Q_{eB} (diagrams in Figures 38 and 39) and PTG operators $Q_{\phi l}^{(1)}, Q_{\phi l}^{(3)}$ and $Q_{\phi e}$ (diagram in Figure 39). There is only one CLFV vertex at the Figures 38 and 39. The CLFV processes are highly suppressed even if they contain only one CLFV vertex. Diagrams containing more than one CLFV vertex are negligible.

Tree level has exhausted all the dimension-six operators but the operators of the $llqq$ class. To be completely general we also have to take the $llqq$ operators into account, even though the $llqq$ operators enter at the loop level. If the fundamental beyond the Standard Model theory does not generate charged lepton flavour violating effective gauge boson couplings the $llqq$ effective vertices might be generated and they are now taken care of. When the New Physics model contains both gauge and $llqq$-effective CLFV vertices the $llqq$ contributions can be dropped since they appear only at the loop level in $l_i \to l_j l_k l_l$. The $llqq$ operators enter at one-loop level through the Feynman diagrams in Figures 40-43.

The diagrams in the Figures 40-43, contain only normal SM gauge boson vertices. We don't take the CLFV gauge boson couplings into the account at the loop level since if the CLFV gauge couplings are significant, the tree level contribution of those couplings will dominate.

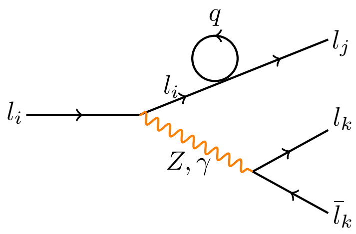

Figure 40: $l_i \to l_j l_k l_l$ mediated by $llqq$ vertex

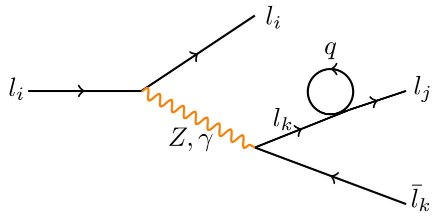

Figure 41: $l_i \to l_j l_k l_l$ mediated by $llqq$ vertex (non-physical)

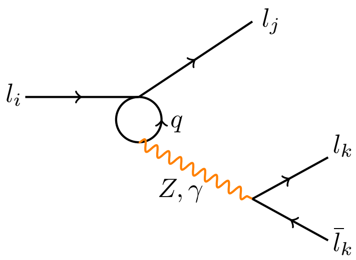

Figure 42: $l_i \to l_j l_k l_l$ mediated by $llqq$ vertex (1PI)

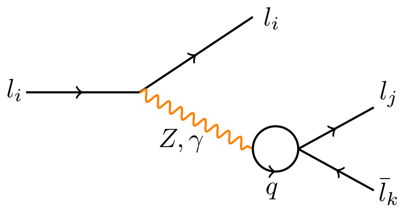

Figure 43: $l_i \to l_j l_k l_l$ mediated by $llqq$ vertex (1PI) (non-physical)

A particle can decay only to particles which are together lighter than the decaying particle itself due to four-momentum conservation and Lorentz invariance. That is in case of 3-body lepton decay $l_i \to l_j l_k l_l$, $m_i > m_j + m_k + m_l$. Since the differences in lepton masses (Table 6) are so big, the decay $l_i \to l_j l_k l_l$ is allowed whenever all the decay products individually are lighter than the decaying particle.

Table 6: Charged lepton masses

Particle	mass
e	$0,511$MeV
μ	$105,6$ MeV
τ	$1,776$ GeV

The diagrams in the Figures 41 and 43 contain $l_i l_i \gamma$ or $l_i l_i Z$ vertices where the initial decaying lepton is involved. This vertex is normal SM vertex and preserves the lepton flavour. This means that the decay products contain the decaying particle itself and that the diagrams in the Figures 41 and 43 are non physical.

The possible charged lepton flavour violating 3-body decays can be classified into three categories [14]:

(A) $\tau^\pm \to e^\pm e^+ e^-$, $\tau^\pm \to \mu^\pm \mu^+ \mu^-$ and $\mu^\pm \to e^\pm e^+ e^-$.

(B) $\tau^\pm \to e^\pm \mu^+ \mu^-$ and $\tau^\pm \to \mu^\pm e^+ e^-$.

(C) $\tau^\pm \to e^\mp \mu^\pm \mu^\pm$ and $\tau^\pm \to \mu^\mp e^\pm e^\pm$.

The category (A) contains processes $l_i^\pm \to l_j^\pm l_j^\pm l_j^\mp$, where the final state contains leptons of the same flavour. All the diagrams 37-39, 40 and 42 are contributing to these processes. The category (B) contains processes $l_i^\pm \to l_j^\pm l_k^\pm l_k^\mp$, with all the three charged leptons present. All the diagrams 37- 39, 40 and 42 are also contributing to these processes. The category (C) contains exotic processes $l_i^\pm \to l_j^\mp l_k^\pm l_k^\mp$ with two leptons of the same electric charge and flavour and one with different flavour and electric charge. Only the four-lepton contact interaction in Figure 37 contributes to the processes in this category, due to the fact that we only allow one effective CLFV vertex per diagram.

It is now possible to calculate the branching ratios to the processes in these categories. The loop diagrams 40 and 42 are neglected. The decaying lepton is always much heavier than it's decay products, so one can neglect the masses of the decay products. When one uses the standard expression for 3-particle phase space [45], the branching ratio can be obtained [14]:

$$
\begin{aligned}
Br(l_i \to l_j l_k l_l) = \frac{N_c M^5}{6144\pi^3 \Lambda^4 \Gamma_{l_i}} \Big[& 4(|C_{VLL}|^2 + |C_{VRR}|^2 + |C_{VLR}|^2 + |C_{VRL}|^2) \\
& + |C_{SLL}|^2 + |C_{SRR}|^2 + |C_{SLR}|^2 + |C_{SRL}|^2 \\
& + 48(|C_{TL}|^2 + |C_{TR}|^2) + X_\gamma \Big], \qquad (27)
\end{aligned}
$$

where $N_c = 1/2$ if two of the final state leptons are the same and $N_c = 1$ otherwise and M is the mass of the initial lepton. The quantities C_X and X_γ are combinations of the Wilson coefficients and their explicit expressions are given in the Appendix G. The quantity Γ_{l_i} is the total decay width of the initial lepton l_i[4].

The quantities C describe the contributions of the Wilson coefficients of the effective operators to the amplitude of the process. When the amplitude to the process in question was calculated in [14], the non-photonic part was expressed in the following basis of quadrilinears:

[4] $\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3}$

$$
\begin{aligned}
&O_{VXY} = \gamma^\mu P_X \times \gamma^\mu P_Y \\
&O_{SXY} = P_X \times P_Y \\
&O_{TX} = \sigma^{\mu\nu} \times \sigma^{\mu\nu} P_X,
\end{aligned}
$$

where the X and Y represent the chiralities L and R. The C are contributions corresponding to those operators.

This concludes our review of the charged lepton flavour violating processes. It is now time to move on and specify the new physics model, in order to make more accurate predictions about the CLFV observables. The supersymmetric models are still one of the most promising candidates for the theories beyond the Standard Model, even though the superpartners have eluded detection so far. We will therefore choose the theory beyond SM to be supersymmetric. Supersymmetric models allow many new potential sources for the charged lepton flavour violation. In the next section we will discuss about charged lepton flavour violation in the supersymmetry context in general and then in section 5.2 we will make a more specific choise.

5 CLFV and SUSY

In the Standard Model of particle physics the lepton flavour is absolutely conserved. If SM is extended to include supersymmetry[5] in a minimal way i.e. that one adds a superpartner for every particle in the theory and also adds a second Higgs doublet, one gets the so called *Minimal Supersymmetric Standard Model* (MSSM). The lepton flavour will be conserved also in the MSSM. However it is known that the neutrinos actually are massive unlike in SM and its extension MSSM. The mass matrix of the neutrinos is not diagonal and the neutrinos are allowed and known to mix with each other, that is the lepton flavour is not absolutely conserved. As explained in the introduction, the mixing of neutrinos or "neutrino oscillations" could also induce flavour violation in the charged lepton sector.

The SM must be extended to include massive neutrinos. If one tried to include the massive neutrino in the SM as a Dirac particle and to suggest that it gets its mass in the spontaneous breaking of the electroweak symmetry, one would run into fine-tuning problem with the neutrino Yukawa coupling: it would have to be of the order of 10^{-12}. A popular way of introducing massive neutrinos without introducing an unnecessary fine-tuning, is the so called *Seesaw Mechanism* (type-I)[6]. In the Seesaw mechanism one assumes that neutrino has a right-handed Majorana mass as well as a Dirac mass. The effective mass of the left-handed neutrino becomes very small (around 1 eV) and the effective mass becomes huge (near $\Lambda_{GUT} \sim 10^{16}$ GeV scale). MSSM can also be extended to have massive neutrinos through seesaw mechanism. When the SM is extended to have massive neutrinos through seesaw mechanism it has two sources of lepton flavour violation: the electron and the neutrino Yukawa couplings $\mathbf{f_e}$ and $\mathbf{f}_\nu$ (both of them are needed for the realization of charged lepton flavour violation).

The MSSM has supersymmetry incorporated in it which means that every particle has a superpartner of different statistics (boson's partner is fermion and fermion's boson). These superpartners have the same masses and quantum numbers as the original particles, except that their spins differ by 1/2. We are not observing this kind of particles so if the supersymmetry is to be a true symmetry of nature, the supersymmetry must be broken. The MSSM does not give the explicit mechanism of SUSY symmetry breaking, but parametrizes it in the so called *soft* SUSY-breaking terms [48]:

$$\begin{aligned}
\mathcal{L}_{soft} = & -\frac{1}{2}(M_3\widetilde{g}\widetilde{g} + M_2\widetilde{W}\widetilde{W} + M_1\widetilde{B}\widetilde{B} + c.c.) \\
& -\left(\widetilde{\bar{u}}\mathbf{a_u}\widetilde{Q}H_u - \widetilde{\bar{d}}\mathbf{a_d}\widetilde{Q}H_d - \widetilde{\bar{e}}\mathbf{a_e}\widetilde{Q}H_d + c.c.\right) \\
& -\widetilde{Q}^\dagger\mathbf{m^2_{\widetilde{Q}}}\widetilde{Q} - \widetilde{L}^\dagger\mathbf{m^2_{\widetilde{L}}}\widetilde{L} - \widetilde{\bar{u}}^\dagger\mathbf{m^2_{\widetilde{\bar{u}}}}\widetilde{\bar{u}} - \widetilde{\bar{d}}^\dagger\mathbf{m^2_{\widetilde{\bar{d}}}}\widetilde{\bar{d}} - \widetilde{\bar{e}}^\dagger\mathbf{m^2_{\widetilde{\bar{e}}}}\widetilde{\bar{e}} \\
& -m^2_{H_u}H_u^*H_u - m^2_{H_d}H_d^*H_d - (bH_uH_d + c.c.). \qquad (28)
\end{aligned}$$

The soft part consists of the gaugino mass terms (M_1, M_2 and M_3), sfermion mass terms ($\mathbf{m^2_x}$, $\mathbf{x} = \widetilde{\mathbf{Q}}, \widetilde{\mathbf{L}}, \widetilde{\bar{\mathbf{u}}}, \widetilde{\bar{\mathbf{d}}}, \widetilde{\bar{\mathbf{e}}}$), $(scalar)^3$ interaction terms ($\mathbf{a_u}$, $\mathbf{a_d}$, $\mathbf{a_e}$) and the supersymmetry breaking contributions to the Higgs potential at the

[5]The basic properties of supersymmetric models are presented in the Appendix B

[6]The non-SUSY seesaw-I mechanism is presented in the Appendix C

last line. The soft terms give the masses to all gaugino and scalar fields in the theory and violate supersymmetry since the terms don't contain fields and also their superpartners. The soft terms are not present at high energy where the supersymmetry is realized. They are created when the supersymmetry breaks at a very high energy.

The soft terms contain many potential sources of LFV at electroweak scale (at high SUSY scale the soft terms vanish and the LFV due to them disappears). The $(scalar)^3$ terms become sfermion mass terms when the Higgs fields acquire VEVs. Without any principle to tell otherwise, the scalar-coupling matrices are off-diagonal and have entries with same order of magnitude. The same goes with the sfermion mass matrices ($\mathbf{m^2_x}$). The off-diagonal terms which are of the same size as the diagonal ones pose a big problem. The transition rates for flavour violating processes would be huge and contradicting the experimental observations. The off-diagonal terms should be almost non-existent to match with the experiments. The problem is that it is known that the flavour violation effects are extremely small and there is no explanation to that even though there are many sources of flavour violation in the soft SUSY-breaking terms. This is called the *flavour problem.*

There are many possible "solutions" to the flavour problem: one can make assumptions of the SUSY-breaking terms, such that the CLFV processes become suppressed. One could allow large off-diagonal soft SUSY-breaking terms if one assumes that the sfermions are heavy. Then the flavour violating processes would be heavily suppressed. In *flavour universality* scenario the sfermion mass matrix is proportional or almost proportional to unit matrix and in the *alignment* scenario the sfermion mass matrix is proportional or almost to the lepton mass matrix. But these are just assumptions. Even though they produce the desired result of small CLFV rates, there should be a mechanism that produces them, i.e. there should be a *supersymmetry-breaking mechanism.*

One of the most popular properties of the SUSY breaking models is that it is *flavour blind* i.e. it produces the flavour universality scenario. In that scenario the only source of flavour violation is due to the Yukawa couplings of the theory, which is also called *Minimal Flavour Violation* (we assumed this in previous sections where we discussed CLFV in effective theories). So in this case the soft terms don't cause any additional contributions to the flavour violation and the various mass and coupling matrices are flavour diagonal in the basis of the fermion mass eigenstates, e.g.,

$$\mathbf{m^2_Q} = m^2_{\tilde{Q}}\mathbf{1}, \qquad \mathbf{m^2_{\bar{u}}} = m^2_{\tilde{\bar{u}}}\mathbf{1}, \qquad \mathbf{m^2_{\bar{d}}} = m^2_{\tilde{\bar{d}}}\mathbf{1}, \qquad \mathbf{m^2_L} = m^2_{\tilde{L}}\mathbf{1}, \qquad \mathbf{m^2_{\bar{e}}} = m^2_{\tilde{\bar{e}}}\mathbf{1}$$

$$\mathbf{a_u} = A_{u0}\mathbf{y_u}, \qquad \mathbf{a_d} = A_{d0}\mathbf{y_d}, \qquad \mathbf{a_e} = A_{e0}\mathbf{y_e},$$

in some high energy scale Q_0, where the supersymmetry breaks. The $\mathbf{y_u}$, $\mathbf{y_d}$ and $\mathbf{y_e}$ are the SM Yukawa couplings.

Even though these are true at the input scale Q_0, they do not hold at the electroweak scale anymore. Renormalization group evolution can make the non-diagonal soft terms to differ from zero even though they were absolutely zero at the input scale. We have a discussion about renormalization group evolution in the Appendix D. Next we will briefly discuss general properties of

SUSY-breaking and then we will review two of the most popular SUSY-breaking models: *gravity-* and *gauge-mediated* supersymmetry-breaking models.

5.1 SUSY-breaking models

In SUSY it is hard to do generic analysis with the free parameters of the theory because there is so many of them (124 [49]). Therefore one has to often rely on analysis using a specific model which restricts the number of free parameters. We will now concentrate our attention to two specific models of supersymmetry-breaking: gauge and gravity mediated supersymmetry breaking mechanisms. In them the SUSY-breaking is spontaneous.

Models in which the spontaneous symmetry breaking of SUSY is due to spontaneous breaking in the F- and/or D-terms are called the "visible sector" SUSY-breaking models[7]. In them the SUSY is broken by known SUSY particles and there is no need of introduction of new particles to break SUSY. These kind of models however fail to give out acceptable particle mass spectrum. Some other kind of a model is therefore needed. Because the SUSY breaking in the visible sector (the known MSSM particles) fails to deliver the viable SUSY partner masses, one could suggest that the SUSY breaking is due to yet unknown particles, called the "hidden sector". There should not be any renormalizable couplings between the visible and the hidden sector. The symmetry breaking is communicated to the visible sector by the "messenger fields". Those interactions should be highly suppressed which could suggest that the scale of the SUSY breaking is larger than the TeV scale. When the SYSY-breaking is communicated to the visible sector by the messenger fields, the soft SUSY-breaking terms, $\mathcal{L}_{soft}$, are generated. The SUSY-breaking scale is so large that one needs to use renormalization group running to obtain the low-energy values for the soft parameters.

The different SUSY breaking mechanisms differ in how the SUSY breaking is mediated to the observable sector. In the gravity mediation the soft terms are generated by couplings which vanish as $m_{pl} \to \infty$, i.e. there is no quantum gravity at the low energy scale. In gauge mediation the soft terms arise due to loop diagrams which contain new fields that couple both to the hidden and to the visible sectors. Let us now look into gravity-mediated SUSY-breaking model more closely.

5.1.1 Gravity mediated supersymmetry breaking

In globally supersymmetric models the parameter of the supersymmetric transformations is the same in every point in space-time. When one promotes the SUSY transformation parameter to be a function of space-time, and demands that the supersymmetry transformation still is a symmetry of the theory, one obtains supersymmetric theory of gravitation. The supersymmetry transformation represents now the general coordinate transformation and by demanding that the physics remains invariant under it, leads to quantum gravity, analogous to classical Einstein's gravity.

One usually assumes that the theory contains only one supersymmetry and then the local supersymmetry is called $N = 1$ supergravity. The $N = 1$ supergravity is not renormalizable however, so it is only an effective field theory of

[7] The F- and D-terms are discussed in more detail in the Appendix B

gravity. Gravity couples to all particles so it mediates the SUSY-breaking from the hidden sector to the visible sector. So in the gravity mediated supersymmetry breaking the supersymmetry is spontaneously broken in the hidden sector and it is communicated to the visible sector by non-renormalizable contact interactions. The true renormalizable theory of gravitation should be realized at the Planck scale so the interactions which mediate the SUSY-breaking must be suppressed by the Planck-scale.

If the supersymmetry is broken by the F-term VEV $\langle F \rangle$ in the hidden sector the soft supersymmetry breaking masses should be approximately:

$$m_{\text{soft}} \sim \frac{\langle F \rangle}{M_{Pl}}.$$

The soft terms should vanish when the supersymmetry is not broken. At Planck scale the supersymmetry must not be broken and indeed when the gravity becomes insignificant ($M_{Pl} \to \infty$) the soft terms vanish (at the limit where the gravity vanishes there is no interaction that could mediate the SUSY-breaking to the visible sector). Also when the supersymmetry is not broken ($\langle F \rangle \to 0$) the soft mass terms vanish.

Let us discuss supergravity in the context of supersymmetry breaking briefly. Gravitational interactions are non-renormalizable. The Lagrangian describing non-renormalizable interactions is more general than the Lagrangian describing only the renormalizable ones. The non-renormalizable Lagrangian describing the gravity will be invariant under (local)supersymmetry and the gauge transformations[8]. Let us assume that one F-term of the hidden sector chiral superfields acquires a VEV. This breaks the supersymmetry. Let the chiral superfield responsible for supersymmetry breaking be X. Then the superpotential, Kähler potential and the gauge kinetic function for supersymmetry breaking scenario are [48]:

$$\begin{aligned} W &= W_{\text{MSSM}} - \frac{1}{M_{Pl}}\left(\frac{1}{6}y^{Xijk}X\Phi_i\Phi_j\Phi_k + \frac{1}{2}\mu^{Xij}X\Phi_i\Phi_j\right) + \ldots && (29)\\ K &= \Phi^{*i}\Phi_i + \frac{1}{M_{Pl}}(n_i^j X + \bar{n}_i^j X^*)\Phi^{*i}\Phi_j - \frac{1}{M_{Pl}^2}k_i^j XX^*\Phi^{*i}\Phi_j + \ldots && (30)\\ f_{ab} &= \frac{\delta_{ab}}{g_a^2}\left(1 - \frac{2}{M_{Pl}}f_a X + \ldots\right). && (31) \end{aligned}$$

The fields Φ_i are the chiral superfields of the MSSM or its extension. Parameters $y^{Xijk}, k_j^i, n_j^i, \bar{n}_j^u$ and f_a are dimensionless couplings and the μ^{Xij} has dimension of mass. When the auxiliary fields of Φ_i's are integrated out and the F-term of X has acquired a VEV the supersymmetry breaking Lagrangian becomes [48]:

[8]The most general Lagrangian describing non-renormalizable supersymmetry, respecting gauge invariant interactions is: $\mathcal{L} = [K(\Phi_i, \widetilde{\Phi}^{*i})]_D + \left(\left[f_{ab}(\Phi_i)\hat{\mathcal{W}}^a\hat{\mathcal{W}}^b + W(\Phi_i)\right]_F + c.c.\right)$. This depends on three quantities: superpotential W, Kähler potential K and the gauge kinetic function $F_{ab}(\Phi_i)$. In general the superpotential is a holomorphic fuction of chiral superfields. In general Kähler potential is a function of chiral and antichiral superfield, and it does not have to be polynomial. The gauge kinetic function is in general a holomorphic function of the chiral superfields.

$$\mathcal{L}_{\rm soft} = \left(-\frac{\langle F_X \rangle}{2M_{Pl}} f_a \lambda^a \lambda^a - \frac{\langle F_X \rangle}{6M_{Pl}} y^{Xijk} \phi_i \phi_j \phi_k - \frac{\langle F_X \rangle}{2M_{Pl}} \mu^{Xij} \phi_i \phi_j \right. \quad (32)$$

$$\left. -\frac{\langle F_X \rangle}{M_{Pl}} n_i^j \phi_j W^i_{\rm MSSM} + c.c. \right) - \frac{|\langle F_X \rangle|^2}{M_{Pl}^2} (k^i_j + n^i_p \bar{n}^p_j) \phi^{*j} \phi_i, \quad (33)$$

where the ϕ_i and λ^a are the scalar and the gaugino fields of MSSM or its extension. One can now compare this to the general soft terms (90) and acquire the soft SUSY-breaking parameters:

$$M_a = \frac{\langle F_X \rangle}{M_{Pl}} f_a, \quad (34)$$

$$a^{ijk} = \frac{\langle F_X \rangle}{M_{Pl}} (y^{Xijk} + n^i_p y^{pjk} + n^j_p y^{pik} + n^k_p y^{pij}), \quad (35)$$

$$b^{ij} = \frac{\langle F_X \rangle}{M_{Pl}} (\mu^{Xij} + n^i_p \mu^{pj} + n^j_p \mu^{pi}), \quad (36)$$

$$(m^2)^i_j = \frac{|\langle F_X \rangle|^2}{M_{Pl}^2} (k^i_j + n^i_p \bar{n}^p_j). \quad (37)$$

The soft SUSY-breaking parameters depend on arbitrary parameters y^{Xijk}, k^i_j, $n^i_j, \bar{n}^u_j, f_a$ and μ^{Xij}, because supergravity is only an effective theory. The fundamental theory of quantum gravity is not known so there is no way of calculating these parameters from the theory. These parameters can therefore have whatever magnitude. This is not phenomenologically viable. The soft SUSY-breaking parameters a^{ijk} and $(m^2)^i_j$ have off-diagonal terms of the sfermion mass matrices, which are sources of flavour violation. Since these off-diagonal terms can be in principle as large as the diagonal ones, the rates of flavour violating processes could be huge. This is clearly violating the observational fact that the flavour violating processes among both leptons and quarks are extremely suppressed. The off-diagonal terms in the squark and the slepton mass matrices should be very close to zero, in order to the predictions match the experiments.

