



DIFFUSION ENHANCEMENT BY THE BEAM-BEAM INTERACTION IN 1-D SIMULATIONS

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In previous papers^{1,2,3} we have shown that enhancement of random diffusion by the beam-beam interaction can occur, and presented a theoretical model which approximately describes this enhancement. In this note we explore this enhancement more completely and systematically. We present results of numerical simulations which explore fractional tunes between 0.0 and 0.5 and tune shifts between 0.0 and 0.10, and obtain a complete mapping of this region in tune, tune shift $(\nu, \Delta\nu)$ space. Our theoretical model is also described and tested systematically, and is found to provide an adequate approximate description of the phenomena.

I. Equations of Motion and Simulation Procedure

In the simulations below, hundreds of particle orbits are tracked through thousands of turns. Transport about one turn is simulated as the product of two matrix multiplications:

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{\text{Final}} = \begin{pmatrix} 1 & 0 \\ \frac{-4\pi\Delta\nu}{\beta} F(x)x & 1 \end{pmatrix} \begin{pmatrix} \cos(2\pi\nu) & \beta \sin(2\pi\nu) \\ -\frac{1}{\beta} \sin(2\pi\nu) & \cos(2\pi\nu) \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}. \quad (1)$$

We have approximated the beam-beam interaction as a "zero-length" - "weak-strong" interaction, where "zero-length" means a truncation of the interaction to a velocity kick and "weak-strong" means the force produced by the opposite "strong" beam is unchanged from turn to turn. The above transformation is equivalent to



integration of the equation of motion:

$$x'' + K(s)x = \frac{-4\pi v}{\beta_0} F(x) \times \delta_p(s), \quad (2)$$

For $F(x)$ we have used the 1-D truncation of the force from a round gaussian beam

$$F(x) = \frac{1 - e^{\frac{-x^2}{2\sigma^2}}}{\frac{x^2}{2\sigma^2}} \quad (3)$$

The parameters $\sigma^2 = (.0816)^2 \text{ mm}^2$ and $\beta_0 = 2 \text{ m}$ are chosen to agree with expected Tevatron $\bar{p}p$ collision parameters, and the simulation approximations also agree with $\bar{p}p$ collision conditions.

A tune at small amplitudes v_0 can be found by noting that as $x \rightarrow 0$, the beam-beam force becomes

$$\Delta x' = \frac{-4\pi \Delta v}{\beta_0} x \quad (4)$$

and the transformations of Equation 1 are now linear. v_0 is found by considering the transformation from the center of an interaction to the next, that is:

$$\begin{pmatrix} \cos 2\pi v_0 & \beta_0 \sin 2\pi v_0 \\ -\frac{1}{\beta_0} \sin 2\pi v_0 & \cos 2\pi v_0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{-4\pi \Delta v}{2\beta_0} & 1 \end{pmatrix} \begin{pmatrix} \cos 2\pi v & \beta \sin 2\pi v \\ -\frac{1}{\beta} \sin 2\pi v & \cos 2\pi v \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{-4\pi \Delta v}{2\beta_0} & 1 \end{pmatrix} \quad (5)$$

which implies

$$\cos 2\pi v_0 = \cos(2\pi v) - 2\pi \Delta v \sin(2\pi \Delta v) \quad (6)$$

$$\beta_0 \sin 2\pi v_0 = \beta \sin 2\pi v. \quad (7)$$

The parameter values for the simulations below are set by the following procedure: β_0 is set equal to the value of 2. m. for all cases. An approximate for v_0 ; v_0^* , is set to a "bench mark" value (0., 0.05, 0.10, 0.45). The parameter Δv is then set to one of the values (0.0, 0.005, 0.01, ... 0.10) and v is set by $v = v_0^* - \Delta v$. The precise value of v_0 is found from Equation 6, and Equation 7 is used to find the transfer matrix value of β .

In each simulation a set of initial particle positions is generated randomly within a bigaussian distribution and then transported through 200,000 turns, with a random diffusion kick on each turn.

