Particles as stable topological solitons

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Abstract. We discuss a model for charged particles and their fields where mass is field energy only, charge appears as a topological quantum number, photons are Goldstone bosons of spontaneous symmetry breaking, spin and chirality are described as topological charge of solitons interacting by Coulomb and Lorentz forces. The model is U(1) gauge invariant. We discuss open problems and add some speculations. The model is as simple as possible, with three degrees of freedom only describing the rotations of Dreibeins (triads) in space-time and dynamics defined by a Lorentz covariant Lagrangian.

1. Introduction

We can ask some naive questions:

- What is the origin of mass?
- Why do all electrons (protons) have the same mass?
- Why is charge quantised?
- What is spin?
- What is the origin of the Pauli principle?
- Where does gauge invariance emerge from?

We will discuss that there are models indicating answers to these questions.

There are several interesting relativistically covariant models with solutions which contain particle-like excitations. Five of them I would like to mention shortly since they are related to the model presented in this paper. The first one was formulated by Dirac, when he aimed to symmetrize electro-dynamics by introducing magnetic currents into the Maxwell equations [1, 2]. Dirac's magnetic monopoles are defined in 3+1D, in three space and one time dimension. They have two types of singularities, line-like singularities, Dirac strings, from the center of the monopole to an antimonopole or to infinity which are gauge dependent and therefore unphysical and point-like singularities in the center, well-known also from classical electron models. Wu and Yang [3, 4, 5] succeeded to formulate magnetic monopoles without the line-like singularities of the Dirac strings by using either a fibre-bundle construction with two different gauge fields, one for the northern and one for the southern hemisphere of the monopole, or by a normalised three-dimensional vector field $\vec{n}(x)$ in 3+1D. These Wu-Yang monopoles still suffer from the point-like singularities in the center. There are monopole solutions without any singularity in the Georgi-Glashow model [6], the 'tHooft-Polyakov monopoles [7, 8]. The Georgi-Glashow model, formulated in 3+1D, has 15 degrees of freedom, an adjoint Higgs field with three degrees of freedom and an SU(2) gauge field with $4 \cdot 3 = 12$ field components. Only one degree of freedom is needed for the Sine-Gordon model [9], a model in 1+1D. It addition to waves it has kink and anti-kink solutions which interact with each other. The kink-antikink configuration is attracting and the kink-kink configuration is repelling. The simplicity of the Sine-Gordon model inspired Skyrme [10, 11, 12] to a model in 3+1D with a scalar SU(2)-valued field, i.e. three degrees of freedom, with stable topological solitons with the properties of particles with a short range interaction, the Skyrmions. The model presented in this paper, also inspired by the simplicity and physical content of the Sine-Gordon model, the model of topological fermions, was first formulated in [13]. It has the same degrees of freedom as the Skyrme model but uses a different Lagrangian. Its relations to electrodynamics and symmetry breaking were discussed in [14, 15, 16, 17]. A python package for numerical calculations was written by Roman Bertle and actual calculations were performed in several diploma works at the Vienna University of Technology by Joachim Wabnig, Maria Hörndl, Julia Fornleitner, Xaver König, Roman Höllwieser and Josef Resch.

2. Formulation of the model

As basic field we use a scalar SO(3)-valued field in three space and one time dimension with coordinates $x^\mu = (ct, \mathbf{x})$, defining a rotation of a Dreibein in three-dimensional space. Below, we call it the soliton field. Objects not connected to the surrounding return after 2π-rotations to the original situation. One can easily show in a nice experiment performed at this conference that objects connected by wires to the surrounding behave differently [18]. Rotating such an object by 2π or 4π, the wires are completely entangled, but after 4π-rotations the wires can be disentangled without moving the object itself. In mathematics this is well known. SU(2) is the double (universal) covering group of SO(3), each pair of elements of SU(2) differing by a rotation by 2π can be mapped to an element of SO(3), and 4π-rotations are smoothly connected to no rotation. For the actual formulation of our basic SO(3)-field it is algebraically simpler to use an SU(2)-field $Q(x)$, keeping in mind that for differentiable fields every SO(3) field configuration corresponds to two SU(2)-field configuration differing by a multiplication with the non-trivial center element $z = -\mathbb{1}_2$ of SU(2). Active rotations Q by an angle $\omega = 2\alpha$ around the axis $\vec{n}$ can be represented by the quaternionic units $\mathbf{i}, \mathbf{j}, \mathbf{k}$ with the famous relation $\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = \mathbf{ijk} = -1$ or by Pauli matrices σ_K with the correspondence $\mathbf{i} = -\mathrm{i}\sigma_1$, $\mathbf{j} = -\mathrm{i}\sigma_2$, $\mathbf{k} = -\mathrm{i}\sigma_3$,

$$Q = q_0 - \mathrm{i}\vec{\sigma}\vec{q} = \begin{pmatrix} q_0 - \mathrm{i}q_3 & -q_2 - \mathrm{i}q_1 \\ q_2 \mathrm{i}q_1 & q_0 + \mathrm{i}q_3 \end{pmatrix} \quad \text{with} \quad \vec{\sigma}\vec{q} = \sigma_K q_K,\ q_0 = \cos\alpha,\ \vec{q} = \vec{n}\sin\alpha \tag{1}$$