In order to get the phenomenologically desirable negligible off-diagonal soft SUSY-breaking terms one has to make assumptions. Usually one assumes that the $k^j_i = k\delta^j_i$ and $n^j_i = n\delta^j_i$, with k and n real; a common $f_a = f$ for all the gauginos and that the couplings y^{Xijk} and μ^{Xij} are proportional to the corresponding superpotential parameters: $y^{Xijk} = \alpha y^{ijk}$ and $\mu^{Xij} = \beta\mu^{ij}$, where the parameters α and β are real and common to all particles. The scenario in which one makes these assumptions to the soft terms is commonly called the *minimal supergravity* (MSUGRA). With these assumptions we find that there are only four parameters describing the soft SUSY-breaking terms. By using equations (34) and our assumptions we can define the following parameters:

$$m_{1/2} \equiv f\frac{F_X}{M_{Pl}} \tag{38}$$

$$m_0^2 \equiv (k+n^2)\frac{|\langle F_X\rangle|^2}{m_{Pl}^2} \tag{39}$$

$$A_0 \equiv (\alpha+3n)\frac{\langle F_X\rangle}{M_{Pl}} \tag{40}$$

$$B_0 \equiv (\beta+2n)\frac{\langle F_X\rangle}{M_{Pl}}. \tag{41}$$

These parameters determine the soft SUSY-breaking masses and couplings in (34). We see that in MSUGRA all the gauginos have same masses and also all the scalars have same masses. The trilinear couplings will be proportional to the corresponding Yukawa couplings and the parameter b will be proportional to the superpotential parameter μ. These can be summarized in the following equations:

$$M_3 = M_2 = M_1 = m_{1/2}, \tag{42}$$

$$\mathbf{m^2_{\tilde{Q}}} = \mathbf{m^2_{\tilde{\bar{u}}}} = \mathbf{m^2_{\tilde{\bar{d}}}} = \mathbf{m^2_{\tilde{L}}} = \mathbf{m^2_{\tilde{\bar{e}}}} = m_0^2\mathbf{1}, \tag{43}$$

$$m^2_{H_u} = m^2_{H_d} = m_0^2, \tag{44}$$

$$\mathbf{a_u} = A_0\mathbf{y_u}, \qquad \mathbf{a_d} = A_0\mathbf{y_d}, \qquad \mathbf{a_e} = A_0\mathbf{y_e}, \tag{45}$$

$$b = B_0\mu. \tag{46}$$

These equations hold at the input scale, from which they have to be RG evolved down to the electroweak scale in order to compare them with the experiments. The input scale is usually taken to be the the GUT scale rather than the Planck scale. Why the input scale is not set to Planck scale then? In MSSM the SM gauge couplings can be unified at the GUT scale, $M_{GUT} \approx 2\times10^{16}$ GeV. The RG evolution is clear up to this point, but not so clear between the GUT scale and the Planck scale. The lack of knowledge is the reason to start the RG evolution from lower energy, even though it gives error proportional to $ln(M_{Pl}/M_{GUT})$.

The explicit supersymmetry-breaking models are used when one wants to explain the smallness of the off-diagonal soft SUSY-breaking terms. If one uses gravity mediated SUSY-breaking models, one does not immediately acquire the wanted flavour violating parameters. One again has to make assumptions about the parameters of the supergravity model used, in order to get the wanted negligible off-diagonal soft terms. So the gravity mediated models by themselves do not ensure the universality (equations (42)-(46)) or the flavourblindness of the SUSY-breaking terms. Maybe in the future the full theory of quantum gravity is known and the soft SUSY-breaking terms can be calculated, but at this moment it is not possible.

The gravity-mediated SUSY-breaking model was the first supersymmetry breaking model (1982)[87]. Even though it successfully breaks the supersymmetry, it leaves the question of flavour problem unanswered. We would like to have a SUSY-breaking model that would naturally produce the negligible off-diagonal soft mass terms of the sfermions. This kind of a model exists, and it is one of the most popular SUSY-breaking models. It is called the gauge-mediated supersymmetry-breaking model and it is our next topic.

5.1.2 Gauge mediated supersymmetry breaking

In gauge mediated supersymmetry-breaking models the supersymmetry is broken by a non-zero scalar VEV in the hidden sector. The hidden sector particles do not interact with the visible sector particles (or interact very weakly). In the gauge mediated models one additional sector is assumed. This sector is called the *messenger sector*. Messenger sector particles couple both to the hidden sector as well as the SM gauge fields. The supersymmetry-breaking is communicated to the visible sector by radiative corrections. The supersymmetry is unbroken in the MSSM sector at the tree-level. This ensures that the mass sum rule (Appendix B, 89) holds at the tree-level and that we don't get problems with the superpartner mass spectrum. The supersymmetry breaking is communicated to the visible sector by radiative corrections involving messenger fields in the loops to the visible sector particle propagators. This is how the soft SUSY-breaking terms are generated. The gaugino masses are generated at the one-loop order and the soft scalar masses are generated at the two-loop order. The trilinear couplings are also generated at the two-loop order.

The gaugino and soft scalar mass terms are generated as radiative corrections to the propagators of those particles. The messenger sector directly couples to the gauginos, which allows them to get mass terms at one-loop level through the following diagram:

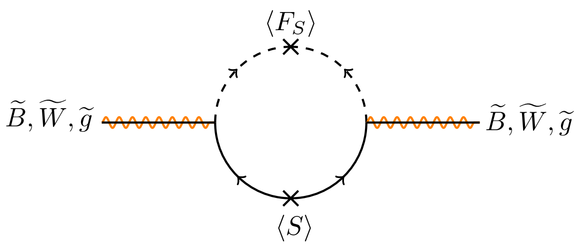

Figure 44: The gaugino propagator

The soft scalar masses are generated at two-loops[9]. The trilinear scalar couplings soft terms are effective operators generated at two-loop level.

In the simplest gauge mediated model the messenger fields are left-handed chiral supermultiplets $q, \bar{q}, l$ and $\bar{l}$ that transform under MSSM gauge group $SU(3)_C \times SU(2)_L \times U(1)$ as:

$$q \sim (\mathbf{3}, \mathbf{1}, -\frac{1}{3}), \qquad \bar{q} \sim (\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3}), \qquad l \sim (\mathbf{1}, \mathbf{2}, \frac{1}{2}), \qquad \bar{l} \sim (\mathbf{1}, \mathbf{2}, -\frac{1}{2}).$$

The fermions and scalars in these supermultiplets are called messenger quarks $\psi_q, \psi_{\bar{q}}$, messenger squarks $\widetilde{q}, \widetilde{\bar{q}}$, messenger leptons $\psi_l, \psi_{\bar{l}}$ and messenger sleptons $\widetilde{l}, \widetilde{\bar{l}}$. These messenger fields couple to the hidden sector gauge-singlet chiral multiples S via the following superpotential [52], [48]:

$$W_{\text{mess}} = y_2 S l \bar{l} + y_3 S q \bar{q}. \qquad (47)$$

[9] The eight contributing diagrams are sketched in [48].

The supersymmetry is spontaneously broken when the scalar component of S and its auxiliary field both acquire VEVs. Since S is chiral superfield,VEV of its auxiliary field corresponds to a F-term breaking. The messenger fields get masses when the supersymmetry is spontaneousy broken[10]. The fermion and scalar messenger masses become:

$$l, \bar{l}: \qquad m^2_{\text{fermion}} = |y_2\langle S\rangle|^2, \qquad m^2_{\text{scalar}} = |y_2\langle S\rangle|^2 \pm |y_2\langle F_S\rangle|,$$
$$q, \bar{q}: \qquad m^2_{\text{fermion}} = |y_3\langle S\rangle|^2, \qquad m^2_{\text{scalar}} = |y_3\langle S\rangle|^2 \pm |y_3\langle F_S\rangle|,$$

Since now the messenger fields are massive, the loop corrections to gaugino and scalar propagators, involving messenger fields, are not zero. When one computes the diagram in Figure 44 one gets the gaugino mass:

$$M_a = \frac{g_a^2}{16\pi^2}\frac{\langle F_S\rangle}{\langle S\rangle}, \qquad (a = 1, 2, 3).$$

There are no one-loop corrections to the MSSM scalars so the scalars get their masses at the two-loop diagrams. When one calculates the loops one gets the scalar masses:

$$m^2_{\phi_i} = 2\frac{\langle F_S\rangle^2}{\langle S\rangle^2}\left[\left(\frac{g_1^2}{16\pi^2}\right)^2 C_1(i) + \left(\frac{g_2^2}{16\pi^2}\right)^2 C_2(i) + \left(\frac{g_3^2}{16\pi^2}\right)^2 C_3(i)\right],$$

where the $C_a(i)$'s are the quadratic Casimir invariants. The scalar squared masses contain extra loop factor $g_a^2/16\pi^2$ compared to the gaugino masses. These are however scalar *squared* masses so the scalar mass has the same order of magnitude as the gaugino mass. Because the gauge interactions are flavour blind the soft mass matrices created are flavour diagonal. The scalar masses are not universal (as in the gravity mediated case) however: the masses of different generations of squarks are the same for example, but that mass is not the same as the mass of the charged sleptons for example.

The trilinear scalar couplings $\mathbf{a_u}, \mathbf{a_d}$ and $\mathbf{a_e}$ are generated at the two-loop level. The trilinear scalar couplings contain one additional loop factor $g_a^2/16\pi^2$ compared to the gaugino masses. One can therefore approximate $\mathbf{a_u} = \mathbf{a_d} = \mathbf{a_e} = 0$ to a good accuracy at the messenger scale. Even though the trilinear couplings are zero at the input scale the RG evolution will generate non-zero trilinear couplings as one runs them from the input scale down to the electroweak scale[11].

The hidden sector also of course couples to the gravity as well. The gravity is however much weaker than the gauge interactions so it can be neglected in the gauge mediated supersymmetry-breaking.

So the gauge-mediation produces completely diagonal sfermion mass matrices. If there is no other source of flavour violation present, the fermion flavour

[10]The superpotential (47) determines the masses of the messengers. Messenger fermion masses are determined from the mass term Lagrangian $\mathcal{L}_{\text{mass}} = -y_2 S\psi_l\psi_{\bar{l}} - y_3\psi_q\psi_{\bar{q}} + c.c.$ The scalar messenger masses are determined by the following scalar potential: $V = \left|\frac{\delta W_{mess}}{\delta l}\right|^2 + \left|\frac{\delta W_{mess}}{\delta \bar{l}}\right|^2 + \left|\frac{\delta W_{mess}}{\delta q}\right|^2 + \left|\frac{\delta W_{mess}}{\delta \bar{q}}\right|^2 + \left|\frac{\delta}{\delta S}(W_{mess} + W_{breaking})\right|^2$

[11]The RG evolution of SUSY parameters is discussed in the Appendix D

will be absolutely conserved at low energies, like in MSSM. However the neutrinos are known to be massive and those masses must be incorporated into the supersymmetric extension of the Standard Model. The addition of neutrino masses does not change the soft SUSY-breaking terms that result from the gauge-mediation, but they might change the renormalization group evolution of the off-diagonal slepton mass terms, making them to deviate from zero at the low energy scale. This is our next topic.

5.2 CLFV in supersymmetric seesaw-I model

As we have said the MSSM does not contain massive neutrinos which we know to exist. In the Standard Model the neutrinos are massless and there is no source of lepton flavour violation. In MSSM the same is true. In supersymmetric theories the soft SUSY-breaking terms contain potential sources for lepton flavour violation in the slepton mass matrices. If the slepton mass matrices are off-diagonal the lepton flavour is violated because in supersymmetric models the leptons couple to the their scalar partners, sleptons, through the *lepton-slepton-gaugino* vertices:

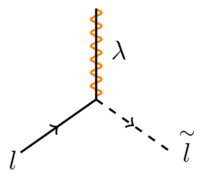

Figure 45: lepton-slepton-gaugino vertex

It is however experimentally known that the lepton flavour violating processes are highly suppressed. This gives stringent constraints to the off-diagonal SUSY-breaking terms, leading to the flavour problem. In order to get out the phenomenologically correct highly suppressed LFV interactions the off-diagonal SUSY-breaking terms must be close to zero or they can be proportional to the diagonal entries as long as the slepton masses are very heavy so that they effectively decouple. There should be some kind of mechanism that produces the wanted phenomenological outcome. There is no consensus about the mechanism which produces the small CLFV parameters.

5.2.1 Extension of MSSM via seesaw-I mechanism

The massive neutrinos can be incorporated into the MSSM using e.g. the seesaw type-I mechanism, though many other ways exist. In this model the massive neutrinos are introduced by adding three right-handed Majorana masses m_{M_1}, m_{M_2} and m_{M_3}. The right handed neutrinos couple to the lepton doublets via a new Yukawa coupling Y_ν. The soft SUSY-breaking terms will now also involve new trilinear coupling $\mathbf{a}_\nu$ and new soft mass matrix, $m^2_{\widetilde{N}}$, for right-handed neutrinos. So new terms must be added to the MSSM superpotential W and to the MSSM soft terms $\mathcal{L}_{soft}$. The leptonic part of the MSSM superpotential W_{MSSM} and the soft terms $\mathcal{L}_{MSSM}$ are:

$$W_{MSSM} = E^c Y_l L H_1$$
$$\mathcal{L}_{MSSM} = -\widetilde{L}^\dagger \mathbf{m}_{\widetilde{\mathbf{L}}} \widetilde{L} - \widetilde{e}^\dagger \mathbf{m}_{\widetilde{\mathbf{e}}} \widetilde{e} + (-\widetilde{e}^\dagger \mathbf{a_e} \widetilde{L} H_1 + h.c.).$$

The new terms needed are:

$$\Delta W = N^c Y_\nu L H_2 + \frac{1}{2} N^c \mathbf{m_M} N^c$$

$$\Delta \mathcal{L} = -\widetilde{N}^c \mathbf{m}_{\widetilde{\mathbf{N}}} \widetilde{N}^{c\dagger} + (-\widetilde{N}^c \mathbf{a}_\nu \widetilde{L} H_2 + h.c.) + (-\frac{1}{2} \widetilde{N}^c \mathbf{b}_\nu \widetilde{N}^c + h.c.).$$

The fields are in the basis where the charged lepton Yukawa matrix Y_l and right-handed Majorana mass matrix $\mathbf{m_M} = diag(m_{M_1}, m_{M_2}, m_{M_3})$ are diagonal. They can be made diagonal by performing unitary transformations of L, E and N. When we choose to diagonalize Y_l and $\mathbf{m_M}$, the neutrino Yukawa coupling Y_ν can not be diagonalized and is therefore off-diagonal in general. This gives rise to lepton flavour violation.

In the seesaw-I mechanism there are two different sources of mass: the Dirac mass term, which appears when the electroweak symmetry breaks spontaneously and the right-handed Majorana mass term. After the electroweak symmetry breaking, the charged lepton and the Dirac neutrino mass matrices can be written in a form [53]:

$$\mathbf{m_l} = \mathbf{Y_l} v_1, \qquad \mathbf{m_D} = \mathbf{Y}_\nu v_2,$$

where v_1 and v_2 are the vacuum expectation values of the neutral Higgs scalars:

$$v_1 = v \cos\beta \qquad v_2 = v \sin\beta, \qquad v = 174 \text{ GeV}.$$

There are six neutrino masses: three Dirac masses and three Majorana masses. Together they form a 6×6 neutrino mass matrix:

$$\mathbf{M}^\nu = \begin{pmatrix} 0 & \mathbf{m_D}^T \\ \mathbf{m_D} & \mathbf{m_M} \end{pmatrix}.$$

Only the right-handed neutrinos are assumed to have Majorana masses, hence the zero in the upper left corner. The physical neutrino masses are the eigenvalues of this mass matrix. There will be three light, ν_i and three heavy, N_i physical neutrinos. In the seesaw-I one assumes that the neutrino Majorana mass is very high, order of grand unification scale 10^{16}GeV. The neutrino Dirac mass is therefore much smaller than the Majorana mass. One can therefore approximate the physical light neutrino masses as [55]:

$$\mathbf{m}_\nu \approx -\mathbf{m_D}^T \mathbf{m_M}^{-1} \mathbf{m_D}. \tag{48}$$

Our basis was so that the $\mathbf{m_M}$ is diagonal so the heavy eigenstates are also diagonal:

$$\mathbf{m_N} = \text{diag}(m_{N_1}, m_{N_2}, m_{N_3}) \approx \mathbf{m_M}.$$

The light neutrino states are those that are observed in the experiments. The heavier neutrinos are so heavy that they effectively decouple and are therefore not seen in the experiments. Since the light neutrino states are physically observable, the light neutrino mass matrix can be diagonalized using the standard *Pontecorvo-Maki-Nakagawa-Sakata* matrix (PMNS matrix) [88],[89]:

$$\mathbf{m}_\nu{}^{\rm diag} = \mathbf{U}_{PMNS}^T \mathbf{m}_\nu \mathbf{U}_{PMNS} = {\rm diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}).$$

The Dirac mass of the neutrino can be solved from the equation (48) [60]:

$$\mathbf{m_D} = i\sqrt{\mathbf{m_N}^{diag}}\mathbf{R}\sqrt{\mathbf{m}_\nu{}^{diag}}\mathbf{U}_{PMNS}^\dagger. \quad (49)$$

The matrix $\mathbf{R}$ is 3×3 complex orthogonal matrix and it represents the possible mixing in the right-handed neutrino sector (The PMNS matrix $\mathbf{U}_{PMNS}$ represents the mixing in the left-handed sector). The matrix $\mathbf{R}$ can be parametrized in terms of three complex angles θ_i, $(i = 1, 2, 3)$ as

$$R = \begin{pmatrix} c_2c_3 & -c_1s_3 - s_1s_2c_3 & s_1s_3 - c_1s_2c_3 \\ c_2s_3 & -c_1c_3 - s_1s_2s_3 & -s_1c_3 - c_1s_2s_3 \\ s_2 & s_1c_2 & c_1c_2 \end{pmatrix}, \quad (50)$$

where $c_i = \cos\theta_i$ and $s_i = \sin\theta_i$.

Now that we have specified our SUSY model, we can study what kind of possibilities it has regarding the charged lepton flavour violation.

5.2.2 CLFV in SUSY seesaw-I

The new terms in the superpotential and the soft terms have significant implications. One sees that the theory contains massive right-handed neutrinos and sneutrinos even though the supersymmetry is not broken. The new terms modify the renormalization group equations so that even though the soft slepton masses are diagonal at the GUT scale, the individual lepton number will not be conserved if there are right-handed neutrinos below the GUT scale. The right-handed neutrinos allow the lepton number to be violated. However the right-handed neutrinos are very heavy and they will decouple at energies much lower than their masses. So if the soft slepton mass matrices are diagonal at GUT scale the presence of right-handed neutrinos modifies the renormalization group equations so that lepton flavour violating off-diagonal slepton mass terms are generated. The effect of right-handed neutrinos is only temporary since at energies much lower than the mass of the lightest right-handed neutrino, the right-handed neutrinos decouple. So the the right-handed neutrinos give contribution to the lepton flavour violation in the region between the GUT scale to the mass scale of the lightest right-handed neutrino. In supersymmetric models the right-handed neutrinos give significant contribution to LFV processes because there are soft slepton masses to which the right-handed neutrinos can affect through renormalization group equations and create lepton flavour violating off-diagonal slepton masses. To see more explicitly how the off-diagonal slepton mass matrices are generated we will examine the renormalization group equations for slepton squared masses. The RG equations for MSSM extended with right-handed neutrinos are presented in the Appendix D. Let us study the left handed scalar squared masses m_L^2. Its renormalization group equation (Appendix D, equation (121)) can be written in a form:

$$\frac{dm^2_{\widetilde{L}}}{dt} = \left(\frac{dm^2_{\widetilde{L}}}{dt}\right)_{Y_\nu=0} + \frac{1}{16\pi^2}(Y_\nu Y_\nu^\dagger m_L^2 + m_L^2 Y_\nu Y_\nu^\dagger + 2Y_\nu m_N^2 Y_\nu^\dagger + 2(m^2_{H_u})Y_\nu Y_\nu^\dagger + 2a_\nu a_\nu^\dagger). \tag{51}$$

The first term in the right-hand side represents the terms that do not contain neutrino Yukawa couplings, i.e. the terms of the MSSM RG equations. The terms in the second line contain neutrino Yukawa couplings and they are additions to the MSSM renormalization group equations due to seesaw-I mechanism we have employed to generate neutrino masses. If we assume that the soft terms are universal at the input scale like in MSUGRA (equations (42)-(46)), the charged slepton masses are the same as the sneutrino masses $m_L = m_\nu = m_0$ and the trilinear scalar couplings are proportional to the corresponding Yukawa couplings, $a_L = A_0 Y_L$ and $a_\nu = A_0 Y_\nu$ at the input scale which can be taken to be M_{gut}. Using these assumptions equation (51) becomes much simpler:

$$\frac{dm^2_{\widetilde{L}}}{dt} = \left(\frac{dm^2_{\widetilde{L}}}{dt}\right)_{Y_\nu=0} + \frac{1}{8\pi^2}(3m_0 + A_0^2)[Y_\nu Y_\nu^\dagger].$$

In the basis where the charged lepton Yukawa couplings are diagonal, the first term on the right-hand side is also diagonal. When one runs the energy scale from the M_{gut} to the lightest right-handed neutrino mass m_{N_i}, the neutrino Yukawa couplings generate the off-diagonal terms to scalar squared mass matrices (the neutrino Yukawa matrices can not be diagonalized simultaneously with the charged lepton Yukawa matrix). To a leading log approximation the off-diagonal terms are

$$(m_L^2)_{ij} \approx -\frac{1}{8\pi^2}\log\left(\frac{M_{gut}}{M_i}\right)(3m_0^2 + A_0^2)[Y_\nu Y_\nu^\dagger]_{ij}, i \neq j. \tag{52}$$

The rest of the charged lepton flavour violating soft parameters, namely the off-diagonal entries in trilinear coupling a_e and the right-handed charged slepton mass squared $m^2_{\widetilde{E}}$, are given in the leading log approximation as:

$$(a_l)_{ij} \approx -\frac{3}{16\pi^2}\log\left(\frac{M_{gut}}{M_i}\right)A_0 Y_{l_i}(Y_\nu^\dagger Y_\nu)_{ij}, \qquad i \neq j, \qquad \text{and} \tag{53}$$

$$(m^2_{\widetilde{E}})_{ij} \approx 0, \qquad i \neq j. \tag{54}$$

The MSSM extended with the seesaw mechanism can not generate lepton flavour violation in $(m^2_{\widetilde{E}})_{ij}$ due to the fact that its RG equation does not contain the neutrino Yukawa term $Y_\nu^\dagger Y_\nu$.

When we study different CLFV processes, different off-diagonal soft terms are relevant. When we are working in a mass insertion approximation [57], [58], only the corresponding matrix elements matter. For example if we study $\mu \to e\gamma$, only the elements $(m_L^2)_{21}$ and $(a_l)_{21}$ matter. One can use the mass insertion approximation to identify the dominant contributions.

Now that we know how the charged lepton flavour violation is introduced in MSSM extended via seesaw-I mechanism (we abbreviate this now on as

MSSMν), we can stydy more closely the processes $l_i \to l_j\gamma$, $l_i \to l_j l_k l_l$ and the l_i-l_j conversion.

6 CLFV reactions in SUSY

In this section we will go through charged lepton flavour violating processes $l_i \to l_j\gamma$, $l_i \to l_j l_k l_l$ and l_i-l_j conversion in the context of MSSM extended with the seesaw-I mechanism (MSSMν). In the following three subsections we will first deal with the general properties of a given process and then we will review different results obtained for these processes in the literature. Before we jump into the processes themselves, let us discuss some of the common properties they have in the MSSMν.

6.1 General properties

In the MSSM extended with the seesaw-I mechanism, the lepton flavour violating processes $l_i \to l_j\gamma$, $l_i \to l_j l_k l_l$ and l_i-l_j conversions all are, at the lowest order, mediated by two kinds of diagrams: penguins and boxes. The penguin diagrams include photon-, Z-boson and Higgs-penguins (in MSSM there are three physical neutral Higgs bosons: two CP-even H_0 and h_0, and one CP-odd A_0). In MSSM there is huge number of free parameters coming from the soft SUSY-breaking terms. It is not practically possible to do numerical analysis for one hundred or so parameters. Therefore the parameters must be constrained somehow. Here we will use couple of different constraints (MSUGRA/CMSSM, NUHM), when we do comparisons between them. There are also other ways of constraining the soft parameters, like *phenomenological minimal supersymmetric standard model* (pMSSM), but we do not consider the further.

Moreover in MSSMν there is only one source of charged lepton flavour violation (neutrino Yukawa coupling). Because of the unique source of flavour violation, it is espected that the LFV observables exhibit some correlation [97].