Random diffusion is simulated by adding a random velocity on each turn

$$x' \rightarrow x' + \Delta \cdot R \quad (8)$$

where R is a random number between -1 and +1 and Δ is an amplitude parameter. In the simulations reported in this note Δ is chosen with a size that doubles rms beam size within a few hundred thousand turns. This is larger than the expected random diffusion from beam-gas scattering, RF noise and power supply ripple in the Tevatron. The larger size is deliberately chosen to display noticeable effects in reasonable computer times.

The diffusion expected from a given value of Δ can be calculated. The change in emittance ϵ is

$$\begin{aligned} \frac{d\epsilon}{dt} &\equiv \frac{d}{dt} 3 \left(\frac{\overline{x^2}}{\beta_0} + \beta_0 \overline{x'^2} \right) \\ &= \beta_0 \Delta^2 \equiv D_0. \end{aligned} \quad (9)$$

In the simulations the quantity

$$\epsilon \equiv 3 \left(\frac{\overline{x^2}}{\beta_0} + \beta_0 \overline{x'^2} \right) \quad (10)$$

is calculated for a set of particle trajectories every 2000 turns. A straight line least squares fit for $\epsilon(t)$ is calculated to find a diffusion constant D which is compared with D_0 by the enhancement factor x_E :

$$D = x_E D_0.$$

In a previous paper¹ we explored the dependence of x_E on tune, tune shift $(\nu, \Delta\nu)$ and D_0 for some selected values. In that paper it was found that x_E is independent of D_0 , in agreement with our theoretical model.² In this note D_0 is kept constant while the dependence of x_E on ν and $\Delta\nu$ is investigated.

II. Simulation Results

In this paper we report two independent sets of simulations: single precision with 500 particle trajectories and double precision with 200 trajectories. In Table I we display x_E for the single precision results. In Table II we present a more detailed compilation of these results. x_E at $t = 50,000, 100,000, 150,000$ and $200,000$ turns is shown. A weak dependence on t can be seen in some simulations.

An rms error in the "least squares" calculation of the slope parameter D has also been calculated and leads to an error in x_E denoted by Δx_E . In Table II " $EN_{\min}$ " is $x_E - \Delta x_E$ and " $EN_{\max}$ " is $x_E + \Delta x_E$.

Accuracy tests were performed for several of these cases. For one case $(\nu = .06, \Delta\nu = .09)$ we perform a reversibility test, in which 500 orbits were transported forward 100,000 turns without diffusion and then reversed. In single precision this lead to errors up to 10^{-4} in position, which is of the same order as a single diffusion kick in these simulations ($\Delta \approx .0002$ mr). Double precision errors are 10^{-14} .

A second test is comparison of single and double precision. Trajectories were found to differ substantially after 50,000 turns. To summarize the accuracy test results, we state that significant deviations in trajectories can develop over 200,000 turns in single precision. If these deviations are random, they will only provide an unimportant jitter to the random diffusion. However it is conceivable that these deviations may add systematically and to eliminate this possibility the simulations were reperformed in double precision.

Tables III and IV display results of double precision simulations for the same parameters as single precision except that only 200 trajectories per simulation are included, and this trajectory set is independent of the corresponding single precision trajectories. No significant deviations from the single precision results are noted, indicating that accuracy errors are not systematic.

In Figures 1,2,3 we graph the double precision enhancement factors. In Figure 1, points with equal ν_0^* are connected. In Figures 2 and 3 points with equal $\Delta\nu$ are connected.

The enhancement factors of Tables I and III can be correlated with the phase space plots of Reference 4. Enhancement factors greatly different from 1 are clearly associated with resonance features in the phase space trajectories. As an example we display the phase space plot of case 108 ($\nu = .24$, $\Delta\nu = .06$)(Fig. 4). The enhancement factor of 6.39 is correlated with a large fourth order resonance.