The SU(2)-manifold is isomorphic to S_3, to a three-dimensional sphere in four dimensions, see Fig. 1. We define an affine connection, a vector field $\vec{\Gamma}_\mu$ and a curvature tensor $\vec{R}_{\mu\nu}$

$$\partial_\mu Q Q^\dagger = -\mathrm{i}\vec{\Gamma}_\mu \vec{\sigma}, \quad \vec{R}_{\mu\nu} = \vec{\Gamma}_\mu \times \vec{\Gamma}_\nu. \tag{2}$$

with simple geometrical interpretations on S_3. From the condition for the function $Q(x)$ to be single-valued, the Schwarz integrability condition $\partial_\mu\partial_\nu Q(x) = \partial_\nu\partial_\mu Q(x)$, follows [13] the Maurer-Cartan equation $\partial_\mu\vec{\Gamma}_\nu - \partial_\nu\vec{\Gamma}_\mu = 2\vec{\Gamma}_\mu \times \vec{\Gamma}_\nu$, allowing to represent the curvature tensor $\vec{R}_{\mu\nu} = \vec{\Gamma}_\mu \times \vec{\Gamma}_\nu$ as $\vec{R}_{\mu\nu} = \frac{1}{2}(\partial_\mu\vec{\Gamma}_\nu - \partial_\nu\vec{\Gamma}_\mu)$ and in a form well-known from gravity and non-abelian gauge theories

$$\vec{R}_{\mu\nu} = \partial_\mu\vec{\Gamma}_\nu - \partial_\nu\vec{\Gamma}_\mu - \vec{\Gamma}_\mu \times \vec{\Gamma}_\nu. \tag{3}$$

The dynamics of our model is given by a Lagrangian density $\mathcal{L}$ which in generalisation of the Sine-Gordon model to 3+1D contains a curvature term $\vec{R}_{\mu\nu}\vec{R}^{\mu\nu}$ and a potential term $\Lambda(q_0)$

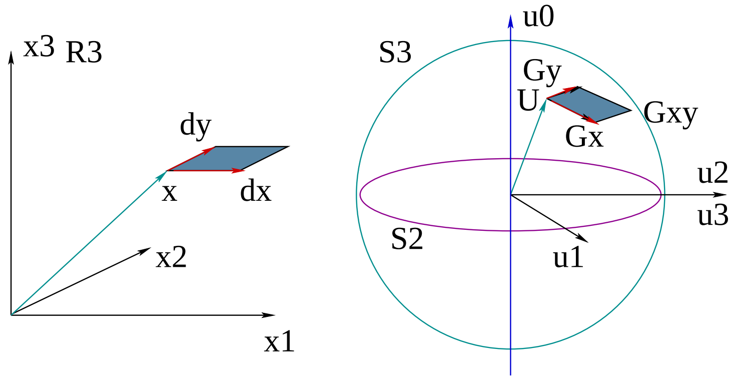


Figure 1. $Q(x)$, $\vec{\Gamma}_\mu \mathrm{d}x^\mu$ and $\vec{R}_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu$ represent points, line and area elements on S_3.

which is a function of q_0 only, with a minimum at $q_0 = 0$ and maxima at $|q_0| = 1$,

$$\mathcal{L} = -\frac{\alpha_f \hbar c}{4\pi}\left(\frac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} + \Lambda(q_0)\right) \quad \text{with} \quad \Lambda(q_0) = \frac{q_0^{2m}}{r_0^4}, \quad m = 1, 2, 3, \cdots . \tag{4}$$

The potential term has no derivatives and the curvature term four derivatives, this forces us to introduce a parameter r_0 for the length scale in Eq. (4). Since we want to compare the solitons emerging from this Lagrangian with electric charges, we have introduced Sommerfeld's fine-structure constant α_f. A relation to physics we get by the identification of the dual field-strength tensor with the curvature tensor ${}^*\vec{F}_{\mu\nu} = -\frac{e_0}{4\pi\varepsilon_0 c}\vec{R}_{\mu\nu}$.