6.2 $\mu - e$ conversion in SUSY

We start with the electron-muon conversion in the vicinity of a nucleus. The muon orbiting the nucleus will interact with the quarks in the nucleus. In previous section we have studied the charged lepton conversion in effective theory context, using effective operators. We don't have to do that now, since we are working in a specific BSM model (MSSMν) where all the interactions are known. In MSSMν the μ–e conversion in the nuclei can happen by the following schematic diagrams at the one loop order:

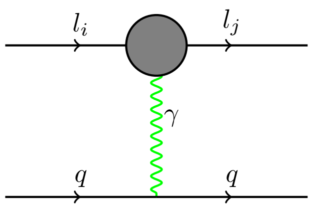

Figure 46: SUSY photon-penguin

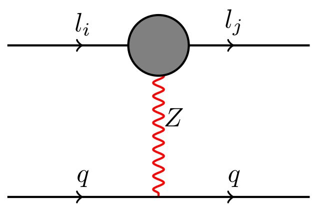

Figure 47: SUSY Z-penguin

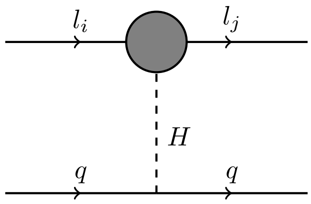

Figure 48: Higgs-penguin

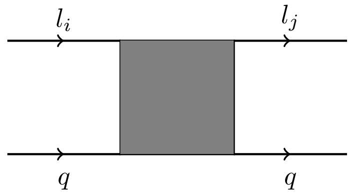

Figure 49: Box-diagram

The explicit diagrams contributing to the conversion are given in the Appendix H. The analytical calculation of this process is done in [53].

It is common in the literature to assume that the photon-penguin contribution is dominant and the Z-boson and Higgs-penguins and the boxes are subdominant. In MSSM with universal SUSY-breaking soft terms this is definitely so ([53], [62]). With different soft parameters the photon-contribution might be rivaled by some of the other contributions. Let us look this more closely. Let us first look into the most popular scenario: MSSM with universal soft parameters.

6.2.1 Constrained MSSM

Let us first study how much different kinds of diagrams, penguins and boxes contribute to the conversion rate of μ–e. Extensive analysis on muon-electron conversion in MSSMν was performed in [53]. They calculated first the conversion rate analytically, taking into account all the contributions from photon-, Higgs- and Z-penguins and the box diagrams to one-loop order. Using the complete analytical result they plotted the muon-electron conversion rate in titanium nuclei for each contribution (γ, Z, H and box) separately using the following assumptions: the heavy neutrinos are hierarchical with the masses $m_{N_i} = (10^{10}, 10^{11}, 10^{14})$GeV and that the light neutrino mixing angle $\theta_{13} = 5°$, and that the other mixing angles (for heavy and light neutrinos) are zero. Using these assumptions the conversion ratio $CR(\mu - e, Ti)$ was plotted in two different ways: first as a fuction of $\tan\beta$ with the universal soft parameters were chosen to be $M_0 = M_{1/2} = 250$ GeV and $A_0 = 0$. The results are plotted in the Figure 50.

It was found that the photon penguin utterly dominates the conversion process: it is at least two orders of magnitude larger than the second largest contribution. The Z- boson and box diagram contributions are observed to be almost constants for whole $\tan\beta$ range considered. The Higgs-penguin contribution experiences rapid growth as $\tan\beta$ grows, almost seven orders of magnitude. This strong dependence is in agreement with the approximation of Higgs contribution:

$$CR(\mu - e, Ti) \simeq \mathcal{O}(10^{-12}) \left(\frac{115\text{GeV}}{m_{H^0}}\right)^4 \left(\frac{\tan\beta}{50}\right)^6 \tag{55}$$

which holds for large $\tan\beta$ [53]. At low $\tan\beta$ the Higgs contribution is the smallest, but it becomes the second largest in the large $\tan\beta$, but it is still far behind the photon-contribution.

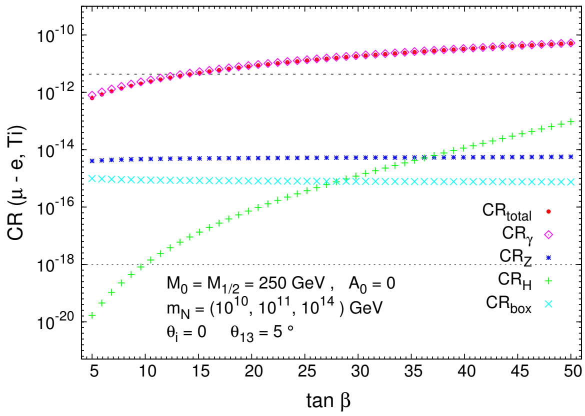

Figure 50: The contributions to $CR(\mu \to e, Ti)$: total, γ-penguins (diamonds), Z-penguins (asterisks), H-penguins (crosses) and box diagrams (times) as a function of $\tan\beta$ [53]

Conversion ratio was also plotted as a function of $M_0 = M_{1/2}$ (Figure 51, assuming that $A_0 = 0$ and that $\tan\beta = 30$.

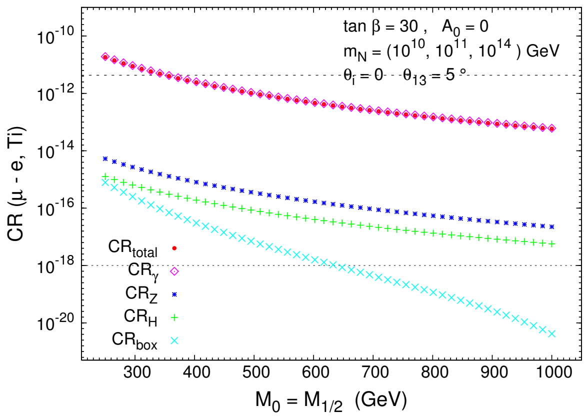

Figure 51: The contributions to $CR(\mu \to e, Ti)$: total, γ-penguins (diamonds), Z-penguins (asterisks), H-penguins (crosses) and box diagrams (times) as a function of SUSY mass scale [53]

The result is that the all the contributions decrease as the SUSY mass scale grows as is expected, the photon-penguin contribution being most dominant: three orders of magnitude larger than the Z-contribution, which is the second largest. One could say that the dependence on $\tan\beta$ is the more interesting one, since it affects the relative order between the magnitudes of the different contributions. So as a conclusion one could state that in the CMSSM case the photon-penguin is the only contribution one has to take into account when one considers conversion μ–e. Because universal soft terms is so simple assumption, the photon domination is also very popular assumption. Let us next look into different case: assume that the SUSY-breaking soft terms are not completely universal.

6.2.2 NUHM MSSM

In CMSSM the photon-penguin clearly dominates the conversion process. This is challenged when one allows some of the soft scalar masses to differ from the rest. There is one scalar mediating the conversion process: the Higgs boson (actually there are two of them, heavy and light; the CP-odd Higgs doesn't contribute to the coherent conversion). If the Higgs mass were to be small compared to the rest of the SUSY masses, the Higgs-penguin would be enhanced with respect to the other contributions, due to smaller suppression coming from the Higgs propagator. The case where all the soft terms are universal, except the Higgs masses, which differ from the rest of the scalar masses, is called the *Non-Universal Higgs Mass* scenario (NUHM). So in the NUHM scenario there is seven parameters: $M_0, M_{1/2}, A_0, \tan\beta, sign(\mu), M_{H_1}$ and M_{H_2}. The M_{H_1} and M_{H_2} are the massparameters of the neutral CP-even Higgses. The departure from universality in NUHM scenario is parametrized in terms of the non-vanishing parameters δ_1 and δ_2:

$$M_{H_1}^2 = M_0^2(1+\delta_1), \qquad M_{H_2}^2 = M_0^2(1+\delta_2).$$

In [53] the different contributions to $CR(\mu - e, Ti)$ were studied with non-universal Higgs masses. Large $\tan\beta$ $(= 50)$ and small Higgs masses were chosen to show the interesting behaviour of the NUHM case. The muon-electron conversion ratio in titanium was plotted as a function of $M_0 = M_{1/2}$ (masses of the SUSY particles other than Higgses) for photon, Z- and Higgs penguins and box diagrams separately assuming that $\theta_{13} = 5°$, $\theta_i = 0$, $\delta_1 = -1.8$ and $\delta_2 = 0$.

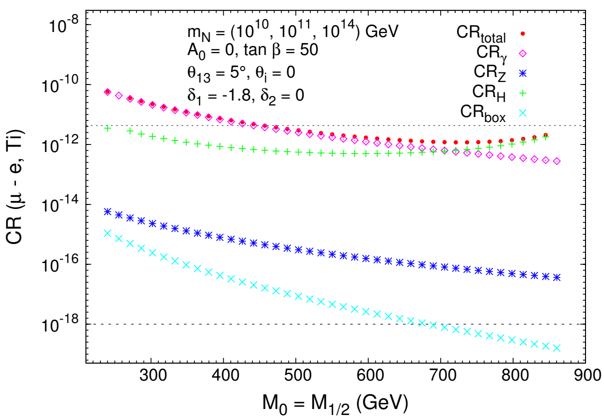

Figure 52: The contributions to the $CR(\mu \to e)$: total, γ-penguins (diamonds), Z-penguins (asterisks), H-penguins (crosses) and box diagrams (times) as a function of SUSY mass scale [53]

Very interesting results were obtained. With small SUSY masses the photon-contribution dominated as in the CMSSM case. All the contributions decrease at first but for large SUSY masses the Higgs contribution starts to increase and Higgs-contribution eventually becomes larger than the photon contribution by an order of magnitude. So in NUHM scenario it is possible for the Higgs to rival the photon contribution and even dominate the whole conversion process!

Similar results were obtained in [62], where the muon-electron conversion was also studied in NUHM scenario. They obtained an approximation for photon- and Higgs-contributions in $\mu - e$ conversion in aluminum for large $\tan\beta$:

$$CR(\mu Al \to eAl)_\gamma \sim \mathcal{O}(10^{-13}) \left(\frac{1000\text{GeV}}{m_S}\right)^4 \left(\frac{\tan\beta}{60}\right)^2, \tag{56}$$

$$CR(\mu Al \to eAl)_{H^0} \sim \mathcal{O}(10^{-13}) \left(\frac{200\text{GeV}}{m_{H^0}}\right)^4 \left(\frac{\tan\beta}{60}\right)^6, \tag{57}$$

where m_{H^0} is the heavy Higgs mass and $m_S \equiv M_0 = M_{1/2}$ is the universal scalar mass. From these we notice the different behaviour of the Higgs- and photon-mediated processes. For $\tan\beta \gtrsim 60$ and $m_{H^0} \ll M_S$ the Higgs-mediation starts to dominate the process. But if $\tan\beta \lesssim 60$ and $m_{H^0} \gg M_S$ the photon-mediation clearly dominates the process.

So according to papers [53] and [62], the photon mediation utterly dominates the μ-e conversion when one assumes universal soft scalar masses in MSSM extended with seesaw-I mechanism and the other contributions could be neglected.

The Higgs mediation can become important however if one relaxes the universality of scalar masses, and allows the Higgs bosons to have different masses. The Higgs mediation can then dominate in the limit of really large $\tan\beta$ and small heavy Higgs mass. We will next study the three-body lepton decay $l_i \to l_j l_j l_j$. It has many similar properties with the charged lepton conversion.

6.3 $l \to l' l'' l'''$ in SUSY

The process $l_i \to l_j l_k l_l$ can proceed at one loop order through box- and penguin-diagrams. For simplicity let us consider process $l_i \to l_j l_j l_j$. It can proceed at one-loop level through the diagrams presented in the Appendix I.

In the literature the photon contribution is usually assumed to be dominant. Let us find out why.

6.3.1 Numerical result for the $l_i \to l_j l_j l_j$

In paper [56] the charged lepton flavour violating processes $l_i \to l_j l_j l_j$ are studied in MSSM extended with seesaw-I mechanism. The soft parameters are assumed to be universal in the MSUGRA scheme. In [56] analytical calculations for $l_i \to l_j l_j l_j$ are conducted, and the results are later utilized in the numerical analysis. The full set of one-loop contributions are considered: photon-, Z- and Higgs-penguins and the box-diagrams. The numerical analysis was done in two kinds of cases: a) the quasi-degenarate light neutrinos and degenerate heavy neutrinos, b) hierarchical light and heavy neutrinos. Let us now review the two cases a) and b).

a): **Degenerate case.** The following processes were studied: $\tau^- \to \mu^-\mu^-\mu^+$, $\tau^- \to e^-e^-e^+$, $\mu^- \to e^-e^-e^+$. The branching ratio of the process $\tau^- \to \mu^-\mu^-\mu^+$ was plotted as the function of $\tan\beta$, with the following assumptions: $m_N = 10^{14}$GeV, $M_0 = 400$GeV, $M_{1/2} = 300$GeV, $A_0 = 0$ and $sign(\mu) > 0$. The result is plotted in the Figure 53.

As is evident from the Figure 53, the photon-penguin utterly dominates the process $BR(\tau^- \to \mu^-\mu^-\mu^+)$: the photon contribution hardly differs from the

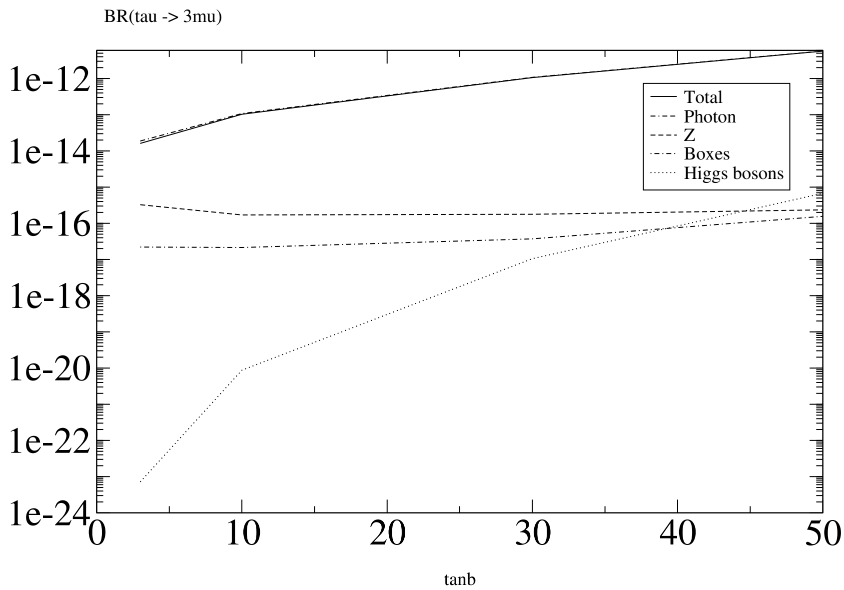

Figure 53: The $BR(\tau \to 3\mu)$ as the function of $\tan\beta$ in the case of degenerate heavy neutrinos [56].

total decay rate. The photon contribution goes approximately as $(\tan\beta)^2$. The Z-penguin contribution is the second largest with the small $\tan\beta$, but still over an order of magnitude smaller than the photon-contribution. Also te Z-penguin and the box contributions are observed not to depend significantly on the $\tan\beta$. They change approximately only an order of magnitude. The contribution of the Higgs-penguins has the most dramatic behaviour however: it becomes the second largest at the large $\tan\beta$ due to the approximate $\tan^6\beta$-dependence. Both the lepton conversion of the previous section and the three-body decay proceed through similar diagrams: penguins and boxes. Due to the similar diagram structure the different contributions, boxes, photon-, Z- and Higgs-penguins have similar behaviour. The Z-penguins and the box diagrams of the lepton conversion and the three-body decay both depend little about the $\tan\beta$. Also the photon- and the Higgs-contributions of the lepton conversion and the three-body decay have the similar behaviour: the photon contributions behave as $(\tan\beta)^2$ and the Higgs-contributions behave as $(\tan\beta)^6$ (equations (56) and (57)).

Even though the Higgs-contribution has significant enhancement at the large $\tan\beta$, it is still four orders of magnitude smaller than the the photon-contribution at the largest $\tan\beta(=50)$ studied. As a summary one could deduce that the leading photon-penguin approximation works extremely well for large $\tan\beta$. We have now established that the photon contribution dominates the process whatever the $\tan\beta$ is. So next in the case of hierarchical heavy neutrinos we set $\tan\beta = 50$.

b): **Hierarchical case.** The same processes as in the case a) are studied here, but now the heavy neutrinos are strictly hierarchical and their masses are chosen to be $(m_{N_1}, m_{N_2}, m_{N_3}) = (10^8, 2\times 10^8, 10^{14})$GeV. They are picked so because this choise generates a proper rate for baryogenesis via leptogenesis in the hierarchical case [63]. The branching ratios were studied as fuctions of neutrino mixing angles $|\theta_1|$, $|\theta_2|$ and $|\theta_3|$. First the $BR(\tau^- \to \mu^-\mu^-\mu^+)$ and $BR(\tau \to \mu\gamma)$ were studied as a fuction of mixing angle $|\theta_2|$ (figure 54). The other mixing angles are kept zero and $Arg(\theta_2) = \pi/4$. The MSUGRA parameters are chosen to be the same as in the degenerate case: $\tan\beta = 50$, $M_0 = 400\text{GeV}$, $M_{1/2} = 300\text{GeV}$, $A_0 = 0$ and $sign(\mu) > 0$.

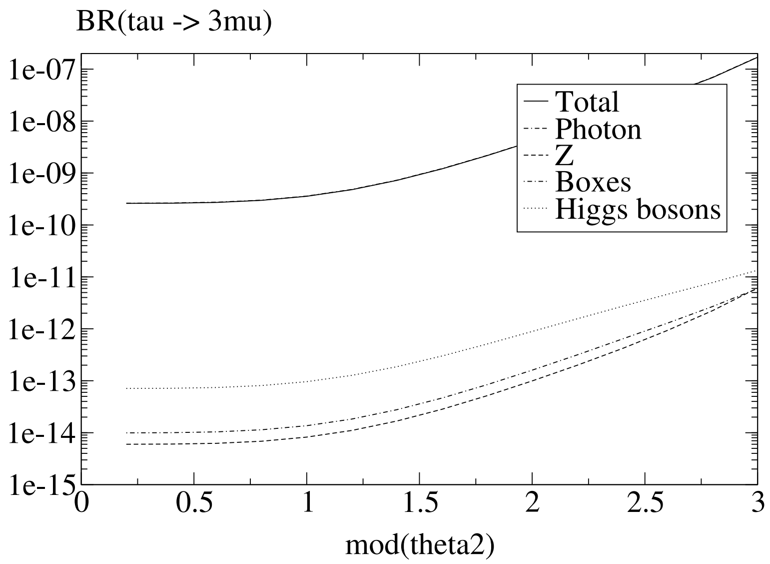

Figure 54: The contributions to $BR(\tau \to 3\mu)$ as the function of $|\theta_2|$ in the case of hierarchical heavy neutrinos [56].

It is again noticed that the photon-contribution completely dominates the process $BR(\tau^- \to \mu^-\mu^-\mu^+)$. The Higgs-contribution is the second largest (because of the choice $\tan\beta = 50$). The relative size of the box- and Z-contributions change with respect to the degenerate case: the Z-contribution is now the smallest contribution. The branching ratio in question stays under the experimental upper limit for all $|\theta_2|$. The branching ratios were also plotted for different phases $Arg(\theta_2)$ for processes $BR(l_j^- \to l_i^- l_i^- l_i^+)$. The branching ratios behave smoothly for complex θ_2, but experience a dip for real θ_2. The branching ratios $BR(l_j^- \to l_i^- l_i^- l_i^+)$ stay within the experimental limits.

Then the $BR(l_j^- \to l_i^- l_i^- l_i^+)$ are studied as a function of $|\theta_1|$. Now we get more severe limits from the experiments. The branching ratios are plotted for different phases of θ_1 as a function of $|\theta_1|$. One notices that the $BR(\mu \to 3e)$ is mostly much larger than the experimental upper limit, except for a small region. The $BR(\tau \to 3e)$ and the $BR(\tau \to 3\mu)$ stay within the upper bounds.

Finally the branching ratios $BR(l_j^- \to l_i^- l_i^- l_i^+)$ are studied as a function of $|\theta_3|$. It was noticed that the branching ratios stay almost constant with the angle θ_3. So for $\tan\beta = 50$, $M_0 = 400\text{GeV}$, $M_{1/2} = 300\text{GeV}$, $A_0 = 0$ and $sign(\mu) > 0$ the branching ratios are approximately: $BR(\tau \to 3\mu) = 2.6\times10^{-10}$, $BR(\tau \to 3e) = 8.8 \times 10^{-15}$ and $BR(\mu \to 3e) = 1.8 \times 10^{-14}$.

6.4 $l \to l'\gamma$ in SUSY

Let us now concentrate on the process $l_i \to l_j\gamma$. This process differs from the decay $l_i \to l_j l_k l_l$ and conversion $\mu - e$ in that it does not contain penguins and boxes whereas previous ones have. It will be related to the two other processes we are considering as we shall see. Let us now discuss about general properties of $l_i \to l_j\gamma$ in MSSMν.

6.4.1 General properties

The general form of the amplitude of $l_i \to l_j\gamma$ is given by [60]:

$$T = \varepsilon^\alpha \bar{l}_j m_{l_j} i\sigma_{\alpha\beta} q^\beta (A_L P_L + A_R P_R) l_i, \tag{58}$$

where A_L is the coefficient of the amplitude when the incoming lepton l_i is left-handed and the A_R is the coefficient of the amplitude when the incoming lepton is right-handed[12]. The branching ratio is then given by

$$BR(l_i \to l_j\gamma) = \frac{12\pi^2}{G_F^2}\left(|A_L|^2 + |A_R|^2\right). \tag{59}$$

The process $l_i \to l_j\gamma$ violates chiral symmetries $l_{Rk} \to e^{i\alpha_k} l_{Rk}$, i.e. the initial and final leptons have different chiralities. We have assumed that the soft terms are universal, so only the Yukawa couplings can be the source of this chirality violation. The A_L must be proportional to the mass of the right-handed lepton, l_{jR}, involved and the A_R must be proportional to the mass of the l_{iR}. The mass of the initial lepton is always much larger than the final lepton, $m_{lj}^2 << m_{li}^2$, so the right-handed amplitude dominates, $|A_L|^2 << |A_R|^2$.

To the one loop order the process $l_i \to l_j\gamma$ is given by the two diagram types presented in the Figures 55 and 56. In those Figures the external photon vertex is not specified. The external photon vertex can be inserted to any propagator of a charged particle, that is to external leptons l_i; intermediate slepton $\widetilde{l}_x$, $x = 1, ..., 6$ or to the intermediate chargino $\widetilde{\chi}_B^-$, $B = 1, 2$, in a similar way as in our R_ξ loop-calculation example in the Appendix A. The *lepton-sneutrino-chargino* vertex and the *lepton-slepton-neutralono* vertex are responsible for the charged lepton flavour violation.

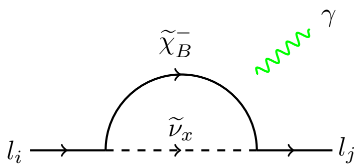

Figure 55: Chargino-sneutrino loop

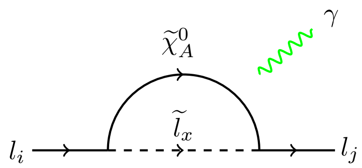

Figure 56: Neutralino-slepton loop

In order to get insight of the physics involved, one can use mass insertion approximation ([57], [58]) to find the dominant contribution to this process. In mass insertion approximation the following diagram describes our process:

[12]The expressions for A_L and A_R can be found in [90].

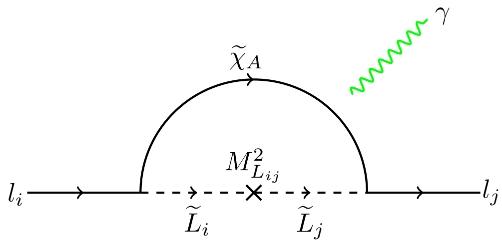

Figure 57: Mass-insertion diagram, contributing to $l_i \to l_j\gamma$. The $\widetilde{L}_i$ is slepton doublet in the basis where the gauge interactions and the charged lepton Yukawa couplings are flavour diagonal.