III. Discussion of Enhancement Factors - Causes of Enhancement

In these simulations, the transport parameters are chosen such that motion of small amplitude particles is approximated by the linear matrix with β -function β_0 , and tune $\nu_0 \cong \nu_0^*$. Very large amplitude particles would have motion corresponding to β , ν and these will not equal β_0 , ν_0 . Other particles would have effective values between β and β_0 , ν and ν_0 .

Our formula for unperturbed diffusion

$$D_0 = \beta_0 \langle \Delta\theta^2 \rangle$$

should be replaced by

$$\bar{D}_0 = \langle \beta \rangle \langle \Delta\theta^2 \rangle \quad (11)$$

where $\langle \beta \rangle$ is a mean β -function value. We expect that x_E would be given by

$$x_E = \frac{\langle \beta \rangle}{\beta_0} . \quad (12)$$

In the simulations $\beta > \beta_0$ for $\nu_0 > .25$ and $\beta < \beta_0$ for $\nu_0 < .25$ (see Table II, Reference 4). As can be seen from Tables I-IV values of x_E for $\nu_0 > 0.25$ and $\Delta\nu$ relatively large show $x_E < 1$ as expected and those with $\nu_0 < .25$ show $x_E > 1$. This distortion is particularly large for $\nu \rightarrow 0.0$ where $\langle \beta \rangle \gg \beta_0$ and for $\nu_0 \rightarrow .5$ where $\langle \beta \rangle \ll \beta_0$. The magnitudes of x_E are in agreement with that expected from Equation 11, except for cases with large resonances.

The appearance of large low order resonances substantially modifies diffusion enhancements, as we have previously noted.^{1,2,3}

We have previously presented a model² to describe this enhancement which we investigate more thoroughly in this note. In this model we note that particle motion is significantly distorted by resonances, leading to changes in particle amplitudes I , where

$$I = \frac{x^2}{2\beta_0} + \frac{\beta_0 x'^2}{2} . \quad (13)$$

Particle amplitudes which reach the lower boundary of a resonance, I_T (see Figure 5) by random diffusion can reach the much larger amplitude $I_T + 2\Delta$ by an infinitesimal diffusion kick. (Δ is the resonance half-width.) The resonance places the amplitudes I_T and $I_T + 2\Delta$ adjacent so that diffusion moves rms particle amplitudes by 2Δ when I_T is reached.

We have defined rms emittance by

$$\epsilon = 3 \left\langle \frac{x^2}{\beta_0} + x'^2 \beta_0 \right\rangle = 6 \langle I \rangle, \quad (14)$$

Diffusion enhancement by resonances modifies D_0 as shown in the equation

$$\dot{\epsilon} = D = D_0 + \left. \frac{\dot{N}}{N} \right|_{I_T} 12 \Delta \quad (15)$$

where $\frac{\dot{N}}{N}$ is the rate at which particles reach the threshold I_T .

$\frac{\dot{N}}{N}$ can be estimated by considering the change in the particle distribution caused by random diffusion. The initial distribution is gaussian:

$$f(x, x') \propto e^{-\frac{x^2}{2\sigma^2}} e^{-\frac{x'^2}{2\sigma'^2}} \quad (16)$$

or

$$f(I) \propto e^{-I/I_0}.$$

To estimate particle flux at I_T we calculate the change in the number of particles with $I < I_T$, obtaining

$$\left. \frac{\dot{N}}{N} \right|_{I_T} \approx -\frac{d}{dt} \int_0^{I_T} f(I) dI = \frac{I_T \dot{I}_0}{I_0^2} e^{-\frac{I_T}{I_0}} \quad (17)$$

