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Special Issue

Quantum Physics including Gravity: Highlights and Novelties

Edited by
Prof. Dr. Norma G. Sanchez

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https://doi.org/10.3390/universe10070281 universe

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Quantized p-Form Gauge Field in D-Dimensional de Sitter Spacetime

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Abstract: In this work, we utilize the dynamic invariant method to obtain a solution for the time-dependent Schrödinger equation, aiming to explore the quantum theory of a p-form gauge field propagating in D-dimensional de Sitter spacetimes. Thus, we present a generalization, through the use of p-form gauge fields, of the quantization procedure for the scalar, electromagnetic, and Kalb–Ramond fields, all of which have been previously studied in the literature. We present an exact solution for the p-form gauge field when $D = 2(p+1)$, and we highlight the connection of the $p = 4$ case with the chiral $N = 2$, $D = 10$ superstring model. We could observe particle production for $D \neq 2(p+1)$ because the solutions are time-dependent. Additionally, observers in an accelerated co-moving reference frame will also experience a thermal bath. This could have significance in the realm of extra-dimensional physics, and presents the intriguing prospect that precise observations of the Cosmic Microwave Background might confirm the presence of additional dimensions.

Keywords: Quantum Fields; particle production; Tensor Fields

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Citation: Dantas, E.W.D.; Alencar, G.; Guedes, I.; Estrada, M. Quantized p-Form Gauge Field in D-Dimensional de Sitter Spacetime. *Universe* **2024**, *10*, 281. https://doi.org/10.3390/universe10070281

Academic Editors: Norma G. Sanchez and Oleg Lunin

Received: 11 April 2024
Revised: 21 June 2024
Accepted: 22 June 2024
Published: 28 June 2024

1. Introduction

The Quantum Field Theory in flat (Minkowski) spacetimes is one of the most successful theories in physics. It serves as the foundation upon which the Standard Model of particles is constructed, and provides a quantum description of the strong, weak, and electromagnetic forces. On the other hand, gravity is described by Einstein's General Theory of Relativity, a classical theory that has also proven to be very successful [1]. However, it is well understood that General Relativity remains an incomplete theory. Attempts to incorporate gravity into the Standard Model have proven to be non-renormalizable, as it requires an infinite number of parameters to do so.

Quantization of gravity is one of the most difficult and arduous challenges in modern physics and mathematics. Various approaches to quantizing gravity have been developed, with the most well-known being String Theory, which achieves quantization along with unification with the three other forces. However, experimental validation of the theory poses a significant obstacle due to the extremely weak quantum effects of gravity. This fact allows for creativity in the search for observable characteristics of the theory [2].

Despite the success of quantum field theory in flat spaces, and some theoretical achievements of string theory, there are still several problems related to the behavior of fields in curved spaces in a quantum theory at cosmological scales. For example, interesting results have been achieved in a background of time-dependent fields such as, for example, black Hawking radiation [3], which predicts the evaporation of black holes. Closely related is the Unruh effect [4], which suggests that an accelerated particle would perceive a thermal bath, and the dynamical Casimir effect [5], which predicts particle creation by an accelerated mirror. Another notable outcome is particle creation by the vacuum, which has been extensively researched [6–12], with the confirmation of the Schwinger effect expected soon [13].

On the other hand, the investigation of de Sitter spacetime gains significance when contemplating a ΛCDM model of the cosmos. In the current epoch, the universe can be roughly characterized by de Sitter spacetime, wherein matter decays with volume while the cosmological constant remains constant. For $t \gg H^{-1}$, the universe tends to be effectively described by the de Sitter model. In this connection, the study of quantum effects of a massive scalar field in de Sitter spacetime was examined in ref. [14], where the authors utilized exact linear invariants and the Lewis and Riesenfeld method [15] to derive the corresponding Schrödinger states based on solutions of a second-order ordinary differential equation. Additionally, they formulated Gaussian wave packet states, and computed the quantum dispersion and correlations for each mode of the quantized scalar field.