3. Solitonic solutions

The Lagrangian (4) has solitonic solutions with the properties of electric charges. The static ansatz for a soliton, see left diagram in Fig. 2,

$$\vec{n}(x) = \frac{\mathbf{r}}{r}, \quad \alpha(x) = \alpha(r), \quad \text{with} \quad \mathbf{r} = (x_1, x_2, x_3) \tag{5}$$

and $\rho = \frac{r}{r_0}$ leads [13] to the energy functional

$$H = \frac{\alpha_f \hbar c}{r_0}\int_0^\infty \mathrm{d}\rho \left[\frac{\sin^4\alpha}{2\rho^2} + (\partial_\rho \cos\alpha)^2 + \rho^2 \cos^{2m}\alpha\right], \tag{6}$$

and by the variation of this functional to the non-linear second order differential equation

$$\partial_\rho^2 \cos\alpha + \frac{(1-\cos^2\alpha)\cos\alpha}{\rho^2} - m\rho^2\cos^{2m-1}\alpha = 0. \tag{7}$$

Solutions of this equation read for $m = 3$

$$\alpha(r) = \arctan\rho, \quad \sin\alpha(r) = \frac{\rho}{\sqrt{1+\rho^2}}, \quad \cos\alpha(r) = \frac{1}{\sqrt{1+\rho^2}}, \tag{8}$$

with a minimum value of the energy functional $H = \frac{\alpha_f \hbar c}{r_0}\frac{\pi}{4}$. Comparing this result with the mass of an electron, $m_e c^2 = 0.511$ MeV, we get $r_0 = 2.21$ fm, a value very close to the classical electron radius $\frac{\alpha_f \hbar c}{m_e c^2} = \alpha_f \frac{\hbar}{m_e c} = 2.82$ fm. For $m = 2$ we get the solution $q_0(\rho) = \frac{1}{1+\rho^2}$ of Eq. (7) by scaling the potential, $\Lambda(q_0) = \frac{7}{2}(\frac{q_0}{r_0})^4$, with the soliton energy $E_1 = \frac{\alpha_f \hbar c}{r_0}\frac{7\pi}{16}$. Implementing the factor $\frac{7}{2}$ in r_0 and again comparing the soliton energy with $m_e c^2$ leads to $\sqrt[4]{2/7}\,r_0 = 2.83$ fm. The three contributions (6) to the radial energy density are shown in the right diagram of Fig. 2.

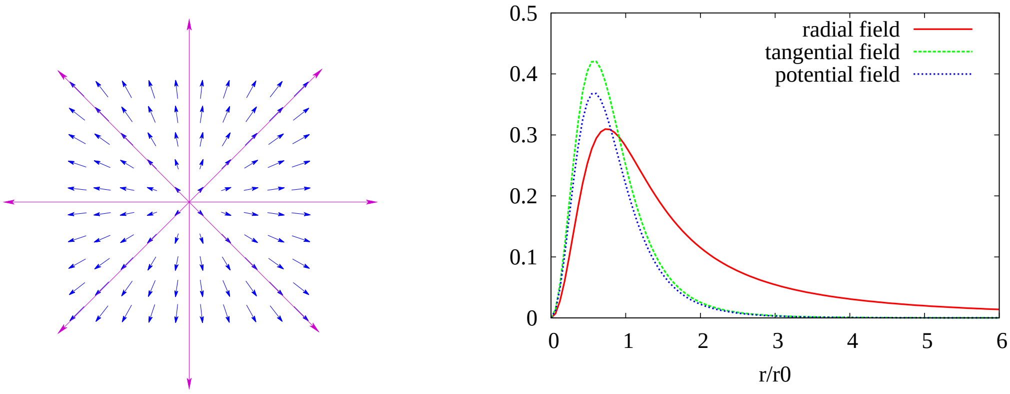


Figure 2. Left: The hedge-hog configuration according to Eq. (5) is depicted by arrows indicating the size of the imaginary part $\vec{q}(x)$ of the soliton field $Q(x)$. The lines connect $\vec{q}$-vectors with the same direction. Right: contributions to the radial energy density in Eq. (6) for $m = 3$ and $q_0(\rho) = \cos\alpha(\rho) = \frac{1}{1+\rho^2}$. For large distances the radial energy density is proportional to $1/\rho^2$.