In the mass insertion approximation one treats the mass terms as interaction terms. In this case we are interested in the charged lepton flavour violation, so the only mass terms contributing are the off-diagonal slepton masses. Using the mass insertion approximation one gets the following branching ratio [60]:

$$BR(l_i \to l_j\gamma) \simeq \frac{12\pi^2}{G_F^2}|A_R|^2 \sim \frac{\alpha^3}{G_F^2}\frac{|m^2_{Lij}|^2}{m_S^8}\tan^2\beta. \tag{60}$$

The branching ratio is highly dependent of the corresponding off-diagonal slepton mass term m^2_{Lij}. The soft SUSY-breaking terms are assumed to be universal, so that the slepton mass matrices are diagonal at the input scale. The RG running, however, generates the off-diagonal terms to the slepton mass matrices as one evolves the parameters from the input scale down to the electroweak scale. Using the equation (52) for the off-diagonal terms one gets:

$$BR(l_i \to l_j\gamma) \sim \frac{\alpha^3}{G_F^2 m_S^8}\left|\frac{-1}{8\pi^2}(3m_0 + A_0^2)\log\frac{M_x}{M}\right|^2 \left|(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)_{ij}\right|^2 \tan^2\beta, \tag{61}$$

where the MSUGRA soft parameters are assumed. The branching ration is enhanced by $\tan\beta$: for large values of $\tan\beta$ the branching ratio can be significant. Let us next review some results in the literature.

6.4.2 Results from numeral calculations in the literature

Let us now review numerical results the literature contains. The properties of $l_i \to l_j\gamma$ are calculated in [56]. The $l_i \to l_j\gamma$ is not numerically that interesting since it has no rivaling contributions as opposed to three-body lepton decay and conversion where there are contributions from photon-, Z- and Higgs penguins and boxes. More interesting is the connection of $l_i \to l_j\gamma$ and the two other process types. Let us nonetheless review some numerical properties of $l_i \to l_j\gamma$. The process is studied in MSSM assuming MSUGRA soft parameters. The branching ratios for $\tau \to \mu\gamma$, $\tau \to e\gamma$ and $\mu \to e\gamma$ are plotted as functions of different SUSY parameters: $\tan\beta$, M_0 and $M_{1/2}$. It was found that the branching ratios for all of the processes grow as the function of $\tan\beta$, which is to be expected by the approximation (61). All of the branching ratios were observed to decrease as the parameters m_0 and $M_{1/2}$ were raised, which again is in line with the approximation (61).

More interesting observations are made when the branching ratios of $l_i \to l_j\gamma$ and $l_i \to 3l_j$ are compared to each other. Under the assumption that the photon-contribution dominates the process $l_i \to 3l_j$, one can get the following relation between the $BR(l_i \to l_j\gamma)$ and $BR(l_i \to 3l_j)$ [56]:

$$\frac{BR(l_j \to 3l_i)}{BR(l_j \to l_i\gamma)} = \frac{\alpha}{3\pi}\left(log\frac{m_{l_j}^2}{m_{l_i}^2} - \frac{11}{4}\right). \tag{62}$$

From this one gets the following approximate values: $\frac{1}{440}$, $\frac{1}{94}$ and $\frac{1}{163}$ for $(l_j l_i) = (\tau\mu), (\tau e)$ and (μe). Therefore one expects the $l_i \to l_j\gamma$ to have higher ratios than the corresponding $l_i \to 3l_j$ in MSSMν.

6.5 Comparison to R-parity violating models

We have now discussed about the charged lepton flavour violating processes $l_i \to l_j\gamma$, $l_i \to l_j l_k l_l$ and μ-e conversion in the nuclei, in the context of MSSM extended to have massive neutrinos via seesaw-I mechanism. We have noticed that photon mediation nearly always utterly dominates each of these processes. In other supersymmetric models this might not be the case. We will now briefly study another supersymmetric model, trilinear R-parity violating model, in the context of CLFV and compare the results to the previous.

In trilinear R-parity violating model the MSSM superpotential is extended by the following lepton flavour violating trilinear terms[13]:

$$W_{\not R} = \frac{1}{2}\lambda_{ijk}\hat{L}_i\hat{L}_j\hat{E}_k^c + \frac{1}{2}\lambda'_{ijk}\hat{L}_i\hat{Q}_j\hat{D}_k^c. \tag{63}$$

The R-parity violating models only add new interactions to the MSSM. This is crucial for our discussion.

It is common in literature to assume that the photon mediation dominates the CLFV processes, and that the other contributions can be safely ignored, except for the large $\tan\beta$ regime where the Higgs-contribution could become important, as noted in the previous papers we reviewed. In MSSM extended only with right-handed neutrinos via seesaw-I mechanism this is so. When trilinear R-parity violation is introduced, things change: Z-boson contribution acquires significant enhancement, so big that it can even surpass the photon-contribution. Let us discuss why this happens.

In MSSM extended via seesaw-I mechanism the processes $l_i \to l_j l_k l_l$ and l_i-l_j conversions can proceed though Z-boson exchange (Appendix I), but these are overshadowed by the photon exchange diagrams. But this is only because of MSSM interactions and particle content (it is okay to add right-handed neutrinos). If either one of them is extended, the Z-boson exchange might prevail.

In MSSM(ν) the amplitudes of photon- and Z-boson penguins for $l_i \to 3l_j$

[13] One could also include R-parity violating term $1/2\lambda''_{ijk}\hat{U}_i\hat{D}_j\hat{D}_k$, but since we are studying lepton flavour violation, this term is neglegted.

are given in the equations (64) and (65).

$$T_{\gamma-\text{penguin}} = \bar{u}_i(p_1)[q^2\gamma_\mu(A_1^L P_L + A_1^R P_R) + im_{l_j}\sigma_{\mu\nu}q^\nu(A_2^L P_L + A_2^R P_R)]u_j(p) \times \frac{e^2}{q^2}\bar{u}_i(p_2)\gamma^\mu v_i(p_3) - (p_1 \leftrightarrow p_2), \tag{64}$$

$$T_{Z^0-\text{penguin}} = \frac{1}{m_Z^2}\bar{u}_i(p_1)[\gamma_\mu(F_L P_L - F_R P_R)]u_j(p) \times \bar{u}_i(p_2)\left[\gamma^\mu\left(Z_L^{(l)}P_L + Z_R^{(l)}P_R\right)\right]v_i(p_3) - (p_1 \leftrightarrow p_2). \tag{65}$$

By dimensional analysis one can understand why the Z-boson exchange could dominate. The total decay width $\Gamma(l_i \to 3l_j)$ is proportional to $m_{l_i}^5$ [64]. One can therefore deduce that the form factors A and F must have dimensions of inverse mass squared. Both photon and the Z-boson contributions have the same mass dimension, so the question is what are the mass scales of the A and F? The mass scale of the F is of course determined by the Z-boson mass so $F \sim m_Z^{-2}$. Because the photon is massless the only mass scale in the form factors A is the mass scale of the supersymmetry, m_{SUSY}. So $A \sim m_{SUSY}$. Since $m_Z \ll m_{SUSY}$ the Z-contribution could very well dominate the photon-contribution. In MSSM the photon exchange dominates the processes in numerical calculations. How can this be? The reason is that there is cancellation among Z-exchange diagrams in MSSM, and that results in the suppression of the Z-boson penguins [65]. There are two things that can spoil the cancellation among the Z-penguins: extended particle content or new couplings. The latter of these happens in trilinear R-parity violating model where new lepton flavour violating interactions (63) are added to those of the MSSM. The new interactions lead to new loop diagrams that don't experience the cancellation as the MSSM ones. A new diagram containing R-parity violating vertices is for example:

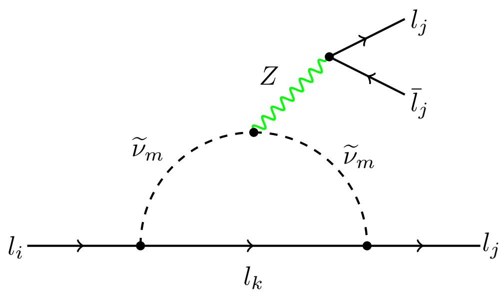

Figure 58: R-parity violating diagram

The lepton conversion near nucleus can proceed through the same kind of a diagram, one only has to change two external leptons to quarks. The processes $l_i \to l_j\gamma$ do not get any enhancement from the Z-boson penguins at one-loop level.

Let us now review the numerical results obtained in [64] for trilinear R-parity violating model. As noted before the photon- and Z-contributions should behave differently as the SUSY mass scale is changed. The photon-contributions should be diminished with respect to Z-contribution as the SUSY mass scale is raised. At one-loop order the $l_i \to 3l_j$ decay and the l_i-l_j conversion acquire boost from the Z-contribution. The decay $l_i \to l_j\gamma$ does not however. Therefore as the SUSY scale is raised the rates of $l_i \to 3l_j$ and l_i-l_j conversion should be increased relative to $l_i \to l_j\gamma$.

This was indeed noticed. The branching ratios of $l_i \to l_j\gamma$ and $l_i \to 3l_j$ were plotted as a function of $\lambda^*_{231}\lambda_{232}$ which is a combination of R-parity violating couplings, with different benchmark points. For a smaller SUSY scale the ratio of $l_i \to l_j\gamma$ was larger. But at larger SUSY mass scale the $l_i \to 3l_j$ slightly surpassed the $l_i \to l_j\gamma$. This is remarkable as in the MSSM $BR(l_i \to l_j\gamma) > l_i \to 3l_j$ [64].

7 Charged lepton flavour violation at LHC

The LHC (Large Hadron Collider) can search charged lepton flavour violation through many different decay modes. In our discussion we divide the searched decay modes into two categories: modes including only SM particles in the final states and modes that include supersymmetric particles in the final states.

7.1 CLFV with only SM particles

The LHC can search for charged lepton flavour violating tau lepton decays, lepton flavour and baryon number violating B- meson decays and charged lepton flavour violating Z-boson decays. Tau is the only charged lepton whose CLFV decay can be detected in LHC. The LHC will produce W- and Z-bosons and B-mesons. Their decays will produce tau-leptons which would yield muons as final state particles. The tau decays with electrons in the final state are also possible of course (e.g. $\tau \to 3e$), but they are experimentally almost impossible to detect:

The LHC produces a large number of B- and D-mesons and W- and Z-bosons in its proton-proton collisions [67]. The taus in the LHC are predominantly produced in the decay of these particles. The tau mass is close to the masses on B- and D-mesons. Therefore the decays of taus from heavy mesons are difficult to trigger due to small transverse momenta of resulting muons.

Tau is so heavy it can decay into hadrons. At LHCb the decays $\tau^- \to \bar{p}\mu^+\mu^-$ and $\tau^- \to p\mu^-\mu^-$ are searched [70]. Also the charged lepton flavour violating B-meson decays, $B^+ \to \pi^-\mu^+\mu^+$ and $B^+ \to K^-\mu^+\mu^+$ are searched at the LHC [71].

The search for CLFV at the LHC is just beginning, but there are already first results obtained from the LHC run at $\sqrt{s} = 7$TeV. The LHCb has reported (2012) of the upper limits obtained for the processes $\tau^- \to \mu^+\mu^-\mu^-$, $\tau^- \to \bar{p}\mu^+\mu^-$ and $\tau^- \to p\mu^-\mu^-$, using data obtained during 2011 at $\sqrt{s} = 7$ TeV by LHCb [70]:

$$\begin{aligned} BR(\tau^- \to \mu^+\mu^-\mu^-) &< 7.8(6.3) \times 10^{-8} \\ BR(\tau^- \to \bar{p}\mu^+\mu^-) &< 4.5(3.4) \times 10^{-7} \\ BR(\tau^- \to p\mu^-\mu^-) &< 6.0(4.6) \times 10^{-7}, \end{aligned}$$

at 97% (90%) confidence level. LHCb has also reported (2012) of limits for B-meson decays $B^+ \to K^-\mu^+\mu^+$ and $B^+ \to \pi^-\mu^+\mu^+$, that were obtained from the data collected during 2010 at the LHCb [71]:

$$\begin{aligned} BR(B^+ \to K^-\mu^+\mu^-) &< 5.4 \times 10^{-8} \\ BR(B^+ \to \pi^-\mu^+\mu^+) &< 5.8 \times 10^{-8}, \end{aligned}$$

at 95% confidence level.

The ATLAS and CMS should be able to search for $\tau \to \mu\gamma$ and $\tau \to \mu\mu\mu$, and LHCb will continue the search for CLFV decays involving hadrons.

7.2 CLFV with supersymmetric particles

The LHC should be able to produce supersymmetric particles when it starts its operation again in early 2015 with higher centre-of-mass energy $\sqrt{s} = 14$TeV.

So LHC can also search for charged lepton flavour violating decays that involve supersymmetric particles in the initial and final states. The possible way to produce the CLFV process with SUSY particles in the LHC is through the following decay chain [69]:

$$\begin{aligned} pp &\rightarrow \widetilde{q}_a\widetilde{q}_b, \widetilde{g}\widetilde{q}_a, \widetilde{g}\widetilde{g}, && (66)\\ \widetilde{q}_a(\widetilde{g}) &\rightarrow \widetilde{\chi}_2^0 q_a(g), && (67)\\ \widetilde{\chi}_2^0 &\rightarrow \widetilde{l}_\alpha l_\beta, && (68)\\ \widetilde{l}_\alpha &\rightarrow \widetilde{\chi}_1^0 l_\beta, && (69) \end{aligned}$$

where a and b run over all squark mass eigenstates and α and β are lepton/slepton mass(flavour) eigenstates. So first squarks ($\widetilde{q}_a$) and gluinos ($\widetilde{g}$) are produced in the proton collision (66). Then the squarks and gluinos decay into quarks and gluons with second lightest neutralino $\widetilde{\chi}_2^0$ (67). The searched charged lepton flavour violation can occur when the $\widetilde{\chi}_2^0$ decays into a slepton-lepton pair with possibly different flavours (68) or when the slepton decays into the lightest neutralino $\widetilde{\chi}_1^0$ and a lepton with possibly a different flavour.

Also one can detect flavoured slepton mass splittings, defined as [73]:

$$\frac{\Delta m_{\widetilde{l}}}{m_{\widetilde{l}}}(\widetilde{l}_i, \widetilde{l}_j) = \frac{|m_{\widetilde{l}_i} - m_{\widetilde{l}_j}|}{\langle m_{\widetilde{l}_i}, m_{\widetilde{l}_j}\rangle}.$$

In many SUSY models the slepton masses are assumed to be universal at the input scale, i.e. the sleptons have equal masses. The renormalization group evolution however changes those masses as one runs the energy scale from the input scale down to the electroweak scale. The slepton masses of different generations evolve differently due to different Yukawa couplings between generations. When one assumes that the neutrino masses are generated by the seesaw-I mechanism the RG equations of the slepton masses get additional contribution from the neutrino Yukawa couplings. The LHC can detect the mass splittings and it can be used to probe whether the seesaw-I is the source of neutrino masses, even if CLFV is not detected [97].

8 Conclusions

The charged lepton flavour violating processes have never been observed, despite numerous experimental searches. The present upper bounds (Table 2) for the CLFV processes are already quite stringent. There are several ongoing and future experiments trying to improve the sensitivity. For $\mu \to eee$ decay, the sensitivity of 10^{-16} is expected [101], which is four orders of magnitude more sensitive than the present bound. The next phase of MEG experiment is expected to reach $BR(\mu \to e\gamma) \leq 6 \times 10^{-14}$ [91], [102]. Also for the μ-e conversion in the nuclei improvements are expected [103]-[107]. Also the LHC should start its next run in early 2015, with higher centre-of-mass energy $\sqrt{s} = 14$TeV.

In this thesis we have studied the CLFV, in effective theories, and as an explicit example, in the context of MSSM extended with the seesaw-I mechanism. This model assumes two unconfirmed things: the existence of supersymmetry and neutrino mass generation via seesaw-I mechanism. The LHC can search for superpartners, and if it finds charged sleptons, it can deduce from their mass splittings whether or not the seesaw-I mechanism is the source of the neutrino masses. But maybe in this case the more important thing would be the confirmation of supersymmetry. Also it would be nice if the existence of CLFV itself could be confirmed in the LHC or in the low energy experiments after over 70 years of search.

The charged lepton flavour violating processes are extremely important due to the restrictions they impose on the BSM theories. Many BSM models like supersymmetric models, contain sources of charged lepton flavour violation. The strict bounds for the CLFV processes give severe restrictions on the parameters of the new physics models. The thing is that charged lepton flavour violation becomes more important if it is not found. The more stringent the bounds on CLFV rates become, the more the parameters of new physics models are constrained. The restrictions the CLFV gives, helps to find the true BSM model. So actually when one studies the charged lepton flavour violation, one actually studies the theories beyond the Standard Model. Charged lepton flavour violation is just a point of view which one has taken. The true explanation of the charged lepton flavour violation (or its absence) is the true theory beyond the Standard Model.

A An example of R_ξ-gauge calculation

I now give an explicit example of a calculation in R_ξ-gauge. I follow the procedure of Cheng&Li [1]. I will calculate the "*scalar diagram*", involving Goldstone bosons, which contributes to the process $\mu \to e\gamma$ in R_ξ-gauge.

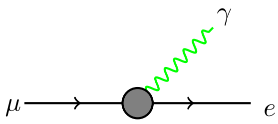

Figure 59: $l_i \to l_j\gamma$

We shall assume that the neutrinos are not massless so that process $\mu \to e\gamma$ is mediated by neutrino oscillations

$$\nu_\alpha = \sum_i U_{\alpha i}\nu_i \qquad \alpha = e, \mu, \tau; i = 1, 2, 3,$$

where ν_α denote weak eigenstates and ν_i denote mass eigenstates. The lowest order diagrams contributing to the process $\mu \to e\gamma$ in R_ξ-gauge are:

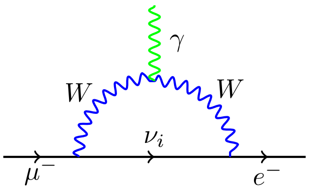

Figure 60: (a)

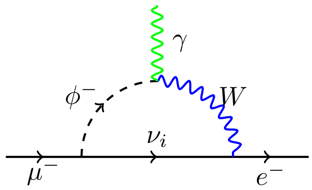

Figure 61: (b)

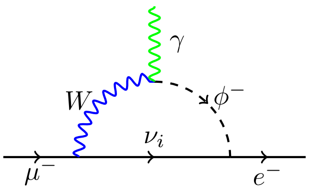

Figure 62: (c)

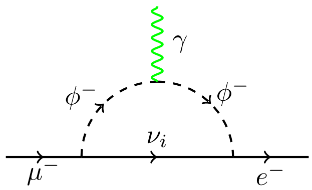

Figure 63: (d)

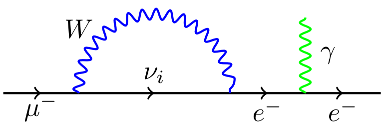

Figure 64: (e_1)

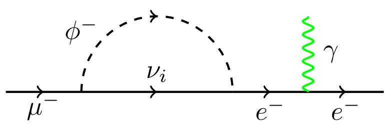

Figure 65: (e_2)

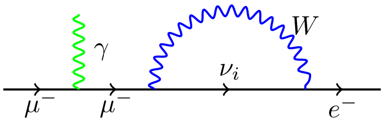

Figure 66: (e_3)

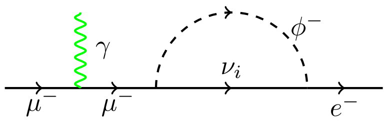

Figure 67: (e_4)

The Feynman amplitude of the general process $\mu \to e\gamma$, presented in the figure (59), can be written in a form

$$\mathcal{M}(\mu \to e\gamma) = \bar{u}_e(p-q)\left[iq^\nu \sigma_{\lambda\nu}(A + B\gamma_5) + \gamma_\lambda(C + D\gamma_5) + q_\lambda(E + F\gamma_5)\right] u_\mu(p)\varepsilon^\lambda,$$

where A, B, C, D, E and F are some constants or "form factors". The object inside the square brackets is the electromagnetic current J^μ_{em}. This most general form can be simplified however. First we can use the gauge invariance of the quantum electrodynamics:

$$\partial_\mu J(x)^\mu_{em} = 0,$$

or rather it's momentum space counterpart:

$$q_\mu J(q)^\mu_{em} = 0.$$

By using that we deduce that $C = D = 0$.

We also notice that because

$$q_\mu \varepsilon^\mu = 0,$$

for a on-shell photon, the "$E - F$"-term in the amplitude vanishes. So we are only left with magnetic transition term:

$$\mathcal{M}(\mu \to e\gamma) = \overline{u}_e(p-q)[iq^\nu \sigma_{\lambda\nu}(A + B\gamma_5)]u_\mu(p)\varepsilon^\lambda. \tag{70}$$

We also make the approximation $m_e = 0$. This means that the outgoing electron must be left-handed. This is only possible if $A = B$. Then the Feynman amplitude for the process is [1]

$$\mathcal{M}(\mu \to e\gamma) = A\overline{u}_e(p-q)(1+\gamma_5)(2p\cdot\varepsilon - m_\mu \gamma\cdot\varepsilon)\overline{u}_\mu(p), \tag{71}$$

where we have used *Gordon decomposition*. We know that the final amplitude must have the form of a magnetic transition (70) so we only have to concentrate on $p \cdot \gamma$ term when calculating the invariant amplitude A. The $\gamma \cdot \varepsilon$ term is canceled by the diagrams (e_1), (e_2), (e_3) and (e_4).

We want to find out the contribution to the invariant amplitude A coming from the "*scalar diagram*" (d). We use the following momentum assignment.

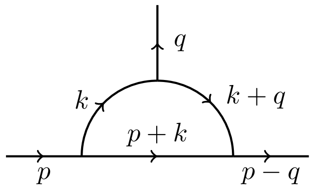

Figure 68: Momentum assignments in diagram (d)

We proceed as follows. First we write down the corresponding Feynman amplitude using generalized Feynman rules in R_ξ-gauge which allow neutrino mixing. Then we extract terms that are proportional to $p \cdot \gamma$ only and discard terms proportional to $\varepsilon \cdot \gamma$. Finally we just compare the result with (71) and recognize the coefficient in front as invariant amplitude A.