The quantization of certain types of fields has been explored in the literature. For the scalar field [16], it has been shown that the conformability of the system is tied to the choice of the curvature parameter. Similarly, the electromagnetic field [17] proves to be conformal in $D = 4$, as expected. Additionally, it has been observed that the Kalb–Ramond field is conformal in $D = 6$ [18]. In [19], the author develops the quantization of anti-symmetric (p-form) gauge fields on D-dimensional spheres using a Lorentz gauge, and shows that the partition function of the edge modes is related to the $(p-1)$-form on the boundary.

On the other hand, various branches of theoretical physics have provided indications of the possible existence of extra dimensions. Examples include string theory, higher-dimensional black holes, or braneworld models. It is worth mentioning that higher-dimensional FRW scenarios (including the de Sitter scenario) and their particle creation have also garnered attention in recent years [20,21].

Typically, in electromagnetism, the reason for characterizing the gauge potential of 0-dimensional charged particles using a differential 1-form, A, is that the trajectory of a charged particle is a 1-dimensional curve in spacetime, known as its worldline. However, if one aims to generalize this representation to the displacement of a $(p-1)$-dimensional manifold, where such displacement traces a path of dimensionality p, the gauge potential is represented by a differential p-form, $A_{\mu_1\cdots\mu_p}$ [22]. In this context, as indicated by reference [23], the p-form gauge fields play a significant role in theories involving extra dimensions. For example, within string theories in 26 dimensions, a low-energy normal mode of the string is represented by a two-form gauge field $A_{\mu\nu}$. However, in four-dimensional spacetimes, the p-forms do not introduce new possibilities. Thus, in this work, we aim to generalize the behavior of p-dimensional gauge fields within the quantum framework by solving the time-dependent Schrödinger equation.

In this study, we will employ the method developed by Lewis and Riesenfeld [15] to find a solution for the equations of motion governing a p-form gauge field in a D-dimensional de Sitter spacetime. Subsequently, we will proceed with its quantization by determining a solution to the 'auxiliary' equation [24,25]. Additionally, we will discuss the potential physical implications of our analysis for various extra-dimensional scenarios of interest in physics.

2. Equations of Motion and Its Decomposition

We will employ the Friedmann–Lemaître–Robertson–Walker (FLRW, for short) metric in D dimensions $(1 + (D-1))$, which has the line element

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x} \cdot d\mathbf{x}. \tag{1}$$

The action is given by

$$S = \frac{1}{2(p+1)!} \int d^D x \sqrt{-g} F^{\mu\mu_1\cdots\mu_p} F_{\mu\mu_1\cdots\mu_p}, \tag{2}$$

where the field strength is $F_{\mu\mu_1\cdots\mu_p} = \partial_{[\mu} A_{\mu_1\cdots\mu_p]}$, and $A_{\mu_1\cdots\mu_p}$ is the p-form gauge field.

Although there are various paths to perform the decomposition of the field in its normal modes, as seen in [16] for example, there are no drawbacks to performing it directly from the equations of motion.

$$\partial_\nu(\sqrt{-g}g^{\mu\nu}g^{\mu_1\nu_1}\cdots g^{\mu_p\nu_p}F_{\mu\mu_1\cdots\mu_p}) = 0. \quad (3)$$

To simplify this equation, we can fix the gauge. A free p-form field has

$$\alpha = \frac{(D-2)!}{p!(D-p-2)!} \quad (4)$$

degrees of freedom. Therefore, we can fix

$$\partial^{i_1}A_{i_1\cdots i_p} = 0 \quad (5)$$

and

$$A_{0i_2\cdots i_p} = 0. \quad (6)$$

From now on, Latin indices $(i_1, i_2, ..., i_p)$ will be used for purely spatial coordinates, while Greek indices $(\mu, \nu, \rho, ...)$ will be for spacetime coordinates. Putting these two conditions into (3) will lead us to the following equation:

$$\ddot{A}_{i_1\cdots i_p} + (D-2p-1)\frac{\dot{a}}{a}\dot{A}_{i_1\cdots i_p} - \frac{1}{a^2}\nabla^2 A_{i_1\cdots i_p} = 0. \quad (7)$$