$\frac{\dot{I}_0}{I_0}$ is the same as $\frac{\dot{\epsilon}_0}{\epsilon_0}$. Equation (15) can be written as

$$\dot{\epsilon} \approx \dot{\epsilon}_0 \left(1 + \frac{12}{\epsilon_0} \frac{I_T}{I_0} e^{-\frac{I_T}{I_0}} \Delta \right) \quad (18)$$

or

$$x_E \approx 1 + \frac{12}{\epsilon_0} \frac{I_T}{I_0} e^{-\frac{I_T}{I_0}} \Delta.$$

x_E can now be calculated provided I_T and Δ can be measured on the phase-space trajectory plots of Reference 4 or calculated in the single resonance model. In Appendix A we describe this calculation. In Table 5 we compare results of these two methods, and compare the measured with calculated enhancement factors. The results of the calculation in Appendix A are that the resonance location I_p is found from the solution of the equation

$$\frac{\nu_p - \nu}{\Delta \nu} - U_0^{N'}(I) = 0, \quad (19)$$

where ν_p is the resonant tune, and the resonance width is found from

$$\Delta = \sqrt{\frac{8 U_m^N(I_p)}{U_0^{N''}(I_p)}} \quad (20)$$

and the threshold value I_T can be expressed as

$$I_T \cong I_p - \Delta. \quad (21)$$

The scaled potential functions $U_m^N(x)$ are defined in Appendix A and displayed graphically in Figures 6 and 7.

These functions are used to calculate resonance locations and widths (I_p, Δ) which can be compared with those obtained from phase space plots as shown in Table V. Agreement is quite good (with ~10-20% errors) as may be expected from higher order corrections to the single resonance model.

The threshold value I_T is not well represented by $(I_p - \Delta)$ for 1/4, 1/6 resonances since the calculated values of Δ are $\geq I_p$, which would imply $I_T \leq 0$. Higher order nonlinearities move I_T to larger values and the measured values of I_T are used in the enhancement factor calculations.

IV. Calculation of Diffusion Enhancement

In Table V enhancement factors for 1/4, 1/6 and 1/8 resonances are calculated and compared with simulation results. In these, the measured I_T

and calculated Δ are used. The calculated x_E are not corrected for the shift in β noted above. This shift increases x_E for $\nu < 0.25$, $\Delta\nu$ large and decreases x_E for $\nu > 0.25$, $\Delta\nu$ large.

The comparison between calculated and simulated x_E is not expected to be extremely accurate, since both values can contain significant inaccuracies.

The calculated x_E contain an inaccuracy of ~10-20% simply from the single resonance approximation. For example, a $1/4$ resonance also contains $2/8$, $3/12$, $4/16$... resonances which could shift the resonance width by ~10%. Also the calculated x_E has a strong dependence on I_T because of the exponential factor e^{-I_T/I_0} . A small error in I_T can change x_E substantially.

The simulation values of x_E can have large statistical inaccuracies because of the fact that resonance enhancement operates through the subset of particle trajectories which lie near resonances. In typical cases this can be ~5 to 10 out of 200 particle orbits. The single precision cases, with 500 orbits, have much better statistics and are much more accurate.

Calculated and simulation x_E agree to within ~10 to 20%, as shown in Table V. This is within expected errors and shows that our theoretical model describes the diffusion enhancement and can predict its magnitude.

This model can be extended to 2-D, where similar behavior is expected. The density of resonances will be larger because of the additional possibility of "coupling" resonances (such as $\nu_x + \nu_y = 1$). 2-D behavior will be explored in future work.

Table 5 Resonance Diffusion Enhancement

(a) 1/4 resonance

ν	$\Delta\nu$	I_p	Δ (calculated)	Δ (measured)	I_T (measured)	x_E (calculated)	x_E (measured)
.20	.10	.0075 m	.0076	.0077	.00152	2.32	2.52
.205	.095	.0085	.0085	.0082	.00185	2.63	2.70
.210	.090	.0093	.010	.0084	.0022	3.05	2.79
.215	.085	.0103	.011	.010	.0025	3.36	2.94
.220	.080	.012	.014	.012	.003	3.76	3.55
.225	.075	.014	.016	.015	.003	4.60	3.73
.230	.070	.017	.021	.020	.004	5.60	3.80
.235	.065	.022	.029	.025	.006	6.08	4.97
.240	.060	.033	.046	.044	.008	6.76	6.92
.245	.055	.064	.102	.097	.013	6.15	6.68

For these cases (ν near 0.25) the approximation $\bar{\beta} \approx \beta_0$ is reasonably accurate.