By using Feynman rules we can write the Feynman amplitude for mass eigenstate i as:

$$T(d) = -i \int \frac{d^4k}{(2\pi^4} \left\{ \overline{u}_e(p-q) \left(\frac{ig}{2\sqrt{2}M} \right) U^*_{ei}[m_i(1+\gamma_5) - m_e(1-\gamma_5)] \right.$$
$$\left. \times i \frac{\not{p} + \not{k} + m_i}{(p+k)^2 - m_i^2} \left(\frac{ig}{2\sqrt{2}M} \right) [m_i(1-\gamma_5) - m_\mu(1+\gamma_5)] u_\mu(p) \right\}$$
$$\times \frac{i}{k^2 - \xi M^2} \frac{i}{(k+q)^2 - \xi M^2} ie(2k \cdot \varepsilon) \qquad (72)$$

To simplify this we set $m_e = 0$, commute $\not{p}$'s to $u_\mu(p)$ and use Dirac equation for a spinor:

$$(\not{p} - m)u(p) = 0$$

In order to get the contribution from all of the intermediate mass eigenstates we sum them over:

$$\sum_i T_i(d) = i \left(\frac{g^2}{8M^2} \right) \int \frac{d^4k}{2\pi^4} \frac{1}{k^2 - \xi M^2} \frac{1}{(k+q)^2 - \xi M^2} e(2k \cdot \varepsilon)$$
$$\times \sum_i U^*_{ei} U_{\mu i} \overline{u}_e(p-q) \frac{2m_i^2(1+\gamma_5)\not{k}}{(p+k)^2 - m_i^2} u_\mu(p). \qquad (73)$$

By using the approximation:

$$\sum_i \frac{U^*_{ei} U_{\mu i}}{(p+k)^2 - m_i^2} \simeq \sum_i \frac{U^*_{ei} U_{\mu i} m_i^2}{(p+k)^2}, \qquad (74)$$

we get that:

$$\sum_i T_i(d) = i \frac{c}{M^2} \int \frac{d^4k}{2\pi^4} \frac{1}{k^2 - \xi M^2} \frac{1}{(k+q)^2 - \xi M^2} \frac{1}{(p+k)^2}$$

$$\times(2k\cdot\varepsilon)\overline{u}_e(p-q)(1+\gamma_5)\not{k}u_\mu(p), \tag{75}$$

where c is:

$$c=\frac{eg^2}{M^2}\sum_i U^*_{ei}U_{\mu i}m_i^2.$$

Next we must calculate the integral over k. We do this by employing *Feynman parametrization*:

$$\frac{1}{(p+k)^2(k^2-\xi M^2)((k+q)^2-\xi M^2)}=2!\int_0^1 dz_1\int_0^{1-z_1}dz_3\frac{1}{D^3}, \tag{76}$$

where denominator D is:

$$\begin{aligned}D&=(p+k)^2z_1+(k^2-\xi M^2)(1-z_1-z_3)+((k+q)^2-\xi M^2)z_3\\&=(k+z_1p+z_3q)^2-(z_1p+z_3q)^2+z_1p^2-\xi M^2(1-z_1)\equiv l^2-a^2.\end{aligned} \tag{77}$$

We take $l=k+z_1+z_3q$ as the new integration variable in (75) and it becomes:

$$\sum_i T_i(d)=\frac{4ic}{M^2}\int_0^1 dz_1\int_0^{1-z_1}dz_3\overline{u}_e(p-q)(1+\gamma_5)\times I\times u_\mu(p), \tag{78}$$

where I is:

$$\begin{aligned}I&=\int\frac{d^4l}{(2\pi)^4}\frac{(\not{l}-(z_1+z_3)m_\mu)(\varepsilon_\nu l^\nu-z_1\varepsilon\cdot p)}{(l^2-a^2)^3}.\\&=\int\frac{d^4l}{(2\pi)^4}\left(\frac{\gamma_\alpha\varepsilon_\nu l^\alpha l^\nu}{(l^2-a^2)^3}+\frac{z_1(z_1+z_3)m_\mu\varepsilon\cdot p}{(l^2-a^2)^3}\right).\end{aligned} \tag{79}$$

We are interested only in terms proportional to $\varepsilon\cdot p$ so we discard the first integral in the second line of (79) since it is proportional to $\varepsilon\cdot\gamma$ [14]. The remaining integral in (79) is well known[15]. Then (78) becomes:

$$\sum_i T_i(d)$$

$$\xrightarrow[\varepsilon\cdot p\text{ only}]{}\frac{cm_\mu(\varepsilon\cdot p)}{8\pi^2M^2}\overline{u}_e(p-q)(1+\gamma_5)\int_0^1 dz_1\int_0^{1-z_1}dz_3\frac{z_1(z_1+z_3)}{a^2}u_\mu(p).$$

In order to calculate the remaining integral over Feynman parameters, we need to approximate a^2. We have defined a^2 in (77). Since $M\equiv M_W\gg m_\mu$ we can make the approximation:

$$\frac{1}{a^2}=\frac{1}{\xi M^2(1-z_1)+(z_1p+z_3q)^2-z_1p^2}\simeq\frac{1}{\xi M^2(1-z_1)}.$$

By using this approximation the integral over Feynman parameters becomes trivial and we get:

[14] symmetry allows one to replace [2]: $l^\mu l^\nu\to\frac{1}{4}l^2g^{\mu\nu}$

[15] [3]: $\int d^4l\frac{1}{(l^2-s)^3}=-\frac{i\pi^2}{2s}$

$$\sum_i T_i(d) = \frac{5c}{96\pi^2}(\varepsilon\cdot p)\overline{u}_e(p-q)(1+\gamma_5)u_\mu(p)\frac{m_\mu}{\xi M^4}.$$

When we compare this to (71), we finally get the invariant amplitude A:

$$A = \frac{c}{36\pi^2}\left(\frac{m_\mu}{M^4}\right)\frac{5}{6\xi}.$$

B Supersymmetry

Supersymmetry is a space-time symmetry which relates bosons and fermions to each other. The particles of supersymmetric theory reside in irreducible representations of supersymmetry algebra, called *supermultiplets*. A supermultiplet contains both boson and fermion states which are said to be *superpartners* of each other. A supersymmetry transformation transforms fermions to their bosonic superpartners and vice versa. Standard Model particles and their supposed superpartners (they have never been observed) fit in two kinds of supermultiplets: *chiral supermultiplets* and *vector supermultiplets*. Chiral supermultiplets contain the SM spin-1/2 fermions, quarks and leptons, and their spin-0 superpartners. Also Higgs boson resides in chiral supermultiplet along with it's spin-1/2 superpartner. Vector supermultiplet contains the gauge bosons and their spin-1/2 superpartners.

In supersymmetry it is convenient to use so called *Weyl spinors* instead of the usual Dirac spinors. The Weyl representation of gamma matrices are

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \qquad \gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \mu\text{=0,1,2,3}$$

where

$$\sigma^\mu \equiv (I_2, \boldsymbol{\sigma})$$
$$\bar{\sigma}^\mu \equiv (I_2, -\boldsymbol{\sigma}) = \sigma_\mu.$$

In this representation Dirac spinor Ψ_D can be written in terms of it's left- and right-handed parts as follows:

$$\Psi_D = \Psi_L + \Psi_R = \begin{pmatrix} \psi_\alpha \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}.$$

The two component objects ψ_α $(\alpha = 1,2)$ and $\bar{\chi}^{\dot{\alpha}}$ $(\dot{\alpha} = \dot{1},\dot{2})$ are called *left-* and *right-handed Weyl spinors* respectively. We have two sets of spinor indices: dotted and undotted. They represent the fact that first two components (undotted) transform inder a different representation of the Lorentz group than the last three components (dotted). The four-component Dirac representation of the Lorentz group is not irreducible, but the two component chiral representation is. One can define:

$$\bar{\psi}_{\dot{\alpha}} \equiv (\psi_\alpha)^*, \qquad \chi^\alpha \equiv (\bar{\chi}^{\dot{\alpha}})^*.$$

Weyl spinors anticommute:

$$\{\psi,\chi\} = \{\bar{\psi},\bar{\chi}\} = \{\psi,\bar{\chi}\}.$$

Simple supersymmetry algebra is:

$$\begin{cases} [Q_\alpha, P^\mu] = [\bar{Q}_{\dot{\alpha}}, P^\mu] = 0 \\ \{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma^\mu_{\alpha\dot{\beta}} P_\mu \\ \{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0. \end{cases}$$

Operator Q_α is Weyl spinor generator. This algebra holds for so called $N = 1$ supersymmetry, where the theory contains only one supersymmetry, i.e. there is only one supersymmetry generator Q_α. In general one could add more supersymmetries in the theory in which case there would be more supersymmetry generators: $Q_{A\alpha}, A = 1, 2, ..., N$. This theory would contain N different supersymmetries. Even though one could add arbitrarily many supersymmetries in the theory, it is not usually wise if one wants to avoid complications. If the N exceeds 8, the theory must contain particles whose spin is greater than two, which is unpleasant.

B.1 Weyl spinor notation

B.1.1 Chiral supermultiplets

Chiral supermultiplets contain all scalar fields of the theory and their spin-1/2 fermionic superpartners (also the vector supermultiplet contains spin-1/2 fermions which are superpartners of the vector bosons). The degrees of freedom of the bosons and the fermions must always match in the supermultiplets, whether they are on-shell or off-shell. In chiral supermultiplet one has to add a non-propagating auxiliary field F in order the bosonic and fermionic degrees of freedom to match.

The free non-interacting part of the Lagrangian for the chiral supermultiplet is:

$$\mathcal{L}_{\text{chiral,free}} = -\partial^\mu \phi^{*i} \partial_\mu \phi_i + i\psi^{\dagger i} \bar{\sigma}^\mu \partial_\mu \psi_i + F^{*i} F_i,$$

where the index i is summed over and it represents the different chiral supermultiplets of the theory.

There are also interactions between the particles of the chiral superfields (and these don't include gauge interactions). The interactions of the supermultiplets are derived from the function called the *superpotential* W, which is a holomorphic function of the scalar fields ϕ_i:

$$W = L^i \phi_i + \frac{1}{2} M^{ij} \phi_i \phi_j + \frac{1}{6} y^{ijk} \phi_i \phi_j \phi_k,$$

where the L^i is a parameter with dimension $[\text{mass}]^2$, M^{ij} is a symmetric mass matrix for the fermion fields and y^{ijk} is a Yukawa coupling of scalar ϕ_k and the fermions ψ_i and ψ_j. The interaction Lagrangian for chiral superfields is:

$$\mathcal{L}_{\text{chiral,int}} = -\frac{1}{2} \frac{\partial^2 W}{\partial \phi_i \partial \phi_j} \psi_i \psi_j - \frac{1}{2} \frac{\partial^2 W^*}{\partial \phi_i^* \partial \phi_j^*} \bar{\psi}_i \bar{\psi}_j + \frac{\partial W}{\partial \phi_i} F_i + \frac{\partial W^*}{\partial \phi_i^*} F_i^*.$$

The complete Lagrangian for chiral supermultiplets is $\mathcal{L}_{\text{chiral}} = \mathcal{L}_{\text{chiral,free}} + \mathcal{L}_{\text{chiral,int}}$. The auxiliary field F is non-propagating field that was introduced just to get right match of degrees of freedom. Since auxiliary field doesn't have derivative term in the Lagrangian it's classical equation of motion is just an algebraic equation:

$$F_i = -\frac{\partial W^*}{\partial \phi_i^*} \qquad F_i^* = -\frac{\partial W}{\partial \phi_i}.$$

The Lagrangian for the chiral supermultiplets therefore is:

$$\begin{aligned} \mathcal{L}_{\text{chiral}} &= -\partial^\mu \phi^{*i} \partial_\mu \phi_i + i\psi^{\dagger i} \bar{\sigma}^\mu \partial_\mu \psi_i \\ & -\frac{1}{2} \frac{\partial^2 W}{\partial \phi_i \partial \phi_j} \psi_i \psi_j - \frac{1}{2} \frac{\partial^2 W^*}{\partial \phi_i^* \partial \phi_j^*} \bar{\psi}_i \bar{\psi}_j - \frac{\partial W}{\partial \phi_i} \frac{\partial W^*}{\partial \phi_i^*}. \end{aligned} \tag{80}$$

Supersymmetry transformation is

$$\begin{aligned} &\delta_\xi \phi_i = \xi \psi_i, && \delta_\xi \phi^{*i} = \bar{\xi} \bar{\psi}^i \\ &\delta_\xi (\psi_i)_\alpha = -i(\sigma^\mu \bar{\xi})_\alpha \partial_\mu \phi_i + \xi_\alpha F_i, && \delta_\xi (\bar{\psi}^i)_{\dot{\alpha}} = i(\xi \sigma^\mu)_{\dot{\alpha}} \partial_\mu \phi^{*i} + \bar{\xi}_{\dot{\alpha}} F^{*i} \\ &\delta_\xi F_i = -i\bar{\xi} \bar{\sigma}^\mu \partial_\mu \psi_i, && \delta_\xi F^{*i} = i\partial_\mu \bar{\psi}^i \bar{\sigma}^\mu \xi, \end{aligned}$$

where ξ is an infinitesimal anticommuting Weyl spinor that characterizes the supersymmetry transformation and ϕ is a complex scalar field, ψ is a left-handed Weyl spinor field and F is a non-propagating complex auxiliary field that ensures that supersymmetry algebra closes off-shell.

B.1.2 Vector supermultiplets

Vector supermultiplets contain all the gauge bosons of the theory and their spin-1/2 superpartners called the *gauginos*. As in the case of chiral supermultiplets, also here the degrees of freedom between fermions and bosons must match both on- and off-shell. One has to add one real pseudoscalar auxiliary field D^a to vector supermultiplet, in order the degrees of freedom to match [11]. The Lagrangian for the vector or gauge multiplet is:

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + i\bar{\lambda}^a \bar{\sigma}^\mu \nabla_\mu \lambda^a + \frac{1}{2} D^a D^a, \tag{81}$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

is the Yang-Mills field strength. The λ^a's are the two-component Weyl spinors describing the gauginos, and the a runs over the adjoint representation of the gauge group in question[16].
The covariant derivative of the gaugino field is:

$$\nabla_\mu \lambda^a = \partial_\mu \lambda^a + g f^{abc} A^b_\mu \lambda^c.$$

The Lagrangian (81) is invariant in the following gauge transformation:

[16] For gluons and gluinos the gauge group is $SU(3)_C$ and $a = 1, 2, ...8$. For $SU(2)_L$ gauge bosons and their superpartners $a = 1, 2, 3$. For $U(1)_Y$ $a = 1$.

$$A^a_\mu \rightarrow A^a_\mu + \partial_\mu \Lambda^a + g^{abc} A^b_\mu \Lambda^c \quad (82)$$
$$\lambda^a \rightarrow \lambda^a + g f^{abc} \lambda^b \Lambda^c. \quad (83)$$

It is also invariant under supersymmetry transformation (up to a total derivative) which we don't state here.

B.1.3 Supersymmetric gauge interactions

We have now reviewed the properties of the chiral and vector multiplets separately. It is now time to put these together. The fermions in the chiral supermultiplet are the SM fermions: quarks and leptons. The quarks know all the interactions and the leptons know the electroweak interactions. The scalar superpartners of the SM fermions have the same quantum numbers as the original particles, with the exception of spin, which differs by 1/2. The scalar superpartners therefore have the same gauge couplings as their SM counterparts and know exactly the same interactions. The gauge interactions of the particles in the chiral supermultiplets must be incorporated to the theory somehow. It can be done by changing the normal derivatives in the chiral Lagrangian (80) into the following covariant derivatives:

$$\nabla_\mu \phi_i = \partial_\mu \phi_i - igA^a_\mu (T^a \phi)_i \quad (84)$$
$$\nabla_\mu \phi^*_i = \partial_\mu \phi^*_i + igA^a_\mu (\phi^* T^a)_i \quad (85)$$
$$\nabla_\mu \psi_i = \partial_\mu \psi_i - igA^a_\mu (T^a \psi)_i. \quad (86)$$

There are also other interactions however. The gauge fields A^a_μ couple to scalars ϕ_i and fermions ψ_i, but so do gaugino fields λ^a and auxiliary fields D^a. So we get the most general supersymmetric Lagrangian when we combine the gauge field Lagrangian (81) , the chiral Lagrangian (80) in which the derivatives are replaced with the covariant derivatives (84), (85), (86), and the possible renormalizable interaction terms involving scalars, their fermionic partners, gauginos and the auxiliary fields D^a:

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{chiral}} \\ &\quad -\sqrt{2}g(\phi^* T^a \psi)\lambda^a - \sqrt{2}g\bar{\lambda}^a(\bar{\psi} T^a \phi) + g(\phi^* T^a \phi)D^a. \quad (87)\end{aligned}$$

The auxiliary field D^a can be expressed algebraically in terms of the scalar fields of the theory as $D^a = -g(\phi^* T^a \phi)$.

Finally we discuss about the scalar potential of the general supersymmetric Lagrangian (87). The scalar potential in the Lagrangian of the chiral multiplet is determined by the auxiliary fields F and F^* as $F^{*i}F^i$. Also the scalar potential coming from the "extra" interaction terms in (87) and (81) is determined by the auxiliary field D^a as $\frac{1}{2}D^a D^a$. So the complete scalar potential of supersymmetry is:

$$V(\phi, \phi^*) = F^*_i F_i + \frac{1}{2}\sum_a D^a D^a = |\frac{\partial W}{\partial \phi_i}|^2 + \frac{1}{2}\sum_a g^2_a (\phi^* T^a \phi)^2. \quad (88)$$

The first term is called the "F"-term and the second term is called the "D"-term. These terms are important when one studies the spontaneous supersymmetry breaking.

B.1.4 Supersymmetry-breaking

Theory can have spontaneously broken symmetry only if scalar potential of the theory has a non-zero VEV. The potential (88) is the most general scalar potential of the supersymmetric theory so if the supersymmetry were to break spontaneously, one of the auxiliary fields must get a non-zero VEV. Model in which the SUSY-breaking happens by the D-term VEV is called *Fayet-Iliopoulos supersymmetry breaking model.* Model in which the SUSY-breaking is due to F-term VEV is called the *O'Raifeartaigh supersymmetry breaking model.* At tree-level the following relation holds for *supertrace* $STr(m^2)$:

$$STr(m^2) \equiv \sum_i (-1)^{2j}(2j+1)Tr(m_j^2) = 0. \tag{89}$$

The supertrace sums over all the particles in the theory. This relation restricts the mass spectrum of the theory. If one applies the F- and D-term breakings to MSSM one notices that the relation (89) requires implausible masses of the superpartners [49]. The supersymmetry therefore can not be spontaneously broken by F- and D-terms of the MSSM fields. The way around this is to assume new field that interact very weakly with the MSSM particles. This new sector is called the *hidden sector*. In hidden sector models the supersymmetry is spontaneously broken by the scalar fields of the hidden sector. The breaking is then communicated to the *visible sector* (MSSM or its extension) by the *messenger fields*. There are many supersymmetry-breaking models involving a separate SUSY-breaking sector. They have different ways of communicating the SUSY-breaking to the visible sector. The most popular models are the *gravity*- and *gauge*-mediated models. These are discussed in the section 5.1.

Whatever the supersymmetry-breaking mechanism is, the result of the breaking can be parametrized in the so called *soft supersymmetry-breaking terms*. The soft SUSY-breaking terms have couplings whose mass dimensions are positive. This maintains the hierarchy between the electroweak scale and the Planck scale [48]. The soft terms for general supersymmetric theory are:

$$\mathcal{L}_{\text{soft}} = -\left(\frac{1}{2}M_a\lambda^2\lambda^2 + \frac{1}{6}a^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}b^{ij}\phi_i\phi_j + t^i\phi_i\right) + c.c. - (m^2)^i_j\phi^{j*}\phi_i. \tag{90}$$

The parameters are gaugino masses M_a, scalar squared masses $(m^2)_i^j$ and b^{ij}, trilinear scalar couplings a^{ijk} and c_i^{jk}, and the "tadpole" coupling t^i. Softly broken supersymmetric theory that has this kind of soft terms, has no quadratic divergences in correction to the scalar squared masses to all orders of perturbation theory, as was shown in [51]. This is why they are called "soft" supersymmetry breaking terms.

When the supersymmetry is spontaneously broken potentially all the scalars and gauginos acquire masses (the mass parameters can in principle be zero). These mass parameters can be quite large, which is in a agreement with the experiments: superpartners have never been observed (at time of writing, early 2015). LHC starts its experimets again in early 2015 with higher 13 TeV collision energy and it should be more than capable of producing superpartners.

In general these soft terms involve sources of lepton flavour violation. The scalar masses and trilinear couplings can in principle contain non-diagonal terms and they are of the same order of magnitude as the diagonal ones if there is no principle to tell otherwise. This is a serious problem since from the experiments we know that the lepton flavour violating processes are extremely suppressed. Large off-diagonal soft terms could give too large transition rates for CLFV processes. To explain the phenomenologically wanted small off-diagonal terms, one can use some explicit supersymmetry breaking model which generates the small off-diagonal soft terms. These kind of models are minimal supergravity models and gauge mediated SUSY-breaking models for example (but to be honest the gravity-mediation does not really produce small off-diagonal terms without radical assumptions).

B.2 Superfield notation

Supersymmetry can also be formulated using more sophisticated *superfield*-notation. In superspace we have linear representation of supersymmetry algebra [48]:

$$\begin{cases} Q_\alpha = i\frac{\partial}{\partial\theta^\alpha} - \sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu \\ \bar{Q}^{\dot{\alpha}} = i\frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}} - \bar{\sigma}^{\mu\dot{\alpha}\alpha}\theta_\alpha\partial_\mu . \end{cases}$$

An ordinary field is a function of the space-time coordinate x^μ only, but a superfield $S(x,\theta,\bar{\theta})$ is also a function of anticommuting *Grassmann variables* θ_α and $\bar{\theta}_{\dot{\alpha}}$. A general superfield can therefore be expanded in a power series in Grassmann variables:

$$\begin{aligned} &S(x,\theta,\bar{\theta}) = f(x) + \theta\phi(x) + \bar{\theta}\bar{\chi} + \theta\theta m(x) + \bar{\theta}\bar{\theta}n(x) \\ &\theta\sigma^\mu\bar{\theta}V_\mu(x) + \theta\theta\bar{\theta}\bar{\lambda}(x) + \bar{\theta}\bar{\theta}\theta\psi(x) + \theta\theta\bar{\theta}\bar{\theta}d(x), \end{aligned}$$

where f, m, n, d are scalar fields, V_μ is a vector field and $\phi, \psi, \bar{\chi}, \bar{\lambda}$ are Weyl spinor fields. This superfield contains more degrees of freedom than chiral and vector supermultiplets. This is therefore a reducible representation of supersymmetry. In order to construct chiral and vector superfields we have to impose constraints on the general superfield.

We define *chiral covariant derivative* as:

$$\begin{aligned} D_\alpha &= \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu \\ \bar{D}^{\dot{\alpha}} &= \frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}} - i\bar{\sigma}^{\mu\dot{\alpha}\alpha}\theta_\alpha\partial_\mu . \end{aligned}$$

This is used to define *left-* and *right-handed chiral superfields* Φ and $\Phi^\dagger$:

$$\begin{aligned} \bar{D}_{\dot{\alpha}}\Phi &= 0 \\ D_\alpha\Phi^\dagger &= 0 \end{aligned}$$

A general left-handed chiral superfield can be written as:

$$\Phi(x^\mu,\theta,\bar{\theta}) = \phi + \sqrt{2}\theta\psi + \theta\theta F + i\bar{\theta}\bar{\sigma}^\mu\theta\partial_\mu\phi - \frac{i}{\sqrt{2}}\theta\theta\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\partial_\mu\partial^\mu\phi,$$

where ϕ and F are complex scalar fields and ψ is a left-handed Weyl spinor field. Product of any left-handed chiral superfields is also a left-handed chiral superfield. The coefficient of $\theta\theta$ is referred as the F-term and the coefficient of $\theta\theta\bar{\theta}\bar{\theta}$ is referred as the D-term. These are important when studying spontaneous breaking of supersymmetry.

A vector superfield V is a superfield that satisfies the condition:

$$V = V^*,$$

which implies

$$f = f^*, \qquad \bar{\phi} = \bar{\xi}, \qquad m = n^*, \qquad V_\nu = V_\nu^*, \qquad \bar{\lambda} = \bar{\psi}, \qquad d = d^*.$$

A general vector superfield therefore is:

$$V(x, \theta, \bar{\theta}) = f(x) + \phi(x)\theta + \bar{\phi}(x)\bar{\theta} + m(x)\theta\theta + m^*(x)\bar{\theta}\bar{\theta}$$
$$\theta\sigma^\mu\bar{\theta}V_\mu(x) + \bar{\lambda}(x)\theta\theta\bar{\theta} + \lambda(x)\bar{\theta}\bar{\theta}\theta + d(x)\theta\theta\bar{\theta}\bar{\theta}.$$

B.3 Charginos and neutralinos

After the electroweak $SU(2)_L \times U(1)_Y$ symmetry breaks spontaneously, the gauge eigenstates no longer are the mass eigenstates. This imposes mixing among the particles with same quantum numbers. This happens in SM with $U(1)_Y$ gauge boson B and the $SU(2)_L$ gauge bosons W^1, W^2 and W^3. The B and W^3 mix together and the mass eigenstates are the neutral photon and the Z^0 boson. Also the W^1 and W^2 mix and the resulting mass eigenstates are the charged W-bosons, W^- and W^+.

The superpartners of B, W^1, W^2 and W^3, namely the Bino $\widetilde{B}$ Winos $\widetilde{W}^1$, $\widetilde{W}^2$ and $\widetilde{W}^3$, experience the similar mixing. However in case of gauge boson superpartners the mixing involves more particles. The certain spin-1/2 superpartners of Higgs bosons $\psi_{h_u^0}$, $\psi_{h_d^0}$, $\psi_{h_u^+}$ and $\psi_{h_d^-}$ share quantum numbers with certain gauginos. The neutral gauginos $\widetilde{B}$ and $\widetilde{W}^3$ mix with the neutral Higgsinos $\psi_{h_u^0}$ and $\psi_{h_d^0}$ to form four mass eigenstates called *neutralinos*, $\widetilde{\chi}_A^0$, $A = 1, 2, 3, 4$. The negatively charged Higgsino $\psi_{h_d^-}$ mixes with the super partner of W^-, $\widetilde{W}^-$, to form negatively charged *charginos*, $\widetilde{\chi}_A^-$, $A = 1, 2$. The positively charged Higgsino in turn mixes with the $\widetilde{W}^+$ to form positively charged charginos, $\widetilde{W}_A^+$, $A = 1, 2$.