Now, we take the standard approach by tackling this equation with the usual normal modes decomposition,

$$A_{i_1\cdots i_p}(\mathbf{x},t) = \sum_{\epsilon=1}^{\alpha}\int \frac{d^{D-1}k}{(2\pi)^{D-1}} f^{\epsilon}_{i_1\cdots i_p}\left(r^{\epsilon}_1(t)e^{i\mathbf{k}\cdot\mathbf{x}} + r^{\epsilon}_2(t)e^{-i\mathbf{k}\cdot\mathbf{x}}\right), \quad (8)$$

with $f^{\epsilon}_{i_1\cdots i_p}$ representing the various polarizations obeying the gauge condition $k^{i_1}f^{\epsilon}_{i_1\cdots i_p} = 0$. Substituting (8) into (7), we finally arrive at our desired equation for the modes:

$$\ddot{r} + (D-2p-1)\frac{\dot{a}}{a}\dot{r} + \frac{k^2}{a^2}r = 0. \quad (9)$$

Here, we have omitted all indices attached to r, as Equation (9) remains the same for all of them.

3. Quantization of the *p*-Form Gauge Field in the de Sitter Spacetime

In this section, we discuss the quantization of the p-form gauge field in a D-dimensional de Sitter background, in light of the Lewis and Riesenfeld (LR) method described in ref. [15]. This technique is very useful for obtaining solutions to the Schrödinger equation for different time-dependent Hamiltonian systems and, for example, is not limited to the time-dependent perturbation theory.

We can observe that the equations of motion (9) have the same form as those of a quantum time-dependent harmonic oscillator, Equation (A2). Given this similarity, we can consider our system as being a time-dependent harmonic oscillator with mass given by $m = a^{D-2p-1}$ and frequency $\omega = \frac{k}{a}$. Therefore, quantizing our p-form field can be attained by quantizing the harmonic oscillator with time-dependent mass.

LR assumed that there is a hermitian Invariant operator, $I(t)$, which satisfies the equation

$$\frac{dI}{dt} = \frac{\partial I}{\partial t} + \frac{1}{i\hbar}[I, H] = 0. \quad (10)$$

Its eigenstates $|n,t\rangle$ are assumed to constitute a complete orthonormal set with time-independent discrete eigenvalues analogous to those of a harmonic oscillator, as in Equation (A4). Thus, the invariant satisfies the following eigenvalue equation:

$$I|n,t\rangle = \lambda_n |n,t\rangle, \tag{11}$$

Lewis and Riesenfeld showed that the solution $|\psi_n\rangle$ for the Schrödinger Equation (A3) are related to $|n,t\rangle$ by

$$|\psi_n\rangle = e^{i\theta_n(t)}|n,t\rangle\,, \tag{12}$$

where the phase $\theta_n(t)$ needs to satisfy the following equation [15]:

$$\hbar\frac{d\theta_n(t)}{dt} = \left\langle n,t\middle|\left(i\hbar\frac{\partial}{\partial t} - H(t)\right)\middle|n,t\right\rangle. \tag{13}$$

The choice of I is not unique. However, it is more suitable to choose an invariant that keeps the same symmetry as the Hamiltonian, i.e., if the Hamiltonian is quadratic in q and $\bar{p}$, we try to find a quadratic invariant in q and $\bar{p}$ as well. With this, it was shown in ref. [26] that $I(t)$ can be written in the form

$$I = \frac{1}{2}\left[\left(\frac{q}{\rho}\right)^2 + (\rho\bar{p} - m\dot{\rho}q)^2\right], \tag{14}$$

where $q(t)$ satisfies Equation (A2), and where $\rho = \rho(t)$ satisfies the generalized Milne–Ermakov–Pinney (MP) [24–26] equation

$$\ddot{\rho} + \frac{\dot{m}}{m}\dot{\rho} + \omega^2\rho = \frac{1}{m^2\rho^3}. \tag{15}$$

We point out the important fact that the classical equation of motion (A2) is identical to the MP equation if we add a source given by $m^{-2}q^{-3}$ and replace $q \to \rho$. This recipe will be useful when we treat the p-form field.

