NEW PERSPECTIVES ON THE ELECTRO-WEAK INTERACTIONS

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ABSTRACT

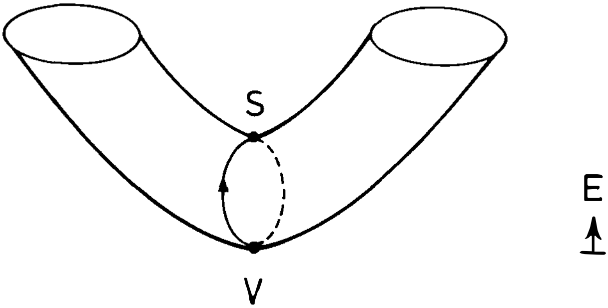
We discuss some classical aspects of the electro-weak standard model and possible implications for cosmology and future particle colliders.

All experiments to date indicate that the the electro-magnetic and weak interactions are described by the so-called standard model, a.k.a. the Weinberg-Salam model. The standard model is certainly not the last word from a theoretical point of view (there are too many free parameters in it), but, clearly, it is a very good effective theory and as such warrants further inspection. The standard model is a Yang-Mills-Higgs theory : a gauge theory with non-Abelian gauge group (specifically $SU(2) \times U(1)$) in which some of the gauge bosons aquire a mass by the Higgs mechanism. The content of the standard model is generally described in terms of its particles : the gauge bosons (massive $W^{\pm}$, Z^0 and a massless photon γ), the Higgs scalar ϕ and the fermions (quarks q and leptons l in specific representations of the gauge group). All these particles can be viewed as small fluctuations about the vacuum. However, this is not the whole story as the following observations make clear :

- configuration space has non-trivial structure, viz. topology (configuration space is the mathematical space of all static, finite energy, configurations of the fields $\vec{W}(\vec{x})$, $\phi(\vec{x})$ etc.);
- this structure is characterized by the sphaleron S (a sphaleron is a static, but unstable, solution of the classical field equations);
- this may lead to significant non-perturbative effects such as the violation of baryon number conservation.

In this contribution we give a brief review of these three points. Our references are far from complete, but may serve as an introduction to the subject.

First we consider the bosonic sector of the standard model . Its configuration space is an infinite dimensional and non-compact manifold. This space is truly huge, but the energy surface over one particular slice looks roughly as follows :



The surface close to the vacuum V is parabola-like, but far away one of these parabolas "tips over". This leads to the existence of a new solution of the classical field equations : the sphaleron S.[1] In a simplified version of the standard model this solution was already known in 1974.[2] Note that a classical solution corresponds to a stationary point on the energy surface : at this special point in configuration space the energy stays constant under an infinitesimal step in an arbitrary direction. As to the curvature, S has only one negative mode ; in all other directions in configuration space the surface curves up (for very large Higgs mass S aquires more negative modes).

We now discuss this sphaleron S in somewhat more detail. S has an extent of order M_W^{-1} and an energy of order $\alpha_{weak}^{-1}\ M_W$. Clearly this is a classical object : the size is much larger than the Compton wavelength. We write the energy as

$$E_S = \epsilon\ (g^2/4\pi)^{-1} M_W,$$

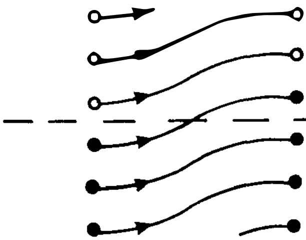
where $\epsilon = \epsilon(\lambda/g^2)$ depends rather weakly on the ratio of the quartic Higgs coupling λ over the $SU(2)$ gauge coupling constant squared g^2. We give ϵ for three representative values of λ/g^2 :

$$\epsilon(0) = 3.010\ ,\ \epsilon(1) = 4.092\ ,\ \epsilon(\infty) = 5.370.$$

These numbers hold for a weak mixing angle $\theta_w = \pi/6$ and are, strictly speaking, only upperbounds, but we beleive them to be within 1% of the true values. [3]

The picture of configuration space sketched above makes clear that the existence of the sphaleron S is related to the non-trivial topology of configuration space : this manifold has holes. The presence of these holes is due to the non-Abelian nature of the gauge group, which leads to self-interaction of the $SU(2)$ gauge bosons, making the theory highly non-linear. In addition, there is the Higgs mechanism which provides the classical theory with a mass scale (M_W). Most likely, there is much more structure in configuration space and associated new sphalerons ($S^{\star}$, $S^{\star\star}$, ...), but probably this will be at higher energies.[3)] Here we continue to look at the simple structure sketched in the figure above.