At low energies the calculations are conducted using the mass eigenstates, the neutralinos and charginos. In supersymmetric theories with exactly conserved R-parity quantum number, like MSSM, there can be no mixing between the Standard Model particles and the Higgs bosons (R-parity +1), and their superpartners (R-parity-1), because the mixing particles must have the exactly the same quantum numbers.

The fermion-sfermion-gaugino vertices are possibly responsible for flavour violations. After the electroweak symmetry breaking the gauginos however mix with the Higgsinos and the possibly flavour violating fermion-sfermion-gaugino vertices become *fermion-sfermion-neutralino* and *fermion-sfermion-chargino* vertices. The Lagrangian for fermion-sfermion-neutralino interaction is [54]:

$$\mathcal{L}_{neutralino} = \bar{f}_i(N_{iAX}^{R(f)}P_R + N_{iAX}^{L(f)}P_L)\widetilde{\chi}_A^0\widetilde{f}_X + h.c.. \qquad (91)$$

The f_i is the fermion in mass eigenstate with the generation index $i = 1, 2, 3$ and $\widetilde{f}_X$ is the sfermion in the mass eigenstate. The $X = 1, 2, 3$ for sneutrinos $\widetilde{\nu}$ (these are partners of the light neurinos; the heavy neutrino states are effectively decoupled at the electroweak scale and don't therefore affect to fermion-sfermion-neutralino vertices). The $X = 1, ..., 6$ for other fermions. The neutralino index A runs from 1 to 4. The coefficients $N_{iAX}^{R(f)}$ and $N_{iAX}^{L(f)}$ depend on the mixing among the neutralinos and among the sfermions. Their explicit form (in the context of right-handed neutrinos) is given in the reference [54]. The Lagrangian for the fermion-sfermion-chargino interaction can be written as [54]:

$$\mathcal{L}_{chargino} = \bar{l}_i(C_{iAX}^{R(l}P_R + C_{iAX}^{L(l)}P_L)\widetilde{\chi}_A^- \widetilde{\nu}_X + \bar{\nu}_i(C_{iAX}^{R(\nu}P_R + C_{iAX}^{L(\nu)}P_L)\widetilde{\chi}_A^+ \widetilde{l}_X \quad (92)$$

$$+\bar{d}_i(C_{iAX}^{R(d}P_R + C_{iAX}^{L(d)}P_L)\widetilde{\chi}_A^- \widetilde{u}_X + \bar{u}_i(C_{iAX}^{R(u}P_R + C_{iAX}^{L(u)}P_L)\widetilde{\chi}_A^+ \widetilde{d}_X + h.c., \quad (93)$$

where the neutralino index a runs from 1 to 2. The explicit form of the coefficients (in the context of right-handed neutrinos) is given in the reference [54].

C Seesaw mechanism

As we have said many times in the main text, the Standard Model treats the neutrinos as massless Dirac particles that is they have distinct antiparticles. Experimentally the neutrinos are known to have masses and the Dirac nature of neutrinos is not proven. These should be clear in the extension of the Standard Model. We will now concentrate on the *seesaw-I mechanism*, which is one of the most popular methods of introducing the neutrino masses in to the simple extensions of the Standard Model. The seesaw-I can also be generalized to the supersymmetric theories to generate the neutrino masses in them (as discussed in the section 5.2.2). Before we can review the seesaw-I, we need to state some notation.

There are two types of mass terms that can be written for a fermion that are renormalizable and gauge invariant: Dirac and Majorana mass terms. Dirac mass term can be written as:

$$\mathcal{L}_D = -m_D\bar{\psi}\psi = -m_D(\bar{\psi}_L + \bar{\psi}_R)(\psi_L + \psi_R) = -m_D(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L).$$

To define Majorana mass terms we have to define *charge conjugation*:

$$\begin{cases} \psi^c \equiv C\gamma^0\psi^* = (-i\gamma^0\gamma^2)\gamma^0\psi^* = i\gamma^2\psi^* \\ \bar{\psi}^c \equiv \psi^T C \end{cases} . \tag{94}$$

The charge conjugation matrix C has the following properties:

$$C^T = C^\dagger = -C = C^{-1}.$$

There are left- and right-handed Majorana mass terms defined as:

$$\mathcal{L}_L^M = -\frac{m_L}{2}\left[\overline{(\psi_L)^c}\psi_L + \bar{\psi}_L(\psi_L)^c\right]$$

$$\mathcal{L}_R^M = -\frac{m_R}{2}\left[\overline{(\psi_R)^c}\psi_R + \bar{\psi}_R(\psi_R)^c\right]$$

It is possible that Dirac and Majorana mass terms exist simultaneously. This combined mass term can be written using identity:[17]

$$\bar{\psi}_1^c\psi_2^c = \bar{\psi}_2\psi_1.$$

Using this we can write Dirac mass term as

$$\mathcal{L}_D = -\frac{m_D}{2}(\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R + \overline{(\psi_L)^c}(\psi_R)^c + \overline{(\psi_R)^c}(\psi_L)^c).$$

The combined mass term can now be written as a product of matrices:

$$\mathcal{L}_m = \mathcal{L}_D + \mathcal{L}_L^M + \mathcal{L}_R^M = -m_D\bar{\psi}_R\psi_L - \frac{m_L}{2}\bar{\psi}_L^c\psi_L - \frac{m_R}{2}\bar{\psi}_R\psi_R^c + h.c.$$

[17]proof: using (94): $\bar{\psi}_1^c\psi_2^c = -(\psi_1)^T\gamma^0\psi_2^* = -(\bar{\psi}_2\psi_1)^T$. The spinor components anticommute [3]: $\{\psi(x), \bar{\psi}(y)\} = iS(x-y)$ (when $x = y$). Now the minus cancels when the left- and right-handed spinors are switched and the transpose is canceled when the spinors are switched again.

$$= -\frac{1}{2} \begin{pmatrix} \overline{(\psi_L)^c} & \bar{\psi}_R \end{pmatrix} \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \begin{pmatrix} \psi_L \\ (\psi_R)^c \end{pmatrix} + h.c.$$

Majorana mass terms are only possible for neutral fermions, whereas Dirac mass terms are allowed to all. In the SM there are only three neutral fermions: the neutrinos. In the Standard Model the neutrinos are assumed to be massless and Dirac particles (i.e. have distinct antiparticles). It is however experimentally known that neutrinos have tiny masses. The upper limit of electron neutrino is experimentally known to be $\lesssim 1eV$. The explanation of such a small mass is a huge problem. If one tries to suggest that the neutrinos gain their masses in ordinary Higgs mechanism like the other fermions, one stumbles to the so called *hierarchy problem*, as explained bellow. When the electroweak symmetry breaks spontaneously the Yukawa interaction terms become fermion mass terms as in equation (17):

$$\mathbf{m}'_i = \frac{v}{\sqrt{2}}\mathbf{f}_i \qquad i = u, d, e. \tag{95}$$

Now if neutrinos were given their masses in similar fashion the electron neutrino mass would be

$$m_{\nu_e} = \frac{v}{\sqrt{2}} f_{\nu_e}$$

where the neutrino Yukawa matrix is assumed diagonal. If one assumes $v =$ 246GeV and that $m_{\nu_e} \lesssim 1eV$, one gets for the electron neutrino Yukawa coupling:

$$f_{\nu_e} \lesssim 10^{-12} \lll f_e.$$

This seems unnatural. The neutrino Yukawa coupling would have to be *fine-tuned* to a precise value by multiple orders of magnitude. The Yukawa coupling should be ~ 1 to be natural. To explain the tiny value of neutrino mass, the Seesaw mechanism was created [7].

In seesaw mechanism one introduces a right-handed neutrino through combined mass term:

$$\mathcal{L}_{seesaw} = \mathcal{L}_D + \mathcal{L}_L^M + \mathcal{L}_R^M =$$
$$-\frac{1}{2} \begin{pmatrix} \overline{(\nu_L)^c} & \bar{\nu}_R \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix} \begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix} + h.c.$$
$$\equiv -\frac{1}{2}\overline{\mathbf{N^c}}\mathbf{M}\mathbf{N} + h.c.$$

The right-handed neutrino has right-handed Majorana mass term, and is a gauge singlet, i.e. it does not know gauge interactions. It is also called *sterile neutrino*, since it does not interact with the Standard Model particles. The left-handed neutrino is not given a left-handed Majorana mass, because it would lead to inconsistencies and one would be required to add new particles to the theory just to get rid of them.

The masses of the physical neutrinos are the eigenvalues of the mass matrix $\mathbf{M}$:

$$\det \begin{pmatrix} -\lambda & m_D \\ m_D & m_R - \lambda \end{pmatrix} = 0$$

$$\Rightarrow \lambda = \frac{1}{2} m_R \left(1 \pm \sqrt{1 + \frac{4m_D^2}{m_R^2}} \right)$$

In seesaw-I mechanism it is assumed that the Majorana mass of the right-handed neutrino, m_R, is much larger than the neutrino Dirac mass, m_D, which is common to both left- and right-handed neutrino: $m_D \ll m_R$. In this case one can approximate the eigenvalues[18]:

$$\lambda_1 \approx m_R, \qquad \text{and} \qquad \lambda_2 \approx -\frac{m_D^2}{m_R}.$$

One notices that as the Majorana mass is raised the other eigenvalue grows, whereas the other grows smaller. Hence the name Seesaw mechanism.

It is usually assumed that the Majorana mass m_R is close to the Grand Unification energy scale $\Lambda_{GUT} \sim 10^{16}$. Then if one assumes that the Dirac mass of the neutrino is at the order of electroweak scale v, the mass of the left-handed neutrino would be:

$$m_{\nu_L} = -\lambda_2 \sim \frac{v^2}{\Lambda_{GUT}} \approx 0.01\text{eV}.$$

This removes the necessity of the fine-tuning of the neutrino Yukawa coupling, it is now allowed to be ~ 1 since the Dirac mass is around the electroweak scale v.

Even though the seesaw mechanism ensures the naturalness of the neutrino Yukawa coupling, it does not solve the *hierarchy problem.* The Dirac mass of the neutrino is of the order of the electroweak scale, $v = 174$ GeV, and the Majorana mass m_R is the order of 10^{16}GeV. There is an enormous gap between the EW scale and the m_R scale leads to the hierarchy problem: tree level Higgs boson has the mass of the order of v, but the radiative corrections drive it to extremely high values, since the physical cut-off must be at least the scale of the Majorana mass m_R. Luckily the supersymmetry will solve this hierarchy problem by introducing superpartners, which will cancel the quadratic corrections to the Higgs mass.

The Standard Model can be extended to include effective operators with mass dimensions greater than four, as we have discussed in section 3. In that section we only considered dimension six operators, discarding the only existing dimension five operator, the Weinberg-operator:

$$Q_{Weinberg} = \epsilon_{ab}\epsilon_{cd}\phi^a\phi^c(l_i^b)^T C l_j^d,$$

since it only contributed to the neutrino masses. But now it is all about neutrino masses. The Weinberg operator is an effective operator which is a left over of some higher theory diagram whose heavy new physics fields have been integrated out. There is only three possible models that produce the Weinberg operator as the heavy fields are integrated out at the tree-level [98]. Those models are called seesaw-I, II and III. In the seesaw-I the SM is extended to have three heavy scalar fields, right-handed neutrinos, to generate small neutrino masses. In seesaw-II the SM is extended to have a heavy scalar triplet (ξ^{++}, ξ^+, ξ^0) [99], [100] and in the seesaw-III the SM is extended with a heavy fermion triplet $(\Sigma^+, \Sigma^0, \Sigma^-)$ [100].

[18] $\sqrt{1+x} \approx 1 + \frac{x}{2}$

D Renormalization group evolution

As we know from standard quantum field theory, masses and coupling constants acquire corrections from the higher orders of perturbation theory. Therefore the masses and the couplings change as the energy varies. One must always use the correct values for the masses and the couplings, i.e. one must use the values at the energy the physics under consideration happens. One can encapsulate the energy dependence of parameters in *renormalization group equations*.

D.1 Renormalization group equations in MSSM extended with massive neutrinos

The MSSM renormalization group equations are as follows [49]. These will include the right-handed neutrinos, so these equations actually describe extended MSSM. Renormalization group equations for the gauge couplings to two-loop order are:

$$\frac{dg_a}{dt} = \frac{g_a^3}{16\pi^2} b_a + \frac{g_a^3}{(16\pi^2)^2} \left[\sum_{b=1}^{3} B_{ab}^{(2)} g_b^2 - \frac{1}{16\pi^2} \sum_{x=u,d,e,\nu} \frac{C_a^x}{16\pi^2} Tr(Y_x^\dagger Y_x) \right],$$

where $t = \ln(Q/Q_0)$ (Q is the renormalization scale in the $\overline{MS}$ scheme and Q_0 is the high energy input scale), $b_a = (\frac{33}{5}, 1, -3)$, and

$$B_{ab}^{(2)} = \begin{pmatrix} \frac{199}{25} & \frac{27}{5} & \frac{88}{5} \\ \frac{9}{5} & 25 & 24 \\ \frac{11}{5} & 9 & 14 \end{pmatrix},$$

and

$$C_a^{u,d,e\nu} = \begin{pmatrix} \frac{26}{5} & \frac{14}{5} & \frac{18}{5} & \frac{6}{5} \\ 6 & 6 & 2 & 2 \\ 4 & 4 & 0 & 0 \end{pmatrix}.$$

The Yukawa matrix Y_ν for neutrinos is of course zero in the MSSM.

The renormalization group equations for the gaugino masses to two-loop order in $\overline{DR}$ scheme[19] are:

$$\frac{dM_a}{dt} = \frac{2g_a^2}{16\pi^2} b_a M_a + \frac{2g_a^2}{(16\pi^2)^2} \sum_{b=1}^{3} B_{ab}^{(2)} g_b^2 (M_a + M_b)$$
$$+ \frac{2g_a^2}{(16\pi^2)^2} \sum_{x=u,d,e,\nu} C_a^x \left(Tr[Y_x^\dagger a_x] - M_a Tr\left[Y_x^\dagger Y_x\right] \right),$$

[19]In SM the most popular regularization method is *dimensional regularization* (DREG). One can not use DREG in supersymmetry however, since when the space-time dimensions are continued from 4 to $4 - 2\epsilon$, the SUSY will be violated because the numbers of gauge boson decrees of freedom and the gaugino degrees of freedom do not match off-shell anymore. The *regularization by dimensional reduction* (DRED) preserves the supersymmetry. In DRED the momentum integrals are performed in $d - 2\epsilon$ dimensions, but the gauge boson field A_μ^a Lorentz, μ, index is still 4-dimensional, not $4 - 2\epsilon$. The modified minimal subtraction in DREG is called $\overline{MS}$ and in DRED it is called $\overline{DR}$.

where the $a_x = A_0 Y_x$ is the trilinear scalar coupling.

The Yukawa coupling RG equations to one-loop order are:

$$\frac{dY_u}{dt} = \frac{1}{16\pi^2}[N_q \cdot Y_u + Y_u \cdot N_u + (N_{H_u}) \cdot Y_u] \quad (96)$$
$$\frac{dY_d}{dt} = \frac{1}{16\pi^2}[N_q \cdot Y_d + Y_d \cdot N_d + (N_{H_d}) \cdot Y_d] \quad (97)$$
$$\frac{dY_\nu}{dt} = \frac{1}{16\pi^2}[N_l \cdot Y_\nu + Y_\nu \cdot N_\nu + (N_{H_u}) \cdot Y_\nu] \quad (98)$$
$$\frac{dY_e}{dt} = \frac{1}{16\pi^2}[N_l \cdot Y_e + Y_e \cdot N_e + (N_{H_d}) \cdot Y_e] \quad (99)$$

where

$$N_q = Y_u Y_u^\dagger - (\frac{8}{3}g_3^2 + \frac{3}{2}g_2^2 + \frac{1}{30}g_1^2)\mathbf{1} \quad (100)$$
$$N_u = 2Y_u^\dagger Y_u - (\frac{8}{3}g_3^2 + \frac{8}{15}g_1^2)\mathbf{1} \quad (101)$$
$$N_d = 2Y_d^\dagger Y_d - (\frac{8}{3}g_3^2 + \frac{2}{15}g_1^2)\mathbf{1} \quad (102)$$
$$N_l = Y_e Y_e^\dagger + Y_\nu Y_\nu^\dagger - (\frac{3}{2}g_2^2 + \frac{3}{10}g_1^2)\mathbf{1} \quad (103)$$
$$N_e = 2Y_e^\dagger Y_e - \frac{6}{5}g_1^2\mathbf{1} \quad (104)$$
$$N_\nu = 2Y_\nu^\dagger Y_\nu \quad (105)$$
$$N_{H_u} = 3Tr(Y_u^\dagger Y_u) + Tr(Y_\nu^\dagger Y_\nu) - (\frac{3}{2}g_2^2 + \frac{3}{10}g_1^2) \quad (106)$$
$$N_{H_d} = 3Tr(Y_d^\dagger Y_d) + Tr(Y_e^\dagger Y_e) - (\frac{3}{2}g_2^2 + \frac{3}{10}g_1^2). \quad (107)$$

The one-loop RG equation for μ-parameter is

$$\frac{d\mu}{dt} = \frac{1}{16\pi^2}[N_{H_u} + N_{H_d}]\mu. \quad (108)$$

The RG equations for soft trilinear couplings a_x are:

$$\frac{da_u}{dt} = \frac{1}{16\pi^2}[N_q \cdot a_u + a_u \cdot N_u + (N_{H_u})a_u + 2P_q \cdot Y_u + 2Y_u \cdot P_u + 2(P_{H_u})Y_u]$$
$$\frac{da_d}{dt} = \frac{1}{16\pi^2}[N_q \cdot a_d + a_d \cdot N_d + (N_{H_d})a_d + 2P_q \cdot Y_d + 2Y_d \cdot P_d + 2(P_{H_d})Y_d]$$
$$\frac{da_\nu}{dt} = \frac{1}{16\pi^2}[N_l \cdot a_\nu + a_\nu \cdot N_\nu + (N_{H_u})a_\nu + 2P_l \cdot Y_\nu + 2Y_\nu \cdot P_\nu + 2(P_{H_u})Y_\nu]$$
$$\frac{da_e}{dt} = \frac{1}{16\pi^2}[N_l \cdot a_e + a_e \cdot N_e + (N_{H_d})a_e + 2P_l \cdot Y_e + 2Y_e \cdot P_e + 2(P_{H_d})Y_e],$$

where

$$P_q = (\frac{8}{3}g_3^2M_3 + \frac{3}{2}g_2^2M_2 + \frac{1}{30}g_1^2M_1)\mathbf{1} + a_uY_u^\dagger + a_dY_d^\dagger \tag{109}$$

$$P_u = (\frac{8}{3}g_3^2M_3 + \frac{8}{15}g_1^2M_1)\mathbf{1} + 2Y_u^\dagger a_u \tag{110}$$

$$P_d = (\frac{8}{3}g_3^2M_3 + \frac{2}{15}g_1^2M_1)\mathbf{1} + 2Y_d^\dagger a_d \tag{111}$$

$$P_l = (\frac{3}{2}g_2^2M_2 + \frac{3}{10}g_1^2M_1)\mathbf{1} + a_eY_e^\dagger + a_\nu Y_\nu^\dagger \tag{112}$$

$$P_e = \frac{6}{5}g_1^2M_1\mathbf{1} + 2Y_e^\dagger a_e \tag{113}$$

$$P_\nu = 2Y_\nu^\dagger a_\nu \tag{114}$$

$$P_{H_u} = (\frac{3}{2}g_2^2M_2 + \frac{3}{10}g_1^2M_1) + 3Tr(Y_u^\dagger a_u) + Tr(Y_\nu^\dagger a_\nu) \tag{115}$$

$$P_{H_d} = (\frac{3}{2}g_2^2M_2 + \frac{3}{10}g_1^2M_1) + 3Tr(Y_d^\dagger a_d) + Tr(Y_e^\dagger a_e). \tag{116}$$

The b term RG equation is:

$$\frac{db}{dt} = \frac{1}{16\pi^2}[(N_{H_u} + N_{H_d})b + 2(P_{H_u} + P_{H_d})\mu]. \tag{117}$$

The renormalization group equations for soft scalar squared-masses are:

$$\begin{aligned}\frac{dm_Q^2}{dt} = \frac{1}{8\pi^2}[&-2(\frac{8}{3}g_3^2|M_3|^2 + \frac{3}{2}g_2^2|M_2|^2 + \frac{1}{30}g_1^2|M_1|^2 - \frac{1}{10}g_1^2S)\mathbf{1} \\ &+(\frac{1}{2}Y_uY_u^\dagger m_Q^2 + \frac{1}{2}m_Q^2Y_uY_u^\dagger + Y_um_U^2Y_u^\dagger + (m_{H_u}^2)Y_uY_u^\dagger + a_ua_u^\dagger) \\ &+(\frac{1}{2}Y_dY_d^\dagger m_Q^2 + \frac{1}{2}m_Q^2Y_dY_d^\dagger + Y_dm_D^2Y_d^\dagger + (m_{H_d}^2)Y_dY_d^\dagger + a_da_d^\dagger)\end{aligned} \tag{118}$$

$$\begin{aligned}\frac{dm_U^2}{dt} = \frac{1}{8\pi^2}[&-2(\frac{8}{3}g_3^2|M_3|^2 + \frac{8}{15}g_1^2|M_1|^2 + \frac{2}{5}g_1^2S)\mathbf{1} \\ &+2(\frac{1}{2}Y_uY_u^\dagger m_U^2 + \frac{1}{2}m_U^2Y_u^\dagger Y_u + Y_u^\dagger m_Q^2Y_u + (m_{H_u}^2)Y_u^\dagger Y_u + a_u^\dagger a_u)\end{aligned} \tag{119}$$

$$\begin{aligned}\frac{dm_D^2}{dt} = \frac{1}{8\pi^2}[&-2(\frac{8}{3}g_3^2|M_3|^2 + \frac{2}{15}g_1^2|M_1|^2 - \frac{1}{5}g_1^2S)\mathbf{1} \\ &+2(\frac{1}{2}Y_dY_d^\dagger m_D^2 + \frac{1}{2}m_D^2Y_d^\dagger Y_d + Y_d^\dagger m_Q^2Y_d + (m_{H_d}^2)Y_d^\dagger Y_d + a_d^\dagger a_d)\end{aligned} \tag{120}$$

$$\begin{aligned}\frac{dm_L^2}{dt} = \frac{1}{8\pi^2}[&-2(\frac{3}{2}g_2^2|M_2|^2 + \frac{3}{10}g_1^2|M_1|^2 + \frac{3}{10}g_1^2S)\mathbf{1} \\ &+(\frac{1}{2}Y_eY_e^\dagger m_L^2 + \frac{1}{2}m_L^2Y_eY_e^\dagger + Y_em_E^2Y_e^\dagger + (m_{H_d}^2)Y_eY_e^\dagger + a_ea_e^\dagger) \\ &+(\frac{1}{2}Y_\nu Y_\nu^\dagger m_L^2 + \frac{1}{2}m_L^2Y_\nu Y_\nu^\dagger + Y_\nu m_N^2Y_\nu^\dagger + (m_{H_u}^2)Y_\nu Y_\nu^\dagger + a_\nu a_\nu^\dagger)\end{aligned} \tag{121}$$

$$\frac{dm_E^2}{dt} = \frac{1}{8\pi^2}[-2(\frac{6}{5}g_1^2|M_1|^2 - \frac{3}{5}g_1^2 S)\mathbf{1} \tag{122}$$
$$+2(\frac{1}{2}Y_e^\dagger Y_e m_E^2 + \frac{1}{2}m_E^2 Y_e^\dagger Y_e + Y_e^\dagger m_L^2 Y_e + (m_{H_d}^2)Y_e^\dagger Y_e + a_e^\dagger a_e)]$$

$$\frac{dm_N^2}{dt} = \frac{1}{8\pi^2}[2(\frac{1}{2}Y_\nu^\dagger Y_\nu m_N^2 + \frac{1}{2}m_N^2 Y_\nu^\dagger Y_\nu + Y_\nu^\dagger m_L^2 Y_\nu + (m_{H_u}^2)Y_\nu^\dagger Y_\nu + a_\nu^\dagger a_\nu)] \tag{123}$$

$$\frac{dm_{H_u}^2}{dt} = \frac{1}{8\pi^2}[-2(\frac{3}{2}g_2^2|M_2|^2 + \frac{3}{10}g_1^2|M_1|^2 - \frac{3}{10}g_1^2 S) \tag{124}$$
$$+3(Tr(Y_u m_Q^2 Y_u^\dagger) + Tr(Y_u m_U^2 Y_u^\dagger) + (m_{H_u}^2)Tr(Y_u Y_u^\dagger) + Tr(a_u a_u^\dagger))$$
$$+(Tr(Y_\nu m_L^2 Y_\nu^\dagger) + Tr(Y_\nu m_N^2 Y_\nu^\dagger) + (m_{H_u}^2)Tr(Y_\nu Y_\nu^\dagger) + Tr(a_\nu a_\nu^\dagger))]$$

$$\frac{dm_{H_d}^2}{dt} = \frac{1}{8\pi^2}[-2(\frac{3}{2}g_2^2|M_2|^2 + \frac{3}{10}g_1^2|M_1|^2 + \frac{3}{10}g_1^2 S) \tag{125}$$
$$+3(Tr(Y_d m_Q^2 Y_d^\dagger) + Tr(Y_d m_D^2 Y_d^\dagger) + (m_{H_d}^2)Tr(Y_d Y_d^\dagger) + Tr(a_d a_d^\dagger))$$
$$+(Tr(Y_e m_L^2 Y_e^\dagger) + Tr(Y_e m_E^2 Y_e^\dagger) + (m_{H_d}^2)Tr(Y_e Y_e^\dagger) + Tr(a_e a_e^\dagger))]$$

where

$$S = m_{H_u}^2 - m_{H_d}^2 + Tr(m_Q^2 - m_L^2 - 2m_U^2 + m_D^2 + m_E^2).$$

D.2 Simple RG equations for MSSM

Let us now look more closely to the physical significance of the renormalization group equations. In order to do this, we well make some simplifications. The RG equations of the previous section can be greatly simplified when one takes into account only one-loop corrections and assumes that only the Yukawa couplings of the third family quarks and leptons are important:

$$\mathbf{Y_u} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix}, \quad \mathbf{Y_d} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix}, \quad \mathbf{Y_e} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_\tau \end{pmatrix}. \tag{126}$$

D.2.1 RG equations for supersymmetry respecting parameters

The supersymmetry respecting MSSM Lagrangian contains SM Yukawa couplings; SM gauge couplings, g_1, g_2, g_3 and the Higgs mass parameter μ as parameters. The one-loop RG equations for Standard Model gauge couplings g_1, g_2 and g_3 are[20] [48]:

$$\frac{d}{dt}g_a = \frac{1}{16\pi^2}b_a g_a^3, \qquad (b_1, b_2, b_3) = \begin{cases} (41/10, -19/6, -7) & \text{SM} \\ (33/5, 1, -3) & \text{MSSM} \end{cases}$$

Both in SM and MSSM the $U(1)$ coupling g_1 has positive β-function so the coupling g_1 grows as the energy scale grows. Also in both SM and MSSM the $SU(3)$ gauge coupling g_3 has negative β-function so the coupling g_3 becomes smaller as the energy scale grows. The $SU(2)$ gauge coupling behaves differently in SM and MSSM. In SM it has negative β-function so it shrinks as the energy scale grows but in MSSM g_2 has positive β-function so it grows as the energy grows.