4. Soliton mechanics

The variation of the soliton-field $Q(x)$ has to respect the unitarity of Q and can be done by $Q \to Q' = \mathrm{e}^{\mathrm{i}\vec{\sigma}\vec{\zeta}}Q$ with a three-component vector $\vec{\zeta}$ which can be considered as a generalised coordinate. The generalised momentum density follows to be $\pi^\mu = \frac{\partial\mathcal{L}}{\partial\partial_\mu\vec{\zeta}} = -\frac{\alpha_f\hbar c}{4\pi}\vec{\Gamma}_\nu \times \vec{R}^{\mu\nu}$ and can also be expressed by $\pi^\mu = \frac{\partial\mathcal{L}}{\partial\vec{\Gamma}_\mu}$. From the variation of $\vec{\zeta}$ follows the equation of motion

$$\partial_\mu[\vec{\Gamma}_\nu \times \vec{R}^{\mu\nu}] + \vec{q}\,\frac{\mathrm{d}\Lambda}{\mathrm{d}q_0} = 0. \tag{9}$$

It corresponds to Newton's second law of motion, where the derivative of the momentum is given by the variation of the potential.

From the Lagrangian (4) we get the canonical energy-momentum tensor by

$$\Theta^\mu{}_\nu(x) = \frac{\partial\mathcal{L}(x)}{\partial(\partial_\mu\alpha_k)}\partial_\nu\alpha_k - \mathcal{L}(x)\,\delta^\mu_\nu = \frac{\partial\mathcal{L}(x)}{\partial\vec{\Gamma}_\mu}\vec{\Gamma}_\nu - \mathcal{L}(x)\,\delta^\mu_\nu. \tag{10}$$

In Maxwell's electrodynamics the canonical energy-momentum tensor is asymmetric and has to be symmetrised by adding an appropriate total divergence [19]. The tensor (10) turns out to be symmetric from the very beginning

$$\Theta^\mu{}_\nu = -\frac{\alpha_f\hbar c}{4\pi}\left\{\left(\vec{\Gamma}_\nu \times \vec{\Gamma}_\sigma\right)\left(\vec{\Gamma}^\mu \times \vec{\Gamma}^\sigma\right)\right\} - \mathcal{L}(x)\,\delta^\mu_\nu. \tag{11}$$

For the trace of the energy-momentum tensor (11) we get

$$\Theta^\mu{}_\mu = 4\mathcal{H}_p. \tag{12}$$

It contributes in the presence of charges only.

The derivative of the energy-momentum tensor defines the force density. It vanishes [13] due to the equation of motion (9)

$$f_\nu = \partial_\mu\Theta^\mu_\nu = 0. \tag{13}$$

This is an obvious consequence of the unified description of charges and fields. Therefore, energy and momentum

$$P^\mu = \frac{1}{c}\int \Theta^{0\mu}\mathrm{d}^3x \tag{14}$$

are conserved quantities.

As expected the mass m_e of a single soliton increases with its velocity $\mathbf{v}$

$$P^0 = \gamma\, m_e c, \quad \mathbf{P} = \boldsymbol{\beta}\,\gamma\, m_e c, \quad \boldsymbol{\beta} = \mathbf{v}/c, \quad \gamma = 1/\sqrt{1-\boldsymbol{\beta}^2}. \tag{15}$$

5. Electrodynamics, Coulomb and Lorentz forces

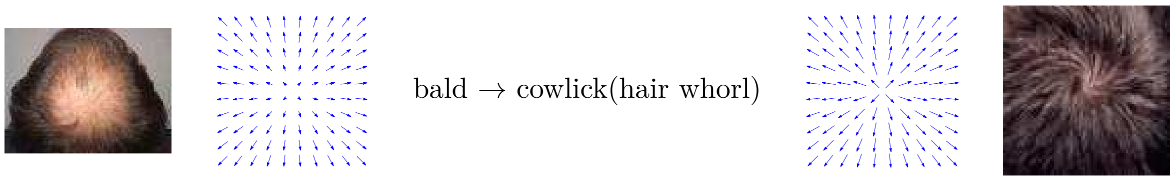


Figure 3. Left: Schematic picture for a regular soliton field $Q(x)$ with no singularity in the center. Right: Singular soliton field of a Wu-Yang monopole $Q(x) = -\mathrm{i}\vec{\sigma}\vec{n}$.