Let us proceed and find the eigenstates of I. Equation (12) shows that the solution of the respective Schrödinger Equation (A3) reduces to two steps: finding the eigenstates of I and solving Equation (15) to find the auxiliary function ρ. Since Equation (14) is similar to that of the harmonic oscillator, we proceed as follows to obtain the eigenfunctions of $I(t)$.

Consider the time-dependent creation $b^\dagger$ and annihilation $b(t)$ operators defined as

$$b^\dagger = \sqrt{\frac{1}{2\hbar}}\left[\left(\frac{q}{\rho} - i(\rho p - m\dot{\rho}q)\right)\right], \tag{16}$$

$$b = \sqrt{\frac{1}{2\hbar}}\left[\left(\frac{q}{\rho} + i(\rho p - m\dot{\rho}q)\right)\right], \tag{17}$$

constructed so that $\left[b, b^\dagger\right] = 1$, with the usual properties (A7) and (A6).

Furthermore, we assume the eigenvalues of I to be discrete. This enables us to represent the eigenvalue equation for Equation (14) along with Equations (11) and (A4).

These assumptions follow the same path we make to quantize the Hamiltonian, so it is safe to assume that the eigenstates for the invariant $I(t)$ are indeed related to the Hamiltonians by means of (12).

Now, following the steps outlined in reference [27], the normalized solution for the time-dependent harmonic oscillator is then written as

$$|\psi_n\rangle(q,t) = e^{i\theta_n}\left(\frac{1}{(\pi\hbar)^{1/2}n!2^n\rho}\right)^{1/2}\exp\left\{\frac{im}{2\hbar}\left[\frac{\dot{\rho}}{2\hbar} + \frac{i}{m\rho^2}\right]q^2\right\}H_n\left(\frac{q}{\rho\sqrt{\hbar}}\right), \tag{18}$$

where H_n are the Hermite polynomials of order n, and the phase $\theta_n(t)$ from (13) now reads

$$\theta_n(t) = -\left(n + \frac{1}{2}\right) \int_{t_0}^{t} \frac{1}{m(t')\rho^2}\, dt'. \tag{19}$$

Hence, quantizing the time-dependent harmonic oscillator hinges on identifying a solution to the corresponding MP Equation (15), which will be incorporated into Equation (18). It is worth mentioning that a solution to this nonlinear equation consists of a nonlinear combination of solutions to the linear case [28]. Notice that the linear form of the MP equation mirrors our classical equation of motion (A2). Hence, discovering solutions to (A2) enables us to uncover the sought-after solution to the problem.

Let us now apply the process of quantizing the p-form gauge field, with $m = a^{D-2p-1}$ and $\omega(t) = \frac{k}{a}$. As said before, the auxiliary Milne–Ermakov–Pinney can be obtained form the classical equation of motion (9) by adding a source $m^{-2}r^{-3}$ and replacing $r \to \rho$. We obtain

$$\ddot{\rho} + (D-2p-1)\frac{\dot{a}}{a}\dot{\rho} + \frac{k^2}{a^2}\rho = \frac{1}{a^{2(D-2p-1)}\rho^3}. \tag{20}$$

As mentioned earlier, in order to obtain the solution for Equation (20), we will initially seek solutions to the classical Equation (9). Now, by means of a change to the conformal time η by setting $dt = a d\eta$ and $r = \Omega\bar{r}$, from (9), we obtain

$$\bar{r}'' + \left(2a\frac{\dot{\Omega}}{\Omega} - \dot{a} + (D-2p-1)\dot{a}\right)\bar{r}' + \left(k^2 + a^2\frac{\ddot{\Omega}}{\Omega} + (D-2p-1)a\dot{a}\frac{\dot{\Omega}}{\Omega}\right)\bar{r} = 0, \tag{21}$$

where prime ($'$) and dot (.) represent differentiation with respect to the conformal time η and t, respectively. If we make the choice $\Omega = a^{-(D-2p-1)/2}$, we obtain

$$\bar{r}'' - \dot{a}\bar{r}' + \left[k^2 - \frac{(D-2p-1)}{2}a\ddot{a} + \frac{(D-2p-1)(D-2p-3)}{4}\dot{a}^2\right]\bar{r} = 0. \tag{22}$$