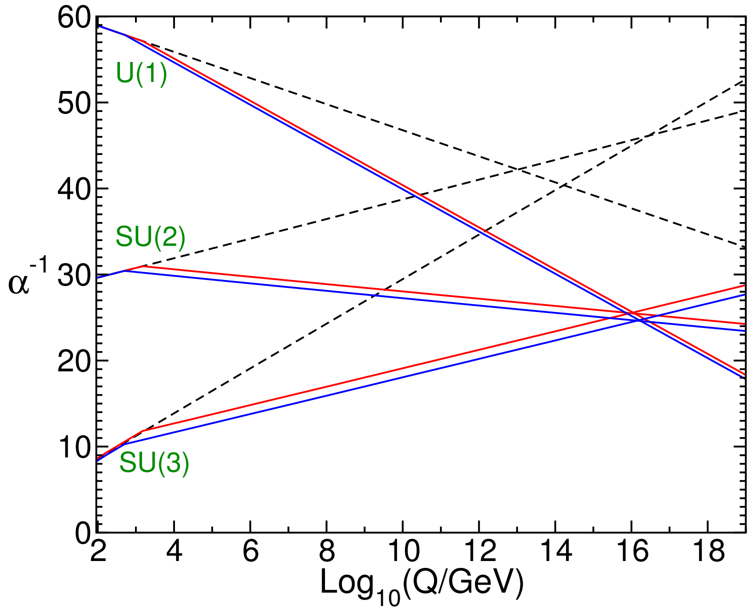

Figure 69: Renormalization group evolution of the inverse gauge couplings $\alpha_a^{-1}(Q)$ in the SM (dashed lines) and in the MSSM (solid lines). [48]

The MSSM coefficients are larger because the MSSM contains more particles and therefore there are more possible loops. The MSSM particle content is such that the three different couplings can unify at a scale $M_U \sim 2 \times 10^{16}$ GeV. Since the couplings seem to unify nicely at high energy one might be motivated to believe that there are no new physics between TeV scale and M_U-scale and that the MSSM will be valid up to M_U. This gap with no new physics between TeV scale and gauge coupling unification scale is called the *desert*. When one uses

[20]The gauge couplings g_1 and g_2 are related to the conventional electroweak couplings g and g' with $e = g\sin\theta_W = g'\cos\theta_W$ as: $g_2 = g$ and $g_1 = \sqrt{5/3}g'$.

RG equations in MSSM one evolves the parameters from the input scale down to electroweak scale assuming that there are no new physics contributions between the input scale and the electroweak scale. Otherwise the evolution would be different (for example in supersymmetric seesaw mechanism massive neutrinos appear between the input scale and the electroweak scale, thus changing the evolution of parameters).

The one-loop renormalization group equations for the relevant third generation Yukawa couplings are [48]:

$$\frac{d}{dt}y_t = \frac{y_t}{16\pi^2}[6y_t^* y_t + y_b^* y_b - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2], \tag{127}$$

$$\frac{d}{dt}y_b = \frac{y_b}{16\pi^2}[6y_b^* y_b + y_t^* y_t + y_\tau^* y_\tau - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2], \tag{128}$$

$$\frac{d}{dt}y_\tau = \frac{y_\tau}{16\pi^2}[4y_\tau^* y_\tau + 3y_b^* y_b - 3g_2^2 - \frac{9}{5}g_1^2]. \tag{129}$$

The Higgs mass parameter obeys the following renormalization group equation:

$$\frac{d}{dt}\mu = \frac{\mu}{16\pi^2}[3y_t^* y_t + 3y_b^* y_b + y_\tau^* y_\tau - 3g_2^2 - \frac{3}{5}g_1^2]. \tag{130}$$

One notices that the β functions of all the supersymmetry respecting parameters is proportional to the parameter itself. This is because of *supersymmetric non-renormalization theorem* [50]. It states that all the quadratic divergences cancel and that the worst divergence is logarithmic. The parameter μ for example will get only small corrections in MSSM in contrast to unacceptably large corrections in SM. The RG equations for the supersymmetry respecting parameters are not affected by the presence of soft supersymmetry-breaking.

D.2.2 RG equations for supersymmetry-breaking parameters

Let us now deal with the RG evolution of the soft supersymmetry-breaking parameters: gaugino masses, trilinear scalar couplings and scalar masses. The one-loop RG equations for gaugino masses are [48]:

$$\frac{d}{dt}M_a = \frac{1}{8\pi^2}b_a g_a^2 M_a, \qquad (b_a = 33/5, 1, -3),$$

where $a = 1, 2, 3$. The coefficient b_a is the same that appears in the RG equation for gauge coupling g_a in MSSM. The rations of gaugino masses and the associated gauge couplings squares

$$\frac{M_a}{g_a^2}, \qquad a = 1, 2, 3$$

turn out to be constants up to a two-loop corrections [48]. The gauge couplings unify at $M_U = 2 \times 10^{16}$ GeV, so it is frequently assumed that also the gaugino masses unify at the scale near the gauge coupling unification. So then at any RG scale

$$\frac{M_1}{g_1^2} = \frac{M_2}{g_2^2} = \frac{M_3}{g_3^2} = \frac{m_{1/2}}{g_U^2},$$

up to two-loop corrections.

One usually assumes that the supersymmetry-breaking is universal i.e. that the soft scalar masses and trilinear couplings obey the following equations at the input scale:

$$\mathbf{m^2_{\tilde{Q}}} = m^2_{\tilde{Q}}\mathbf{1}, \qquad \mathbf{m^2_{\tilde{u}}} = m^2_{\tilde{u}}\mathbf{1}, \qquad \mathbf{m^2_{\tilde{d}}} = m^2_{\tilde{d}}\mathbf{1}, \qquad \mathbf{m^2_{\tilde{L}}} = m^2_{\tilde{L}}\mathbf{1}, \qquad \mathbf{m^2_{\tilde{e}}} = m^2_{\tilde{e}}\mathbf{1}$$

$$\mathbf{a_u} = A_{u0}\mathbf{y_u}, \qquad \mathbf{a_d} = A_{d0}\mathbf{y_d}, \qquad \mathbf{a_e} = A_{e0}\mathbf{y_e}, \tag{131}$$

The trilinear terms are initially proportional to the Yukawa couplings and the RG evolution does not change that [48]. So using (126) and (131) the trilinear couplings can be written as:

$$\mathbf{a_u} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & a_t \end{pmatrix}, \qquad \mathbf{a_d} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & a_b \end{pmatrix}, \qquad \mathbf{a_e} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & a_\tau \end{pmatrix},$$

at any RG scale. The couplings a_t, a_b and a_τ, and the parameter b will obey the following RG equations [48]:

$$\begin{aligned}
16\pi^2 \frac{d}{dt} a_t &= a_t[18 y_y^* y_t + y_b^* y_b - \frac{16}{3} g_3^2 - 3 g_2^2 - \frac{13}{15} g_1^2] + 2 a_b y_b^* y_t \\
&\quad + y_t[\frac{32}{3} g_3^2 M_3 + 6 g_2^2 M_2 + \frac{26}{15} g_1^2 M_1] && (132) \\
16\pi^2 \frac{d}{dt} a_b &= a_b[18 y_b^* y_b + y_t^* y_t + y_\tau^* y_\tau - \frac{16}{3} g_3^2 - 3 g_2^2 - \frac{7}{15} g_1^2] + 2 a_t y_t^* y_b \\
&\quad + 2 a_\tau y^* y_b + y_b[\frac{32}{3} g_3^2 M_3 + 6 g_2^2 M_2 + \frac{14}{15} g_1^2 M_1] && (133) \\
16\pi^2 \frac{d}{dt} a_\tau &= a_\tau[12 y_\tau^* y_\tau + 3 y_b^* y_b - 3 g_2^2 - \frac{9}{5} g_1^2] + 6 a_b y_b^* y_\tau \\
&\quad + y_\tau[6 g_2^2 M_2 + \frac{18}{5} g_1^2 M_1] && (134) \\
16\pi^2 \frac{d}{dt} b &= b[3 y_t^* y_t + 3 y_b^* y_b + y_\tau^* y_\tau - 3 g_2^2 - \frac{3}{5} g_1^2] \\
&\quad + \mu[6 a_t y_t^* + 6 a_b y_b^* + 2 a_\tau y_\tau^* + 6 g_2^2 M_2 + \frac{6}{5} g_1^2 M_1]. && (135)
\end{aligned}$$

We see that the β-functions of the trilinear couplings and parameter b are not proportional to the coupling itself. For supersymmetry respecting parameters the β-function is always proportional to parameter itself, but since the trilinear scalar couplings and the parameter b violate the supersymmetry they are not protected by non-renormalization theorem, and can have sizable quantum corrections. Even if the scalar couplings a_t, a_b and a_τ are zero at the input scale (as is the case in gauge mediated SUSY-breaking), the RG evolution makes sure that they are not zero at the electroweak scale. The terms proportional to gaugino masses in equations (132), (133), (134) and (135) do not vanish when the parameters them selves are zero.

Let us finally consider the RG evolution of the soft scalar masses. We will use the approximation (126), which means that the first and second generation only know the gauge interactions and the third generation knows also the Yukawa interactions. So the RG evolution of the first two generations differs from that of the third one. We assume that the supersymmetry-breaking is flavour diagonal so that (131) holds. So the scalar masses are initially the same among generations. Initially at the input scale the scalar mass matrices are proportional to unit matrix, but the RG evolution deviates the masses of first and second families from the third generation. In MSSM there are no flavour violating effects between the input scale and the electroweak scale so the RG evolution will maintain the diagonality of the scalar mass matrices (Yukawa couplings contain non-diagonal elements but they are negligible. Actually the Yukawa matrices can be diagonalized simultaneously. In case of majorana masses of neutrinos that is not the case however.). So the scalar mass matrices for sleptons at any RG scale can be written approximately in the form

$$\mathbf{m_L^2} \approx \begin{pmatrix} m_{L_1}^2 & 0 & 0 \\ 0 & m_{L_1}^2 & 0 \\ 0 & 0 & m_{L_3}^2 \end{pmatrix}, \qquad \mathbf{m_{\bar{e}}^2} \approx \begin{pmatrix} m_{\bar{e}_1}^2 & 0 & 0 \\ 0 & m_{\bar{e}_1}^2 & 0 \\ 0 & 0 & m_{\bar{e}_3}^2 \end{pmatrix}.$$

The scalar mass matrices for quarks are of a same form. The first two generations of sfermions obey the following RG equations.

$$16\pi^2 \frac{d}{dt} m_{\phi_i}^2 = -\sum_{a=1,2,3} 8C_a(i) g_a^2 |M_a|^2 + \frac{6}{5} Y_i g_1^2 S,$$

where ϕ_i's are the first and second generation squarks and sleptons, $C_a(i)$ is the Casimir invariant[21] and

$$S \equiv Tr[Y_j m_{\phi_j}^2] = m_{H_u}^2 - m_{H_d}^2 + Tr[\mathbf{m_Q^2} - \mathbf{m_L^2} - 2\mathbf{m_{\bar{u}}^2} + \mathbf{m_{\bar{d}}^2} + \mathbf{m_{\bar{e}}^2}].$$

In most realistic models the S-term is small [48]. The terms proportional to the gaugino masses M_a will then dominate. The β-function will then be negative, because the terms proportional to the gaugino masses are negative. This means that the masses of the first two generations of sfermions will grow as the RG-scale is lowered from the input scale sown to the electroweak scale.

The third family sfermions and the Higgs scalars get contribution from the gauge interactions, but from the Yukawa couplings as well. The RG equations for the third family sfermion squared masses are:

[21] Quadratic Casimir invariants $C_a(i)$ of the group are defined in terms of the corresponding Lie algebra generators T^a: $(T^a T^a)i\ ^j = C_a(i)\delta_i^{\ j}$. The MSSM Casimirs are: $C_3(i) = 4/3$ for $Q, \bar{u}, \bar{d}$; $C_3(i) = 0$ for $L, \bar{e}, H_u, H_d$; $C_2(i) = 3/4$ for Q, L, H_u, H_d; $C_2(i) = 0$ for $\bar{u}, \bar{d}, \bar{e}$ and $C_1(i) = 3Y_i^2/5$ for each field with hypercharge Y_i.

$$
\begin{aligned}
16\pi^2 \frac{d}{dt} m^2_{Q_3} &= X_t + X_b - \frac{32}{3} g_3^2 |M_3|^2 - 6 g_2^2 |M_2|^2 - \frac{2}{15} g_1^2 |M_1|^2 + \frac{1}{5} g_1^2 S, \\
16\pi^2 \frac{d}{dt} m^2_{\bar{u}_3} &= 2X_t - \frac{32}{3} g_3^2 |M_3|^2 - \frac{32}{15} g_1^2 |M_1|^2 - \frac{4}{5} g_1^2 S, \\
16\pi^2 \frac{d}{dt} m^2_{\bar{d}_3} &= 2X_b - \frac{32}{3} g_3^2 |M_3|^2 - \frac{8}{15} g_1^2 |M_1|^2 + \frac{2}{5} g_1^2 S, \\
16\pi^2 \frac{d}{dt} m^2_{L_3} &= X_\tau - 6 g_2^2 |M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2 - \frac{3}{5} g_1^2 S, \\
16\pi^2 \frac{d}{dt} m^2_{\bar{e}_3} &= 2X_\tau - \frac{24}{5} g_1^2 |M_1|^2 + \frac{6}{5} g_1^2 S,
\end{aligned}
$$

where

$$
\begin{aligned}
X_t &= 2|y_t|^2 (m^2_{H_u} + m^2_{Q_3} + m_{\bar{u}_3}) + |a_t|^2, && (136) \\
X_b &= 2|y_b|^2 (m^2_{H_d} + m^2_{Q_3} + m_{\bar{d}_3}) + |a_b|^2, && (137) \\
X_\tau &= 2|y_\tau|^2 (m^2_{H_d} + m^2_{L_3} + m_{\bar{e}_3}) + |a_\tau|^2. && (138)
\end{aligned}
$$

The terms (136), (137) and (138) describe the contribution from the Yukawa couplings and the soft trilinear couplings.

The terms containing the gaugino masses are negative in the beta functions of the sfermion RGEs, whereas the terms involving the Yukawa couplings are positive. The negative terms drive the sfermion masses up and the positive drive them down as one runs the energy scale from the input scale down to the electroweak scale. The sleptons do not know the strong interaction as the squarks do. Therefore the squarks have more more negative beta functions compared to that of sleptons and the squark masses will be larger than the slepton masses at the electroweak scale. The left-handed first generation squark will be the heaviest, since it knows all the gauge interactions and has smallest Yukawa coupling among the squarks. The lightest charged sfermion is probably a right-handed stau, since it couples only to $U(1)$ and has the largest Yukawa coupling among the sleptons.

The Higgs scalar squared masses satisfy the following RG equations.

$$
\begin{aligned}
16\pi^2 \frac{d}{dt} m^2_{H_u} &= 3X_t - 6 g_2^2 |M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2 + \frac{3}{5} g_1^2 S && (139) \\
16\pi^2 \frac{d}{dt} m^2_{H_d} &= 3X_b + X_\tau - 6 g_2^2 |M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2 - \frac{3}{5} g_1^2 S && (140)
\end{aligned}
$$

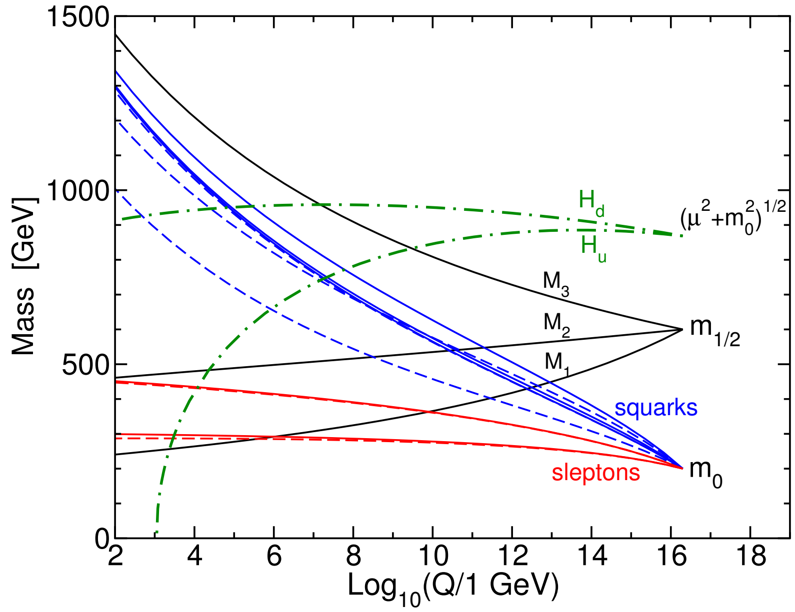

Figure 70: The RGE running in MSSM [48]

The terms X_t, X_b and X_τ are positive, so they effectively decrease the Higgs scalar masses as the RG scale is run down from the input scale to the electroweak scale. The top quark mass is the largest of the SM fermions. This suggests that the top Yukawa coupling is the largest Yukawa coupling. This means that X_t is much larger than X_b and X_τ. The $m^2_{H_u}$ will therefore become much smaller than the $m^2_{H_d}$ as the RG scale is evolved down from the input scale to the electroweak scale. It is even possible that $m^2_{h_u}$ becomes negative near the electroweak scale, which would help the linear combination of H_u and H_d to acquire VEV and thus explain the electroweak symmetry breaking.

There is a dangerous possibility with the squared masses of the third family sfermions: what if they run negative as the RG scale is evolved down to the electroweak scale and acquire VEV's? (If the scalar fermions would get VEV's there would then be vertices where the electric charge would not be conserved, which would be disturbing.) The positive terms X_t, X_b and X_τ have the effect of making the sfermion mass squares smaller at smaller energies. Luckily the terms proportional to gaugino masses are negative, and they affect to the evolution of sfermion mass squares so that they don't become negative at the electroweak scale.

E Explicit effective operators

The effective operators represented in Tables 4 and 5 give the effective vertices for the effective theory of the New Physics. In order to obtain the vertices we have to expand the effective operators in terms of the fields. We will use relations (4), (3) and notations

$$\phi^\dagger i \overset{\leftrightarrow}{D}_\mu \phi \equiv i\phi^\dagger \left(D_\mu - \overset{\leftarrow}{D}_\mu \right) \phi \qquad \text{and} \qquad \phi^\dagger i \overset{\leftrightarrow}{D}{}^I_\mu \phi \equiv i\phi^\dagger \left(\tau^I D_\mu - \overset{\leftarrow}{D}_\mu \tau^I \right) \phi,$$

with $\phi^\dagger \overset{\leftarrow}{D}_\mu \phi \equiv (D_\mu \phi)^\dagger \phi$, where D_μ is the covariant derivative of the Higgs field as in (12).

We will consider the charged lepton flavour violation after spontaneous symmetry breaking so we assume that the Higgs field has acquired it's vacuum expectation value. The effective operators written explicitly in terms of the fields are given in the Tables 7, 8 and 9.