(b) 1/6 resonance, 1/8 resonance

ν	$\Delta\nu$	I_p	Δ (calculated)	Δ (measured)	I_T (measured)	x_E (calculated)	x_E (measured)
.15	.10	.032	.026	.029	.013	2.21	2.86
.155	.095	.046	.043	.047	.015	2.14	2.36
.160	.09	.079	.089	.097	.022	1.46	2.11
.10	.10	$\frac{1}{6}$.004	.001	----	.003	1.47	1.81
		$\frac{1}{8}$.020	.006	.0063	.014		
.105	.095	$\frac{1}{6}$.004	.001	.001	.002	1.42	1.83
		$\frac{1}{8}$.025	.009	.0085	.016		
.110	.090	$\frac{1}{6}$.005	.0012	.001	.002	1.63	1.91
		$\frac{1}{8}$.033	.015	.016	.016		
.115	.085	$\frac{1}{6}$.005	.0013	.001	.002	1.94	2.10
		$\frac{1}{8}$.048	.029	.029	.016		
.120	.080	$\frac{1}{6}$.0055	.0015	.0016	.004	1.34	1.65
.125	.075	$\frac{1}{6}$.0063	.0019	.002	.0044	1.40	1.41

ν	$\Delta\nu$	I_p	Δ (calculated)	Δ (measured)	I_T (measured)	x_E (calculated)	x_E (measured)
.13	.07	.007	.0023	.002	.0041	1.48	1.37
.135	.065	.0078	.0027	.0035	.0045	1.54	1.34
.14	.06	.0093	.0036	.004	.0056	1.68	1.54
.145	.055	.011	.0048	.0065	.0060	1.82	1.59
.15	.05	.014	.0071	.0086	.0077	2.98	1.98
.155	.045	.019	.0018	.0148	.0095	2.17	2.08
.160	.04	.033	.0265	.0296	0.0146	1.87	2.04
.31	.04	.0055	.0015	.002	.0048	1.31	1.21
.315	.035	.007	.0023	.002	.0065	1.38	1.22
.32	.03	.0092	.0036	.003	.0077	1.50	1.41
.325	.025	.0128	.0061	.008	.0095	1.60	1.49
.33	.02	.0325	.0265	.028	.016	1.63	1.59

References

1. D. Neuffer and A. Ruggiero, "Enhancement of Diffusion by a Nonlinear Force", FN-325, Proc. of the Beam-Beam Interaction Seminar, SLAC, May 22-23, 1980 (SLAC PUB-2624, p. 332).
2. D. Neuffer, A. Riddiford and A. Ruggiero, "A Model to Describe Diffusion Enhancement by the Beam-Beam Interaction", TM-1007, August 8, 1980
3. D. Neuffer, A. Riddiford and A. Ruggiero, IEEE Trans. on Nuclear Science, NS-28, p. 2494 (1981)
4. D. Neuffer, A. Riddiford and A. Ruggiero, "Exploration of Phase Space Trajectories in Simulations of the Beam-Beam Interaction", TM-1054 (1981)
5. S. Snowdon, Fermilab Note TM-185, May 8, 1969

Appendix A

Resonance Widths in the Single Resonance Model

The matrix multiplications of Section I are equivalent to integration of the equation of motion

$$x'' + K(s)x = \frac{-4\pi \Delta v}{\beta_0} \begin{pmatrix} \frac{-x^2}{2\sigma^2} \\ \frac{1-e}{x^2} \\ \frac{x^2}{2\sigma^2} \end{pmatrix} \times \delta_p(s) \quad (A-1)$$

which can be rewritten as the equation

$$x'' + K(s)x = - \frac{\partial U(x)}{\partial x} \delta_p(s) \quad (A-2)$$

where U is a potential function.