In order to derive Coulomb and Lorentz forces we have to artificially separate particles and their fields. In the limit

$$r_0 \to 0 \quad \Longleftrightarrow \quad q_0 = \cos\alpha = 0 \quad \Longleftrightarrow \quad \alpha = \frac{\pi}{2}, \tag{16}$$

see Fig. 3 for a descriptive picture, solitons transform to dual Dirac-monopoles in the Wu-Yang description [5]. Their remain only two degrees of freedom, a normalised three-dimensional vector field $\vec{n}(x)$ with

$$Q(x) = -\mathrm{i}\vec{\sigma}\vec{n}(x), \quad \vec{\Gamma}_\mu(x) = \vec{n}(x)\times\partial_\mu\vec{n}(x), \quad \vec{R}_{\mu\nu}(x) = \partial_\mu\vec{n}(x)\times\partial_\nu\vec{n}(x). \tag{17}$$

Since $\vec{R}_{\mu\nu}(x)$ points in $\vec{n}$-direction there is only one essential component left, $\vec{R}_{\mu\nu}\vec{n}$. The field-strength tensor and the Lagrangian reduce to

$$^*F_{\mu\nu}(x) = -\frac{e_0}{4\pi\varepsilon_0 c}\vec{R}_{\mu\nu}\vec{n} = -\frac{e_0}{4\pi\varepsilon_0 c}\vec{n}(x)[\partial_\mu\vec{n}(x)\times\partial_\nu\vec{n}(x)], \quad \mathcal{L}_{\mathrm{ED}} = -\frac{1}{4\mu_0}{}^*F_{\mu\nu}(x)^*F^{\mu\nu}(x). \tag{18}$$

Since solitons can not move faster than light, the singularities can be enclosed in two-dimensional surfaces, e.g. parametrised by u and v, leading to the definition of electric charge

$$Q_{\mathrm{el}}(\mathcal{S}) = -\frac{e_0}{4\pi}\oint_{\mathcal{S}(u,v)} \mathrm{d}u\mathrm{d}v\,\vec{n}[\partial_u\vec{n}\times\partial_v\vec{n}]. \tag{19}$$

In Minkowski-space the singularities form closed world-lines, the electric charge is conserved,

$$j^\mu = -e_0 c\sum_{i=1}^{N}\int \mathrm{d}\tau_i \frac{\mathrm{d}X^\mu(\tau_i)}{\mathrm{d}\tau_i}\,\delta^4(x - X(\tau_i)) = (c\rho, \mathbf{j}). \tag{20}$$

After this artificial splitting of the soliton field in electric currents and electro-magnetic fields, we can also split the total force density (13) in two parts

$$f_\nu = \partial_\mu \Theta^\mu{}_\nu = f^\mu_{\text{charges}} + \partial^\nu T^\mu{}_\nu = 0, \tag{21}$$

Coulomb and Lorentz forces appear

$$f^0_{\text{charges}} = \frac{1}{c}\mathbf{jE}, \quad \mathbf{f}_{\text{charges}} = \rho\mathbf{E} + \mathbf{j}\times\mathbf{B}. \tag{22}$$

It is well known from QCD that the tensor (3) is gauge-invariant. In the case of the soliton model this means invariant against rotations of the local coordinate systems on S^3. Since q_0 is also invariant, the Lagrangian (4) is invariant against SU(2)-gauge transformations. In the electrodynamic limit the soliton-field is restricted from the Q-field to the $\vec{n}$-field, from S^3 to S^2. The tangential spaces of S^2 are two-dimensional, the gauge-invariance reduces to a U(1)-invariance, well known from Maxwell's electrodynamics.

6. Topological charge

In addition to electric charge which turned out to be a topological quantum number explaining the quantisation of charge there is a further quantum number, the topological charge which counts the number of coverings of S^3

$$\mathcal{Q} = \frac{1}{V(\mathrm{S}^3)}\int_0^\infty \mathrm{d}r \int_0^\pi \mathrm{d}\vartheta \int_0^{2\pi} \mathrm{d}\varphi\, \vec{\Gamma}_r(\vec{\Gamma}_\vartheta \times \vec{\Gamma}_\varphi), \quad V(\mathrm{S}^3) = \int_{S^2} \mathrm{d}^2 n \int_0^\pi \mathrm{d}\alpha \sin^2\alpha = 2\pi^2. \tag{23}$$

The hedge-hog configuration (5) covers half of S^3, therefore it is characterised by $|\mathcal{Q}| = \frac{1}{2}$. It turns out that $\mathcal{Q}$ is a conserved quantity [13], $\frac{\mathrm{d}\mathcal{Q}(t)}{\mathrm{d}t} = 0$.