Table 7: Effective dimension-six operators with leptons only

$llll$
$Q_{ll} = (\bar{l}_i \gamma_\mu l_j)(\bar{l}_k \gamma^\mu l_l)$ $= (\bar{\nu}_i \gamma_\mu \nu_j)(\bar{\nu}_k \gamma^\mu \nu_l) + (\bar{\nu}_i \gamma_\mu \nu_j)(\bar{e}_{Lk} \gamma^\mu e_{Ll})$ $+ (\bar{e}_{Li} \gamma_\mu e_{Lj})(\bar{\nu}_k \gamma^\mu \nu_l) + (\bar{e}_{Li} \gamma_\mu e_{Lj})(\bar{e}_{Lk} \gamma^\mu e_{Ll})$
$Q_{ee} = (\bar{e}_i \gamma_\mu e_j)(\bar{e}_k \gamma^\mu e_l) = (\bar{e}_{Ri} \gamma_\mu e_{Rj})(\bar{e}_{Rk} \gamma^\mu e_{Rl})$
$Q_{le} = (\bar{l}_i \gamma_\mu l_j)(\bar{e}_k \gamma^\mu e_l) = (\bar{\nu}_i \gamma_\mu \nu_j)(\bar{e}_{Rk} \gamma^\mu e_{Rl}) + (\bar{e}_{Li} \gamma_\mu e_{Lj})(\bar{e}_{Rk} \gamma^\mu e_{Rl})$

Table 8: Effective dimension-six operators with photons

$llX\phi$
$Q_{eW} = (\bar{l}_i \sigma^{\mu\nu} e_j) \tau^I \phi W^I_{\mu\nu}$ $= v\,(\bar{\nu}_i \sigma^{\mu\nu} e_{Rj}) \left(\partial_\mu W^+_\nu - igW^+_\mu (cos\theta_W Z_\nu + sin\theta_W A_\nu) \right)$ $-v\,(\bar{\nu}_i \sigma^{\mu\nu} e_{Rj}) \left(\partial_\nu W^+_\mu - igW^+_\nu (cos\theta_W Z_\mu + sin\theta_W A_\mu) \right)$ $-\frac{v}{\sqrt{2}} (\bar{e}_{Ri} \sigma^{\mu\nu} e_{Rj}) [\partial_\mu (cos\theta_W Z_\nu + sin\theta_W A_\nu) - \partial_\nu (cos\theta_W Z_\mu + sin\theta_W A_\mu)$ $+gW^+_\mu W^-_\nu - igW^-_\mu W^+_\nu]$
$Q_{eB} = (\bar{l}_i \sigma^{\mu\nu} e_j) \phi B_{\mu\nu}$ $= \frac{v}{\sqrt{2}} (\bar{e}_{Li} \sigma^{\mu\nu} e_{Rj}) [\partial_\mu (-sin\theta_W Z_\nu + cos\theta_W A_\nu) - \partial_\nu (-sin\theta_W Z_\mu + cos\theta_W A_\mu)$

Table 9: Effective dimension-six operators without photons

$ll\phi\phi D$
$Q_{\phi l}^{(1)} = (\phi^\dagger i \overleftrightarrow{D}_\mu \phi)(\bar{l}_i \gamma^\mu l_j) = \frac{v^2}{2} g_2 \frac{1}{cos\theta_W} Z_\mu (\bar{\nu}_i \gamma^\mu \nu_j + \bar{e}_{Li} \gamma^\mu e_{Lj})$
$Q_{\phi l}^{(3)} = (\phi^\dagger i \overleftrightarrow{D}^I_\mu \phi)(\bar{l}_i \tau^I \gamma^\mu l_j) = -\frac{v^2}{\sqrt{2}} g_2 (W_\mu^+ \bar{\nu}_i \gamma^\mu e_{Lj} + W_\mu^- \bar{e}_{Li} \gamma \nu_j)$ $- \frac{v^2}{2} g_2 \frac{1}{cos\theta_W} Z_\mu (\bar{\nu}_i \gamma^\mu \nu_j - \bar{e}_{Li} \gamma^\mu e_{Lj})$
$Q_{\phi e} = (\phi^\dagger i \overleftrightarrow{D}_\mu \phi)(\bar{e}_i \gamma^\mu e_j) = \frac{v^2}{2} g_2 \frac{1}{cos\theta_W} Z_\mu (\bar{e}_{Ri} \gamma^\mu e_{Rj})$

We notice that $llX\phi$-operators are the only ones to contain photon field. In $ll\phi\phi D$-operators the photon field is canceled away, which cannot be seen directly from the compact form.

F Effective form factors for $l_i \to l_j\gamma$

Involving Z boson:

$$F_{TL}^{Z\ fi} = \frac{4e\left[\left(C_{\phi l}^{(1)\ fi} + C_{\phi l}^{(3)\ fi}\right) m_f(1+s_W^2) - C_{\phi e}^{fi} m_i(\frac{3}{2} - s_W^2)\right]}{3(4\pi)^2}$$

$$F_{TR}^{Z\ fi} = \frac{4e\left[\left(C_{\phi l}^{(1)\ fi} + C_{\phi l}^{(3)\ fi}\right) m_i(1+s_W^2) - C_{\phi e}^{fi} m_f(\frac{3}{2} - s_W^2)\right]}{3(4\pi)^2}$$

Involving W boson:

$$F_{TL}^{W\ fi} = -\frac{10 e m_f C_{\phi l}^{(3)\ fi}}{3(4\pi)^2}$$

$$F_{TR}^{W\ fi} = -\frac{10 e m_i C_{\phi l}^{(3)\ fi}}{3(4\pi)^2}$$

From four-fermion contact interaction:

$$F_{TL}^{4f\ fi} = -\frac{16e}{3(4\pi)^2} \sum_{j=1}^{3} C_{lequ}^{(3)\ \ fiff*} m_{u_j} \left(\Delta - \log \frac{m_{u_j}^2}{\mu^2}\right)$$

$$F_{TR}^{4f\ fi} = -\frac{16e}{3(4\pi)^2} \sum_{j=1}^{3} C_{lequ}^{(3)\ \ fiff} m_{u_j} \left(\Delta - \log \frac{m_{u_j}^2}{\mu^2}\right)$$

From four-lepton contact interaction:

$$F_{TL}^{4l\ fi} = \frac{2e}{(4\pi)^2} \sum_{j=1}^{3} C_{le}^{fjji} m_j$$

$$F_{TR}^{4l\ fi} = \frac{2e}{(4\pi)^2} \sum_{j=1}^{3} C_{le}^{jifj} m_j$$

G Wilson coefficient combinations for $l_i \to l_j l_k l_l$

The coefficients C_X in the branching ratio expression (27) are linear combinations of the Wilson coefficients.

G.1 (A)

$$
\begin{aligned}
C_{VLL} &= 2\left((2s_W^2-1)\left(C_{\phi l}^{(1)ji}+C_{\phi l}^{(3)ji}\right)+C_{ll}^{jijj}\right)\\
C_{VRR} &= 2(2s_W^2 C_{\phi e}^{ji}+C_{ee}^{jijj})\\
C_{VLR} &= -\frac{1}{2}C_{SRL} = 2s_W^2\left(C_{\phi l}^{(1)ji}+C_{\phi l}^{(3)ji}\right)+C_{le}^{jijj}\\
C_{VRL} &= -\frac{1}{2}C_{SLR} = (2s_W^2-1)C_{\phi e}^{ji}+C_{le}^{jjji}\\
C_{SLL} &= C_{SRR} = C_{TL} = C_{TR} = 0\\
C_{\gamma L} &= \sqrt{2}C_{\gamma}^{ij*}\\
C_{\gamma R} &= \sqrt{2}C_{\gamma}^{ij}\\
X_{\gamma}^{(A)} &= -\frac{16v}{M}Re\left[\left(2C_{VLL}+C_{VLR}-\frac{1}{2}C_{SLR}\right)C_{\gamma R}^{*}+\left(2C_{VRR}+C_{VRL}-\frac{1}{2}C_{SRL}\right)C_{\gamma L}\right]\\
&\quad +\frac{64v^2}{M^2}\left(\log\frac{M^2}{m^2}-\frac{11}{4}\right)(|C_{\gamma L}|^2+|C_{\gamma R}|^2)
\end{aligned}
$$

G.2 (B)

$$
\begin{aligned}
C_{VLL} &= \left((2s_W^2-1)\left(C_{\phi l}^{(1)ji}+C_{\phi l}^{(3)ji}\right)+C_{ll}^{jikk}\right)\\
C_{VRR} &= 2s_W^2 C_{\phi e}^{ji}+C_{ee}^{jikk}\\
C_{VLR} &= 2s_W^2\left(C_{\phi l}^{(1)ji}+C_{\phi l}^{(3)ji}\right)+C_{le}^{jikk}\\
C_{VRL} &= (2s_W^2-1)C_{\phi e}^{ji}+C_{le}^{jkki}\\
C_{SLR} &= -2C_{le}^{jkki}\\
C_{SRL} &= -2C_{le}^{jikk}\\
C_{SLL} &= C_{SRR} = C_{TL} = C_{TR} = 0\\
C_{\gamma L} &= \sqrt{2}C_{\gamma}^{ij*}\\
C_{\gamma R} &= \sqrt{2}C_{\gamma}^{ij}-\frac{16v}{M}Re[(C_{VLL}+C_{VLR})C_{\gamma R}^{*}+(C_{VRR}+C_{VRL})C_{\gamma L}^{*}]\\
&\quad +\frac{32v^2}{M^2}\left(\log\frac{M^2}{m^2}-3\right)(|C_{\gamma L}|^2+|C_{\gamma R}|^2)
\end{aligned}
$$

G.3 (C)

$$
\begin{aligned}
C_{VLL} &= 2C_{ll}^{kikj} \\
C_{VRR} &= 2C_{ee}^{kikj} \\
C_{VLR} &= -\frac{1}{2}C_{SRL} = C_{le}^{kikj} \\
C_{VRL} &= -\frac{1}{2}C_{SRL} = C_{le}^{kjki} \\
C_{SLL} &= C_{SRR} = C_{TL} = C_{TR} = 0 \\
C_{\gamma L} &= C_{\gamma R} = 0 \\
X_{\gamma}^{(C)} &= 0
\end{aligned}
$$

H Diagrams contributing to e-μ conversion in MSSM

Here are the diagrams contributing to the e-μ conversion in MSSM. The $\widetilde{l}_X$ ($X = 1, .., 6$) represents the charged sleptons, $\widetilde{\nu}_x$ ($x = 1, 2, 3$) the sneutrinos, $\widetilde{\chi}_A^-$ ($A = 1, 2$) the charginos, $\widetilde{\chi}_A^0$ ($A = 1, .., 4$) the neutralinos and H_p ($p = h^0, H^0, A^0$) the Higgses.

H.1 Photon-penguins contributing to e-μ conversion

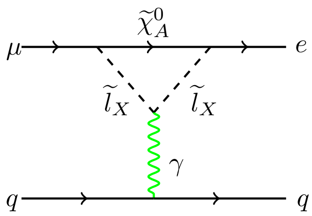

Figure 71: Photon 1

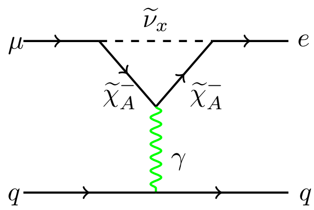

Figure 72: Photon 2

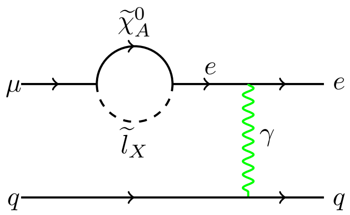

Figure 73: Photon 3

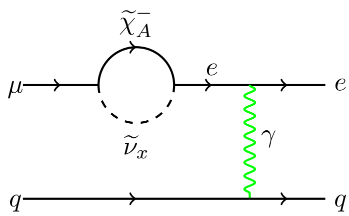

Figure 74: Photon 4

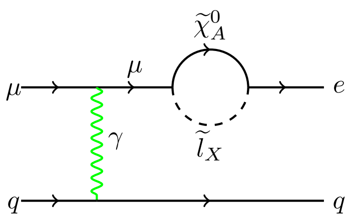

Figure 75: Photon 5

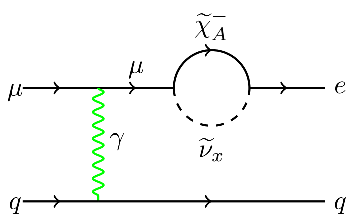

Figure 76: Photon 6

H.2 Z-penguins contributing to e-μ conversion

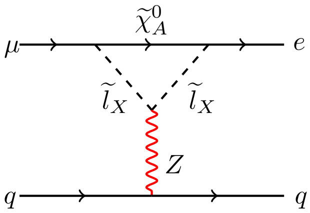

Figure 77: Z 1

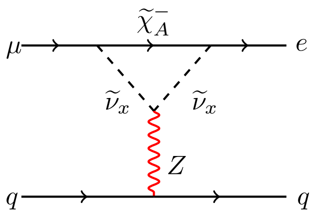

Figure 78: Z 2

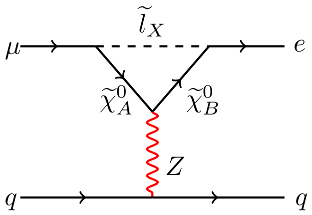

Figure 79: Z 3

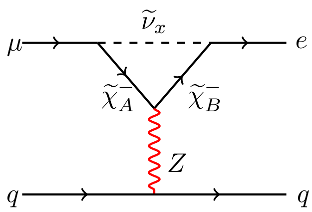

Figure 80: Z 4

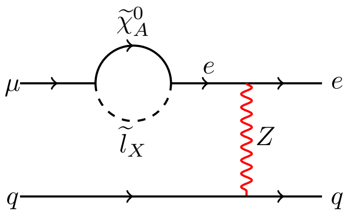

Figure 81: Z 5

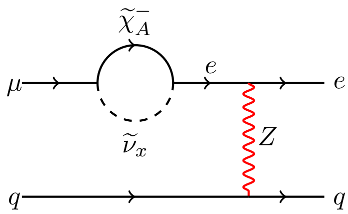

Figure 82: Z 6

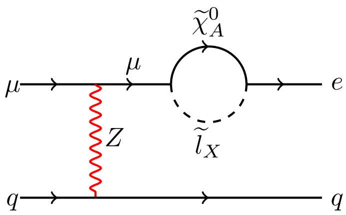

Figure 83: Z 7

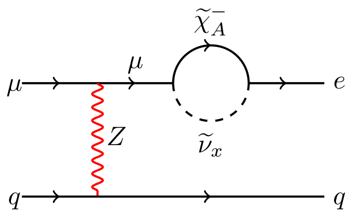

Figure 84: Z 8

H.3 Higgs-penguins contributing to e-μ conversion

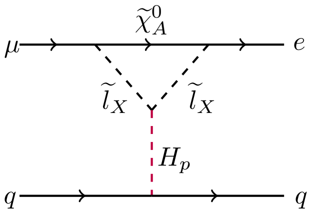

Figure 85: Z 1

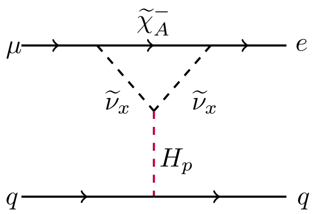

Figure 86: Z 2

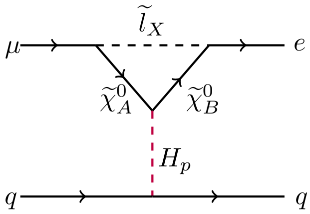

Figure 87: Z 3

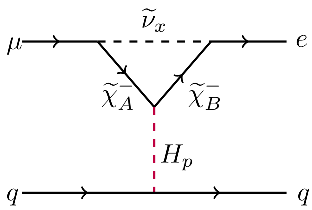

Figure 88: Z 4

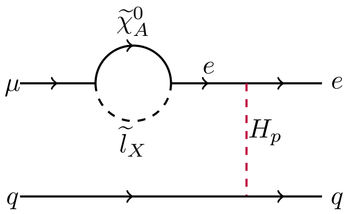

Figure 89: Z 5

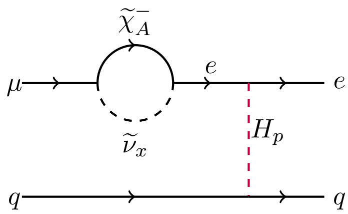

Figure 90: Z 6

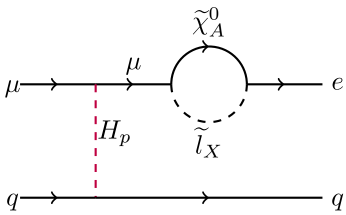

Figure 91: Z 7

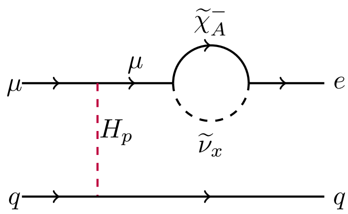

Figure 92: Z 8

H.4 Box diagrams contributing to e-μ conversion

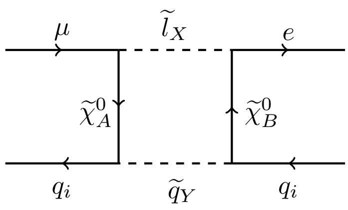

Figure 93: Box 1

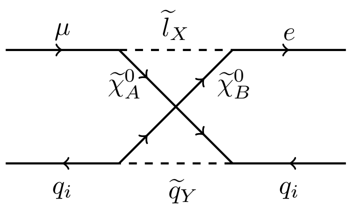

Figure 94: Box 2

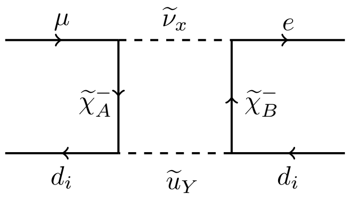

Figure 95: Box 3

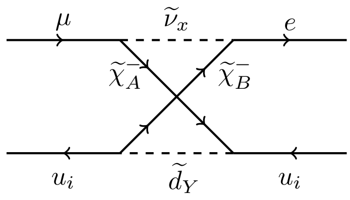

Figure 96: Box 4

I Diagrams contributing to $l_i \to 3l_j$ in MSSM

Here are the diagrams contributing to the process $l_i \to 3l_j$ in MSSM. The $\widetilde{l}_X$ ($X = 1, .., 6$) represents the charged sleptons, $\widetilde{\nu}_x$ ($x = 1, 2, 3$) the sneutrinos, $\widetilde{\chi}_A^-$ ($A = 1, 2$) the charginos, $\widetilde{\chi}_A^0$ ($A = 1, .., 4$) the neutralinos and H_p ($p = h^0, H^0, A^0$) the Higgses.

I.1 Photon-penguins contributing to $l_i \to 3l_j$

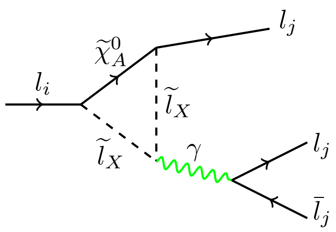

Figure 97: Photon 1

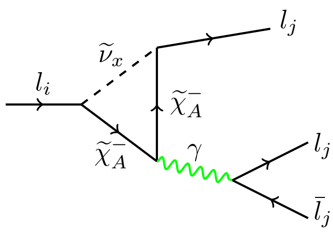

Figure 98: Photon 2

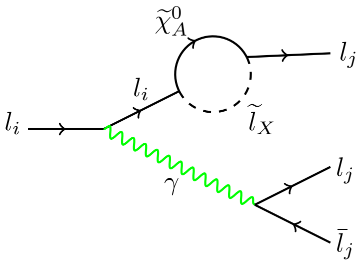

Figure 99: Photon 3

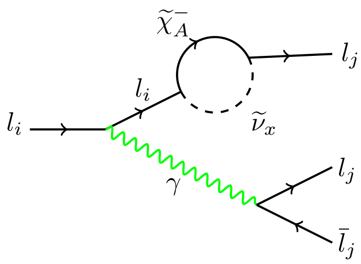

Figure 100: Photon 4

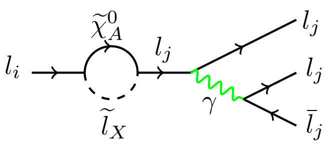

Figure 101: Photon 5

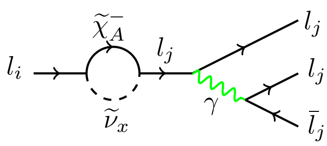

Figure 102: Photon 6

I.2 Z-penguins contributing to $l_i \to 3l_j$

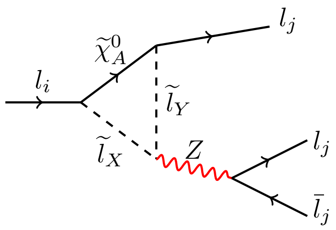

Figure 103: Z 1

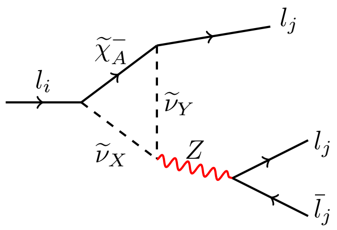

Figure 104: Z 2

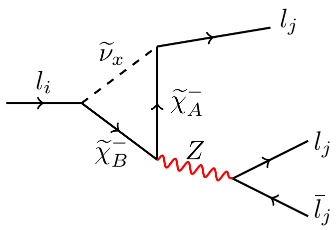

Figure 105: Z 3

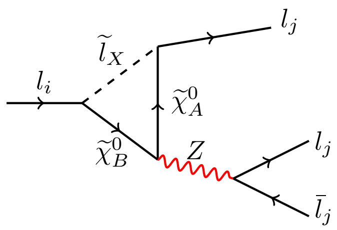

Figure 106: Z 4

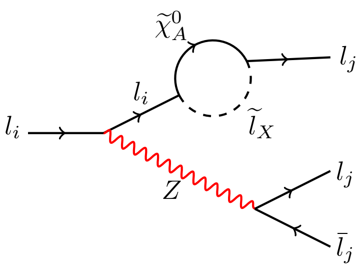

Figure 107: Z 5

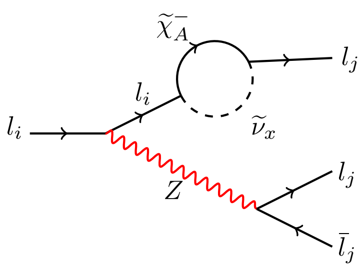

Figure 108: Z 6

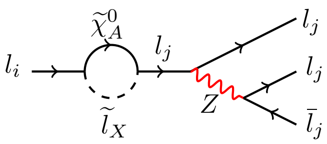

Figure 109: Z 7

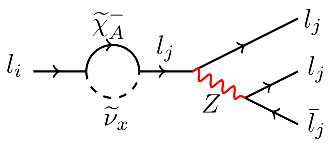

Figure 110: Z 8

I.3 Higgs-penguins contributing to $l_i \to 3l_j$

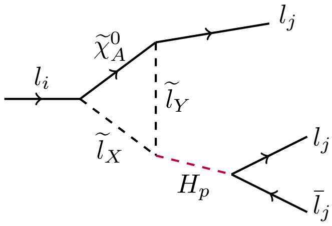

Figure 111: Higgs 1

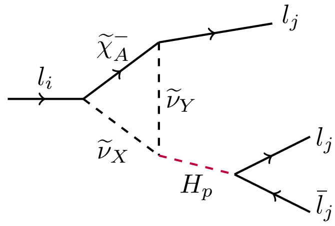

Figure 112: Higgs 2

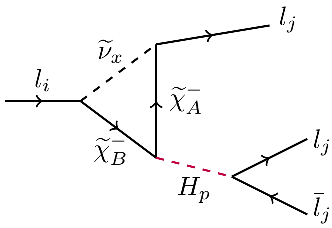

Figure 113: Higgs 3

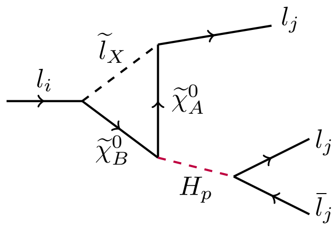

Figure 114: Higgs 4

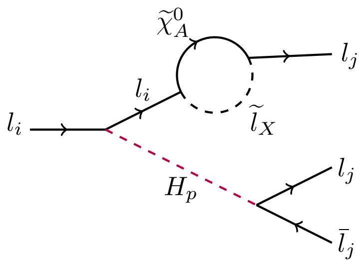

Figure 115: Higgs 5

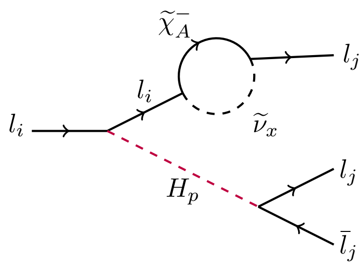

Figure 116: Higgs 6

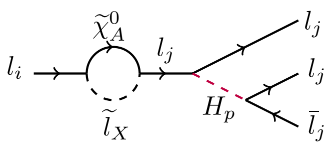

Figure 117: Higgs 7

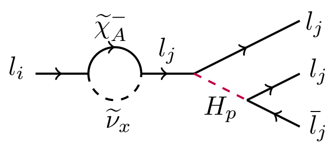

Figure 118: Higgs 8

I.4 Box diagrams contributing to $l_i \to 3l_j$

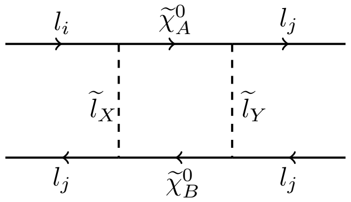

Figure 119: Box 1

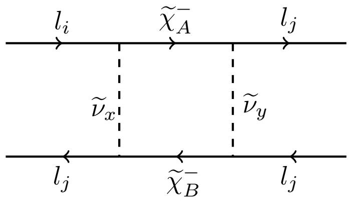

Figure 120: Box 2

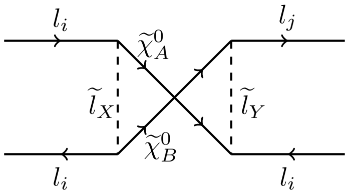

Figure 121: Box 3

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