The Hamiltonian associated with (19) is:

$$H = \frac{1}{2}(p^2 + K(s)x^2) + U(x) \delta_p(s). \quad (A-3)$$

To obtain the single resonance form we apply three successive canonical transformations: a Courant-Snyder transformation, an action-angle transformation and a slow variable transformation. These can be combined in a single canonical transformation⁵ with the generating function

$$S(x, \psi, s) = \frac{x^2}{2\beta(s)} \left[\tan\phi(s, \psi) - \frac{\beta'(s)}{2} \right] \quad (A-4)$$

where

$$\phi = \psi + \nu_p \frac{s}{R} + \int_0^s \left(\frac{1}{\beta} - \frac{\nu}{R} \right) ds' \quad (A-5)$$

and β is the Courant-Snyder β -function, $2\pi R$ is the storage ring circumference, ν_p is a resonant tune (see below), and ψ is a betatron phase. From the

generating function, we find:

$$x = \sqrt{2I\beta} \cos\phi \quad (A-6)$$

$$p = \sqrt{\frac{2I}{\beta}} \sin\phi - \frac{\beta'}{2} \cos\phi.$$

The new Hamiltonian is:

$$H(I, \psi, \theta) = (\nu - \nu_p)I + \delta_p(\theta) U(\sqrt{2I\beta(0)} \cos\phi) \quad (A-7)$$

where $\theta = s/R$, our new independent variable.

The beam-beam perturbation $\delta_p(\theta) U(x)$ is expanded as a double Fourier series in θ and ϕ :

$$\delta_p(\theta) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} e^{in\theta} \quad (A-8)$$

$$\begin{aligned} U(\sqrt{2I\beta(0)} \cos\phi) &= U_0(I) + 2 \sum_{m=1}^{\infty} U_m(I) \cos m\phi \\ &= \sum_{m=-\infty}^{\infty} U_m(I) e^{im\phi} \end{aligned} \quad (A-9)$$

where

$$U_m(I) = \frac{1}{2\pi} \int_0^{2\pi} U(\sqrt{2I\beta} \cos\phi) \cos m\phi d\phi. \quad (A-10)$$

The "slow approximation" is used, which means ignoring fast changing parameters. In

$$\phi = \psi + \nu_p \theta + \int_0^{R\phi} ds' \left(\frac{1}{\beta(s')} - \frac{\nu}{R} \right) \quad (A-11)$$

ψ is the slowly changing variable, and ν_p is some resonant tune $\frac{n}{m}$ and the integral over s' is a fast oscillation, and all terms in the fourier

expansion except those with $-\nu_p = \frac{n}{m}$ are fast.

Keeping only the lowest order slow terms, we obtain

$$\begin{aligned} & \delta_p(\theta) U(\sqrt{2I\beta(0)} \cos \phi) \\ & \approx \frac{U_0(I)}{2\pi} + \frac{U_m(I)}{\pi} \cos m\psi \end{aligned} \quad (A-12)$$

and the Hamiltonian is

$$H \approx (\nu - \nu_p)I + \frac{U_0(I)}{2\pi} + \frac{U_m(I)}{\pi} \cos m\psi \quad (A-13)$$