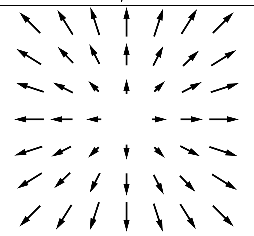
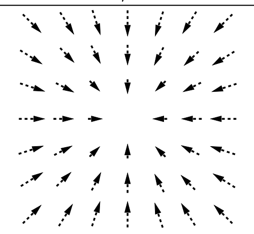
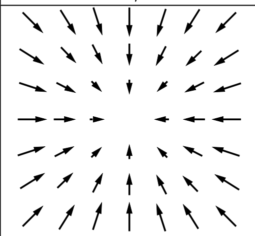
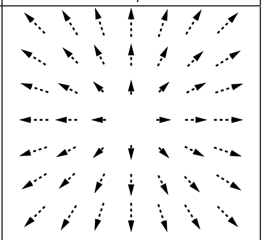
Transf.	1	z	Π_n	$z\Pi_n$
$\vec{n}$	$\vec{r}/r$	$-\vec{r}/r$	$-\vec{r}/r$	$\vec{r}/r$
q_0	≥ 0	≤ 0	≥ 0	≤ 0
Q_{el}/e_0	-1	1	1	-1
$\mathcal{Q} = \chi \times s$	$1/2$	$1/2$	$-1/2$	$-1/2$
diagram				

Table 1. Schematic pictures of the topologically different single-soliton configurations and their quantum numbers. Length and direction of arrows depict the value of the imaginary part $\vec{q}(x)$ of the soliton field $Q(x)$. The sign of q_0 is indicated by the line type of the arrow, full lines for $q_0 > 0$ and dashed lines for $q_0 < 0$.

Investigating the possible types of solitons, the physical meaning of the topological charge gets obvious. There are two transformations changing the homotopy type of the configuration characterised by the electric charge Q_{el} and the topological charge $\mathcal{Q}$. These are the transformation of the soliton field $Q(x)$ with the non-trivial center element $z = -\mathbb{1}_2$ and a reversal Π_n of the $\vec{n}$-direction. Applying these transformations, as shown in Table 1, we get four single-soliton configurations. It can be seen that arrows pointing outward (inward) define negative (positive) charges. Traversing the soliton, the arrows indicate a rotation by 2π. The topological charge reflects the chirality χ of this rotation, whether it is right or left-handed. Lines of constant $\vec{n}$-field, e.g. shown in the left part Fig. 2, connect solitons to the surroundings. Therefore, as argued in the beginning of section 2, solitons return only after a 4π-rotation to the original state. This is characteristic of particles with spin quantum number $s = 1/2$, of fermions. Since solitons are characterised by $|\mathcal{Q}| = \frac{1}{2}$ it is quite natural to interpret $\mathcal{Q}$ as the product of χ and s

$$\mathcal{Q} = \chi \times s. \tag{24}$$

This relation may answer a question, posed by T.D.Lee around 30 years ago, when he gave a talk at the University of Vienna: "Why does the mass violate chiral symmetry?"

7. Interacting solitons

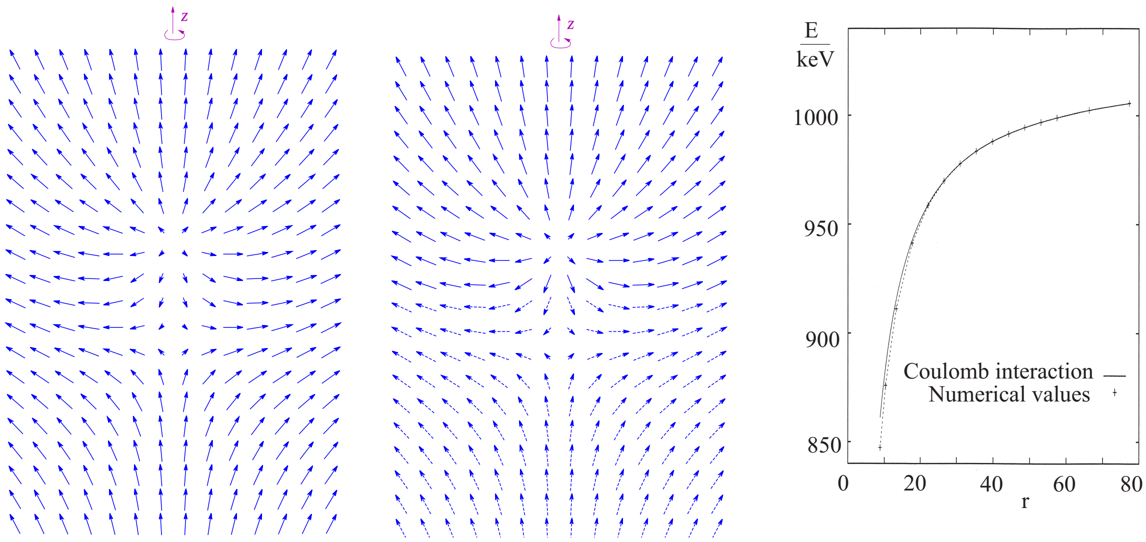


Figure 4. Soliton-antisoliton configurations, two opposite charges, left: $\mathcal{Q} = S = 0$, middle: $\mathcal{Q} = S = 1$, observe the dashed vectors in the lower half. Right: Total energy of an "electron-positron"-system in the $S = 0$-state for $m = 3$ in units of keV (crosses) as a function of distance r in lattice units $a = 3r_0$ for $r_0 = 2.21$ fm in comparison to the expectation for point-like charges (full line), from [20].