We would like to emphasize that when $D = 2p+1$, certain terms cancel out, leading to a simplified equation contingent upon the choice of parameter a. Let us consider the de Sitter spacetime, where $a = e^{Ht}$. The expressions for a, $\dot{a}$ and $\ddot{a}$ are reduced to

$$a = -\frac{1}{H\eta}, \quad \dot{a} = -\frac{1}{\eta}, \quad \ddot{a} = -\frac{H}{\eta}, \tag{23}$$

and we finally obtain

$$\frac{d^2\bar{r}}{d(k\eta)^2} + \frac{1}{(k\eta)}\frac{d\bar{r}}{d(k\eta)} + \left(1 - \frac{\nu^2}{(k\eta)^2}\right)\bar{r} = 0, \tag{24}$$

where $\nu = \frac{D-2p-1}{2}$. This equation is Bessel's equation, which has two linearly independent solutions, given by $J_\nu(k|\eta|)$ and $Y_\nu(k|\eta|)$, which are the Bessel functions of the first and second kind, respectively. Now, employing our earlier redefinition of $r = \Omega\bar{r}$, with $\Omega = a^{-(D-2p-1)/2}$, we obtain that two linearly independent solutions for r are:

$$r = \begin{cases} a^{-(D-2p-1)/2} J_\nu(k|\eta|) \\ a^{-(D-2p-1)/2} Y_\nu(k|\eta|) \end{cases} \tag{25}$$

Following references [29,30], a particular solution of Equation (20) is:

$$\rho = -(H\eta)^{(D-2p-1)/2}\left[AJ_\nu^2 + BY_\nu^2 + 2\left(AB - \frac{\pi^2}{4H^2}\right)^{1/2} J_\nu Y_\nu\right]^{1/2}, \tag{26}$$

It is important to mention that the choice of constants A and B is crucial from a physical standpoint, as the determination of these constants is intricately tied to our vacuum selection. This arises from the non-uniqueness in constructing particle states and selecting the vacuum in curved spaces like the one employed in this scenario. This holds significance because the generation of particles can only be deduced once we have selected a vacuum for comparison with our physical solution. In this way, solution (26), being related to the solution of Schrödinger's Equation (18), suggests that an eigenstate of the latter may be associated with particle generation under the choice of a specific vacuum.

In our scenario, a suitable choice corresponds to the Bunch–Davies vacuum, which aligns with the adiabatic vacuum for very early times ($t \to -\infty$) or the adiabatic vacuum for wavelengths much smaller than the de Sitter horizon H^{-1}. With these assumptions, the values of the constants are $A = B = \pi/2H$ [29], and ρ is given by

$$\rho = (H|\eta|)^{(D-2p-1)/2}\sqrt{\frac{\pi}{2H}}\left(J_\nu^2 + Y_\nu^2\right)^{1/2}. \tag{27}$$

Thus, an eigenstate of Schrödinger's Equation (18), related to solution (27), may be associated with particle generation in a Bunch–Davies vacuum which, as mentioned earlier, corresponds to the adiabatic vacuum in the physical scenarios described in the preceding paragraph.

Now that we have finally found the general solution to the Milne–Ermakov–Pinney Equation (20) in a de Sitter scenario, we can substitute it into the expression for the solution of the harmonic oscillator with mass and frequency dependent on time (18). This concludes the quantization of the p-form gauge field in a D-dimensional de Sitter background.