We now reintroduce the fermions and ask how they respond to these particular gauge and Higgs configurations. Specifically, we look at the eigenvalues of the Dirac operator $\gamma^{\mu}(\partial_{\mu} + A_{\mu})$ as we move along a "large" loop starting and ending at the vacuum (note that loops with the lowest possible maximal energy have to pass through the sphaleron S). The Dirac eigenvalues, of which there are an infinite number, are the same at the start and end of the loop, but their evolution in between is non-trivial. This phenomenon is called spectral flow :



Filling the Dirac sea for the configuration at the beginning of the loop this corresponds to particle creation or annihilation at the end. In the context of the electro-weak standard model this process violates $B + L$ conservation.

The next question is the rate for such fermion number violating processes.

Historically, the first mechanism considered was quantum tunneling through the potential barrier and, as expected, the rate (calculated via instantons) was very small [4)]

$$\Gamma^{quantum}_{\Delta(B+L)} \propto e^{-2S_{Inst}/\hbar} = e^{-16\pi^2/g^2} \sim 10^{-170}.$$

Instead of tunneling one can go over the barrier classically, for example by a thermal fluctuation [5,6)] . In that case the rate has a Boltzmann suppression factor determined by the height of the barrier, which is precisely the sphaleron energy E_S

$$\Gamma^{thermal}_{\Delta(B+L)} \propto e^{-E_S/kT}.$$

We consider these thermal fluctuations in the context of the early Universe. At temperatures $T \sim 300$ GeV, just below the phase transition, the Boltzmann factor is still quite small (less so if we take into account the running of $M_W(T)$, which decreases for $T \uparrow T_c$). Small as the rate may be (but not as small as 10^{-170}) it has to be compared to the Hubble expansion which is even smaller

$$\Gamma_{Universe} \sim (T/M_{Planck})T,$$

where $M_{Planck} \equiv (\hbar c^5/G)^{1/2} = 1.22\ 10^{19}$GeV. Detailed calculations give for $\lambda/g^2 \sim 1$ a ratio [5)]

$$\Gamma^{thermal}_{\Delta(B+L)}/\Gamma_{Universe} \sim 10^8,$$

so that any pre-existing $B + L$ is washed out. The only way to prevent having $B = 0$ today is to require $B - L \neq 0$. This is illustrated by the following example : for a comoving volume which had initially $B = L = 3$, say, the sphaleron transitions give $B = L = 0$, but for a volume which had initially $B = 4$ and $L = 2$ the sphaleron transitions can reduce it only to $B = 1$ and $L = -1$ (note that $B - L$ is conserved). The requirement of non-zero $B - L$ at $T \geq 300$GeV constitutes a new indirect observation of the early Universe. Recall that there are, basically, only two other cosmological observations : at T=O(eV) from the last scattering surface of the cosmic background radiation and at T=O(MeV) from the nucleosynthesis of ^{4}He. There is also the possibility that the presently observed baryon number was actually created at these temperatures of 100-1000 GeV. This would require some additions to the standard model , but it is an interesting prospect that the resolution of such a fundamental cosmological problem as the matter excess could result from new physics at these relatively modest energies.

A different question is wether or not similar (non-perturbative) effects could show up in particle collider experiments with center of mass energy $\sqrt{s} \sim E_S \sim 10$ TeV. The total fermion number violating cross section ($q+q \rightarrow 7\bar{q}+3\bar{l}+$ many W, Z, ϕ) has the following form according to recent instanton calculations [8,9)]

$$\sigma_{\Delta(B+L)} \propto e^{-16\pi^2/g^2 F(x)},$$

where the function F of $x \equiv (\sqrt{s}/E_S)^{2/3}$ is only known by the first few terms of a perturbative expansion

$$F(x) = 1 - c_2 x^2 + c_3 x^3 - O(x^4).$$

The crucial, open, question is how close $F(1)$ is to zero. Note that there may also be "non-perturbative" effects at energies of order E_S in the $B+L$ conserving sector.

To summarize, we have seen that the classical field theory of electro-weak interactions has an energy scale set by the Sphaleron $E_S = O(M_W/\alpha)$. Up till now we have probed small fluctuations around the vacuum at energies $\sim \alpha E_S$. Under extreme conditions the non-trivial structure of configuration space may become important. A prime example of this may be strong $B+L$ violation in the early Universe. Wether or not similar conditions could be created in high energy particle colliders remains to be seen.

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