Solitons interact with electromagnetic forces. Therefore, static two-soliton configurations are in general not solutions of the equations of motion. Nevertheless, in a static approximation one can fix the soliton centers and determine the energy of the field configuration. In the left and middle diagram of Fig. 4 we show the two topological different soliton-antisoliton configurations, the left one corresponds to the two-particle spin quantum number $S = 0$, the middle diagram to $S = 1$. Both diagrams are rotationally symmetric around the z-axis. It should be noted that the antisolitons in the lower half of the diagrams are rotated by π around the symmetry-axis. In the left diagram there is no resistance when the two solitons approach each other, they attract each other and annihilate. In the middle diagram there is also attraction until the two "balds" in the center approach closely but can not annihilate since they belong to different half-spheres of S^3, indicated by the dashed arrows in the lower half. The right diagram in Fig. 4 is from the diploma work of Joachim Wabnig at the Vienna University of Technology [20] where he compared the energy of an "electron-positron" system for $m = 3$ and $S = 0$ in units of keV as a function of distance r in lattice units a with $r_0 = 3a = 2.21$ fm with the energy of point-like charges. It can be seen that the energy of the soliton-pair agrees with the Coulomb law for distances large compared to r_0 until the soliton cores of the two particles get close enough and increase the attraction. The energy density in a plane through the soliton centers is shown in Fig. 5. It is remarkable that the energy density is everywhere finite.

In scattering processes these solitons react almost like point-like particles do, they have no sub-structure which could be revealed. However, the strength of the interaction starts to change at distances of the order r_0. This behaviour is reminiscent of the running coupling in quantum field theory.

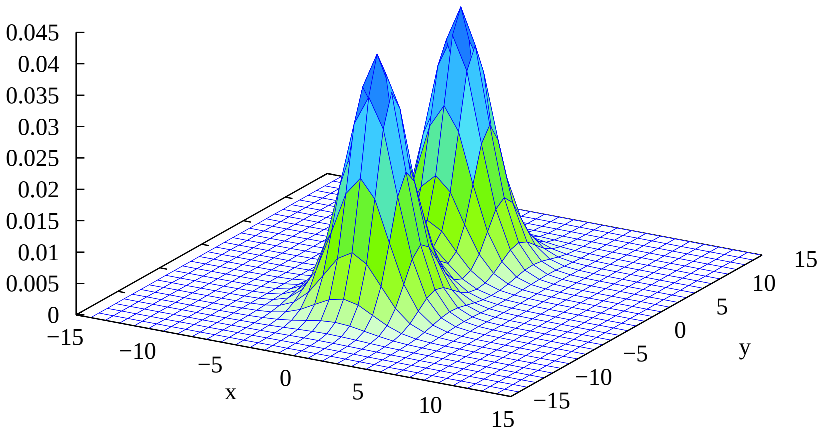


Figure 5. The energy density for a particle anti-particle solution

8. Conclusions and speculations

We can summarise our results:

- Mass is field energy
- Particles are topological solitons
- Charge and spin are topological quantum numbers
- Only integer multiples of elementary charge are possible
- Distinction between charges and their field is artificial
- There are two types of massless excitations, $\vec{n}$-waves
- Pauli principle has topological origin
- U(1) gauge invariance = invariance against rotations of a Zweibein (dyad) around $\vec{n}$
- Cosmological constant Λ $\rightarrow$ Cosmological Function $\Lambda(t, \vec{r})$

The above soliton model describes charged particles as topological solitons. The mass of these particles is field energy only. There is no Higgs particle necessary to give mass to particles. Due to their topological nature all free particles with the same topology have the same mass.

Charge is quantised in units of an elementary charge, it is a topological quantum number. There are only integer multiples of the elementary charge.