4. Concluding Remarks

In this work, we have presented a generalization of the quantization procedure, through the use of p-form gauge fields, for the scalar, electromagnetic, and Kalb–Ramond fields, all of which have been previously studied and referenced [16–18]. In this connection, we have obtained a solution to the Schrödinger equation using the method developed by Lewis and Reisenfeld [15], applied to the quantization of the p-form field in a D-dimensional de Sitter spacetime. A general solution for Equation (21) is found to depend on the scale factor present in the FLRW spacetime, and was obtained in the particular case of de Sitter spacetime, which is significant because our Universe today can be approximated as such, and in the far future, it would fully become one.

We can check that Equation (27) is constant for $D = 2(p+1)$. This is in agreement with the previous works for $D = 4$ and $p = 1$ [17,30]. Thus, for $D = 2(p+1)$, for a (massless) photon, the initial adiabatic vacuum persists indefinitely, resulting in zero photon production within de Sitter spacetime, while its energy undergoes the redshift characteristic of radiation.

An interesting case where $D = 2(p+1)$ arises for $p = 4$ in a 10-dimensional spacetime, where $A_{\mu_1\cdots\mu_p}$ corresponds to a 4-form gauge field, while the field strength corresponds to a 5-form. In this scenario, there will be no production of particles, and a co-moving accelerated observer will not experience a thermal bath. This specific dimension value indicates that the 4-form exhibits conformal invariance, allowing for the straightforward solution of the time-dependent harmonic oscillator (18) in a de Sitter spacetime. Since our field strength is going to be a 5-form, it will be dual to itself $F_{\mu_1\cdots\mu_5} = *F_{\mu_1\cdots\mu_5}$. The case where $D = 10$ has garnered attention as it represents the critical dimension in superstring theory, and the 5-form field strength naturally appears as a first-order approximation for the gravitational coupling constant of chiral $N = 2$ $D = 10$ supergravity [31].

For $D \neq 2(p+1)$, it becomes evident that we can no longer have a constant solution for (27), regardless of our chosen cosmological model. In a future work, a more general solution could be computed for the case of de Sitter spacetime, which undoubtedly has more interesting consequences. Specifically, we can observe particle production for $D \neq 2(p+1)$

because the solutions are time-dependent. Additionally, observers in an accelerated co-moving reference frame will also experience a thermal bath.

As mentioned in the introduction, gauge p-form fields in certain extra-dimensional scenarios exhibit physical properties that are not visible in four dimensions. Thus, an intriguing implication arises in the realm of extra-dimensional physics, which has garnered significant attention in certain scenarios involving de Sitter spacetimes. For instance, in higher-dimensional FRW scenarios and their associated particle creation [20,21], or in de Sitter braneworld models [32,33]. Braneworld models where FRW branes possess a temperature have been investigated in reference [34]. In braneworld models, our universe is conceptualized as a brane existing within a five-dimensional space. Consequently, in such a setup, a de Sitter spacetime would lead to particle production and the presence of a thermal bath for observers moving within this expanded space. Consequently, this could potentially contribute to an effective temperature within the membrane, suggesting the intriguing possibility that precise measurements of the Cosmic Microwave Background could reveal the presence of extra dimensions.

Author Contributions: Investigation and writing, E.W.D.D., G.A., I.G. and M.E. All authors have read and agreed to the published version of the manuscript.

Funding: Milko Estrada is funded by the FONDECYT de Iniciación 2023 folio 11230247.

Data Availability Statement: No new data were created or anlyzed in this study. Date sharing is not applicable to this article.

Acknowledgments: E.W.D. Dantas, I. Guedes and G. Alencar thank the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES).

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A. The Quantum Time-Dependent Harmonic Oscillator

The time-dependent harmonic oscillator Hamiltonian is given by

$$H(t) = \frac{\bar{p}^2}{2m(t)} + \frac{m(t)\omega(t)^2}{2}q^2. \tag{A1}$$

where $\bar{p}$ and q represent the coordinate and momentum in a quantum framework.

The equations of motion are trivially obtained, and are given by

$$\ddot{q} + \frac{\dot{m}}{m}\dot{q} + \omega^2 q = 0, \tag{A2}$$

Furthermore, it is important to mention that the Hamiltonian Equation (A1) satisfies the following eigenvalue equation:

$$i\hbar\partial_t\psi(q,t) = H(t)\psi(q,t). \tag{A3}$$

where the eigenvalues of the Hamiltonian operator, related to the system's energy in a time-independent background, are given by:

$$\lambda_n = (n + \frac{1}{2})\hbar. \tag{A4}$$

where n corresponds to the eigenvalues of the number operator, which satisfies

$$\langle n|n'\rangle = \delta_{nn'}. \tag{A5}$$

On the other hand, it is important to mention the existence of the raising and lowering operators, whose effect on the eigenstates of the number operator is:

$$b|n,t\rangle = \sqrt{n}|n-1,t\rangle, \tag{A6}$$

$$b^\dagger|n,t\rangle = \sqrt{n+1}|n+1,t\rangle, \tag{A7}$$

where the number operator is given by

$$\hat{n}|n,t\rangle = b^\dagger b|n,t\rangle = n|n,t\rangle, \tag{A8}$$

References

1. Will, C.M. The Confrontation between General Relativity and Experiment. *Living Rev. Rel.* **2014**, *17*, 4. [CrossRef] [PubMed]
2. Hossenfelder, S.; Smolin, L. Phenomenological Quantum Gravity. *Phys. Canada* **2010**, *66*, 99–102.
3. Hawking, S.W. Black hole explosions. *Nature* **1974**, *248*, 30–31. [CrossRef]
4. Padmanabhan, T. Gravity and the thermodynamics of horizons. *Phys. Rept.* **2005**, *406*, 49–125. [CrossRef]
5. Leonhardt, U. Wave correlations and quantum noise in cosmology. *J. Phys. A* **2023**, *56*, 024001.
6. Haouat, S.; Chekireb, R. On the Creation of Scalar Particles in a Flat Robertson-Walker Space-time. *Mod. Phys. Lett. A* **2011**, *26*, 2639–2651. [CrossRef]
7. Grib, A.A.; Levitskii, B.A.; Mostepanenko, V.M. Particle creation from vacuum by non-stationary gravitational field in the canonical formalism. *Teor. Mat. Fiz.* **1974**, *19*, 59–75. [CrossRef]
8. Basler, M.; Kampfer, B. On the dynamics of cosmological particle production. *Gen. Rel. Grav.* **1989**, *21*, 881–898. [CrossRef]
9. Pavlov, Y.V. Exact solutions for a scalar field in homogeneous isotropic cosmological models, and particle creation. *Uch. Zapiski Kazan Univ.* **2011**, *153*, 65–71. [CrossRef]
10. Parker, L. Particle creation in expanding universes. *Phys. Rev. Lett.* **1968**, *21*, 562–564. [CrossRef]
11. Parker, L. Quantized fields and particle creation in expanding universes. I. *Phys. Rev.* **1969**, *183*, 1057–1068. [CrossRef]
12. Parker, L. Quantized fields and particle creation in expanding universes. II. *Phys. Rev. D* **1971**, *3*, 346–356; Erratum in *Phys. Rev. D* **1971**, *3*, 2546. [CrossRef]
13. Berdyugin, A.I.; Xin, N.; Gao, H.; Slizovskiy, S.; Dong, Z.; Bhattacharjee, S.; Kumaravadivel, P.; Xu, S.; Ponomarenko, L.A.; Holwill, M.; et al. Out-of-equilibrium criticalities in graphene superlattices. *Science* **2022**, *375*, 430–433. [CrossRef]
14. Lopes, C.E.F.; Pedrosa, I.A.; Furtado, C.; de M. Carvalho, A.M. Gaussian wave packet states of scalar fields in a universe of de Sitter. *J. Math. Phys.* **2009**, *50*, 083511. [CrossRef]