The spin quantum number is the absolute value of topological charge and can take only multiples of 1/2. The angular momentum properties of spin appear only when particles move around each other. Let's take the important example of a light negative charge rotating around a much heavier positive charge. By their common soliton field, see left and middle diagram of Fig. 4, the two charges are connected like two cogwheels of equal size. For a static heavy particle the light particle is forced to rotate twice around itself in a full orbit, as it should be for a gyromagnetic spin factor $g_s = 2$.

Charges and their fields are not distinguishable, they are build from the same field. The artificial separation between charges and fields as formulated in Maxwell's electrodynamics is a clever way to get a linear theory from an originally combined non-linear system.

The $\vec{n}$-field describing massless excitations has two degrees of freedom corresponding to the two polarisation directions of electromagnetic waves. Photons are realised as Goldstone bosons of the symmetry breaking of the vacuum described by a fixed direction of the $\vec{n}$-field.

A rudimentary form of the Pauli principle may be that two solitons in a $S = 1$ state can not be at the same position.

The local coordinate systems in the tangential space of S^3 are arbitrary. The area covered on S^3 by the Q-field is independent of the choice of this coordinate system. There is consequently an SU(2) gauge invariance of the field strength tensor. In the electrodynamic limit the elements $Q \in \mathrm{SU}(2)$ reduce to the normalised three dimensional vector field $\vec{n} \in \mathrm{S}^2$. There is only an invariance of the local coordinate system on S^2 left. This is a U(1) gauge-invariance also well known from QED.

The potential term $\Lambda(x)$ in the Lagrangian has some similarity to the cosmological constant Λ in the gravitation theory. In the presence of charged matter $\Lambda(x)$ is non-zero inside of solitons. It contributes with 25% to the soliton energy, see [13].

The model has also several properties which at first look differ from our present understanding of nature. The equations of motion allow for magnetic currents. These currents are non-topological [14] and act as sources of electric and magnetic fields. Magnetic currents are unknown in Maxwell theory. Further, there exist excitations in the α-degree of freedom. For $m = 1$ these excitations are realised as vibrational states of the solitons [17]. But for the electron such vibrational excitation are not known. For higher values of m there are no vibrational excitations, but α-waves with non-zero mass propagate in the vacuum. Such waves are also unknown. Since the potential term $\Lambda(x)$ in the Lagrangian has a dimension-full coupling constant the model is perturbatively non-renormalisable. Quantum theory can be included via a path integral formulation. Due to the smallness of the fine-structure constant α_f the quantum system should be in the strong coupling regime, at zero temperature it should be disordered. But there may exist a finite temperature phase transition from this disordered to an ordered phase. There is no definite answer to this question yet.

We can add some speculations about possible solutions of the above problems. To solve the problem on non-topological magnetic currents and α-waves we can claim that all particles which we measure in our detectors are topological solitons. Non-topological currents and waves escape our detectors. We measure them as electric and magnetic fields only. These non-topological fields could contribute to dark-matter. The potential term $\Lambda(x)$ in analogy to the cosmological constant Λ could be called a cosmological function and possibly contribute to dark energy. There are strong indications that in the early universe there was a period called inflation with a huge energy transfer and consequently fast expansion of the universe. One can speculate that the early universe was characterised by a vacuum with $Q(x) = 1$, $\alpha = 0$ and maximal potential energies, then a transition happened to a minimum of $\Lambda(x)$ with $\alpha = \pi/2$. Since this minimum is a two-dimensional manifold the vacuum is degenerate, spontaneous symmetry breaking happened and photons as Goldstone bosons appeared.

Finally, some words on the main topics of this conference, i.e., on the emergence of quantum mechanics. This model seems to favor a scenario where dark matter disturbs the motion of solitonic electrons when they orbit around nuclei. These statistical fluctuations are influenced by the density of dark matter and not by temperature, they induce zero-point motion and lead to probability distributions for the electrons.

These are some ideas which could be worked out and compared with experiment. The presented model is rather uncomplicated, it has three degrees of freedom only and gives a nice insight in many electro-dynamical phenomena and phenomena which are described by quantum field theory, like mass generation, particle-antiparticle annhihilation and the running of the coupling.

This model is a geometric model, the algebra is used as a tool only to describe geometry. It has a strong similarity with gravitation which can be formulated as translations of Vierbeins (tetrads). This model is based on local rotations of Dreibeins (triads). John A. Wheeler has formulated "Space-time tells matter how to move, matter tells space-time how to curve". We can add "$\cdots$ and charge tells space how to rotate". This means the phenomenology of gravitation and electro-dynamics can possibly be understood by the properties of space and time only. For the description of strong and weak forces the model should probably be extended to include further dimensions, i.e. further degrees of freedom.

I think that physics is geometry and not algebra. We should use the algebra only to describe the geometry.

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