15. Lewis, H.R.; Riesenfeld, W.B. An Exact quantum theory of the time dependent harmonic oscillator and of a charged particle time dependent electromagnetic field. *J. Math. Phys.* **1969**, *10*, 1458–1473. [CrossRef]
16. Alencar, G.; Guedes, I.; Landim, R.R.; Costa Filho, R.N. Quantum Scalar Field in D-dimensional de Sitter Spacetimes. *EPL* **2012**, *98*, 11001. [CrossRef]
17. Alencar, G.; Guedes, I.; Landim, R.R.; Costa Filho, R.N. An Exact Solution to the Quantized Electromagnetic Field in D-dimensional de Sitter Spacetimes. *Int. J. Mod. Phys. A* **2012**, *27*, 1250177. [CrossRef]
18. Alencar, G.; Guedes, I.; Landim, R.R.; Costa Filho, R.N. Quantum Kalb-Ramond Field in D-dimensional de Sitter Spacetimes. *Int. J. Mod. Phys. A* **2013**, *28*, 1350011. [CrossRef]
19. Mukherjee, J. Entanglement entropy and the boundary action of edge modes. *J. High Energy Phys.* **2023**, *2024*, 113. [CrossRef]
20. Aygün, S.; Aktaş, C.; Yılmaz, I.; Şahin, M. Higher dimensional FRW universe solutions with quark and strange quark matter in creation field cosmology. *Chin. J. Phys.* **2016**, *54*, 810–819. [CrossRef]
21. Alfedeel, A.H.A. The Evolution of a Higher-Dimensional FRW Universe with Variable G and Λ and Particle Creation. *Universe* **2023**, *9*, 255. [CrossRef]
22. Henneaux, M.; Teitelboim, C. p-form electrodynamics. *Found. Phys.* **1986**, *16*, 593–617. [CrossRef]
23. Weinberg, S. *The Quantum Theory of Fields. Vol. 1: Foundations*; Cambridge University Press: New York, NY, USA, 2005. [CrossRef]
24. Milne, W.E. The Numerical Determination of Characteristic Numbers. *Phys. Rev.* **1930**, *35*, 863–867. [CrossRef]
25. Pinney, E. The nonlinear differential equation $y'' + p(x)y + cy^{-3} = 0$. *Proc. Am. Math. Soc.* **1950**, *1*, 681. [CrossRef]
26. Pedrosa, I.A.; Serra, G.P.; Guedes, I. Wave functions of a time-dependent harmonic oscillator with and without a singular perturbation. *Phys. Rev. A* **1997**, *56*, 4300–4303. [CrossRef]
27. Carinena, J.F.; De Lucas, J. Applications of Lie systems in dissipative Milne–Pinney equations. *Int. J. Geom. Methods Mod. Phys.* **2009**, *06*, 683–699. [CrossRef]
28. Prince, G.E.; Eliezer, C.J. Symmetries of the time-dependent N-dimensional oscillator. *J. Phys. Math. Gen.* **1980**, *13*, 815. [CrossRef]
29. Bertoni, C.; Finelli, F.; Venturi, G. Adiabatic invariants and scalar fields in a de Sitter space-time. *Phys. Lett. A* **1998**, *237*, 331–336. [CrossRef]
30. Finelli, F.; Gruppuso, A.; Venturi, G. Quantum fields in an expanding universe. *Class. Quant. Grav.* **1999**, *16*, 3923–3935. [CrossRef]
31. Schwarz, J.H.; West, P.C. Symmetries and Transformations of Chiral N = 2 D = 10 Supergravity. *Phys. Lett. B* **1983**, *126*, 301–304. [CrossRef]
32. Das, S.; Maity, D.; SenGupta, S. Cosmological constant, brane tension and large hierarchy in a generalized Randall-Sundrum braneworld scenario. *JHEP* **2008**, *05*, 042. [CrossRef]

33. Aros, R.; Estrada, M. Embedding of two de-Sitter branes in a generalized Randall Sundrum scenario. *Phys. Rev. D* **2013**, *88*, 027508. [CrossRef]
34. Aros, R.; Estrada, M. Study of branes with variable tension. *Commun. Theor. Phys.* **2017**, *68*, 595. [CrossRef]