Fixed points occur where $\psi' = 0$ and $I' = 0$

$$\begin{aligned} \psi' = \frac{\partial H}{\partial I} &= \nu - \nu_p + U_0'(I) + \frac{U_m'(I)}{\pi} \cos m\psi \\ I' = \frac{-\partial H}{\partial \psi} &= \frac{U_m}{\pi} m \sin m\psi \end{aligned} \quad (A-14)$$

or

$$\psi = 0, \pi/m, \dots, \frac{2(m-1)}{m}\pi$$

and

$$\nu - \nu_p + \frac{U_0'}{2\pi} \pm \frac{U_m'}{\pi} = 0.$$

If $U_m' \ll U_0'$, fixed points are at $I = I_p$ the solutions of

$$\nu - \nu_p + \frac{U_0'(I)}{\pi} = 0. \quad (A-15)$$

The Hamiltonian in the region near I_p ($I \approx I_p$) can be written as

$$H \approx H_0 + \frac{U_0'''(I)}{4\pi} (I - I_p)^2 + \frac{U_m(I_p)}{\pi} \cos m_0\psi \quad (A-16)$$

which is recognizable as the Hamiltonian for a pendulum. A resonance half-width can be found from the boundary of the separatrix:

$$\Delta^2 = (I - I_p)_{\max}^2 = \frac{8 U_m(I_p)}{U_0''(I_p)} . \quad (A-17)$$

The threshold value I_T is simply given by

$$I_p = I_T - \Delta.$$

The calculated values of I_T , I_p and Δ can be used to estimate diffusion factors.

The scaled potential function $U(x)$ and the Fourier components $U_m^N(I)$ can be calculated numerically. In Reference 2 results of numerical calculation for potentials from our gaussian charge distribution are presented. The scaled functions

$$U_m^N(x) = \int_0^x x F(x') dx'$$

and

$$U^N(I) = \frac{1}{2\pi} \int_0^{2\pi} U^N(\sqrt{2I\beta} \cos\phi) \cos m\phi d\phi$$

are calculated, where $F(x)$ is given by Equation (3), and σ and β have the "Tevatron" values: .0816 mm and 2.0 m. In Figures 6 and 7 we display some of these results graphically.

TABLE I Single precision enhancement values after 200,000 turns; 500 particles.

Δv										
0.100	6.02	1.47	1.81	2.80	2.52	1.00	0.60			
0.095	2.59	1.52	1.83	2.38	2.70	0.92	0.74			
0.090	2.34	1.17	1.91	2.11	2.79	0.90	0.81			
0.085	1.77	1.46	2.10	1.40	2.94	1.05	0.76			
0.080	1.75	1.50	1.65	1.19	3.55	0.97	0.70			
0.075	1.41	1.58	1.41	1.16	3.73	1.02	0.75			
0.070	1.37	1.54	1.37	0.93	3.80	1.01	0.83			
0.065	1.28	1.16	1.34	1.08	4.97	0.99	0.72			
0.060	1.32	1.62	1.54	1.10	6.92	0.74	0.76			
0.055	1.34	1.28	1.59	1.18	6.68	1.07	0.75			
0.050	4.08	1.18	1.17	1.98	1.13	1.33	0.90	0.96	0.42	
0.045	1.86	1.15	1.00	2.08	0.96	0.93	1.21	0.91	0.56	
0.040	1.70	1.12	1.19	2.04	1.10	0.97	1.21	0.96	0.56	
0.035	1.37	1.04	1.37	1.45	0.92	0.91	1.22	0.92	0.78	
0.030	1.26	1.18	1.39	1.18	1.14	0.94	1.41	1.01	0.70	
0.025	1.30	1.04	1.92	1.11	0.99	0.81	1.49	0.97	0.68	
0.020	1.16	1.40	1.09	1.05	1.08	0.96	1.59	0.94	0.81	
0.015	1.10	1.01	1.00	0.92	1.20	1.22	0.98	0.85	0.89	
0.010	1.09	1.03	0.96	1.01	1.09	1.12	0.90	1.01	0.80	
0.005	1.05	1.16	0.88	1.10	1.06	0.92	1.01	1.00	0.88	
0.000	0.93	0.82	0.98	1.25	0.99	1.20	1.05	0.98	0.81	0.98
$v_0^* = 0.0001$	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	

TABLE 2 Single precision enhancement values; 500 particles.

Fig. _{Rev 4}	Δv	Tune Spread v_y v_z	50,000 Turns			100,000 Turns			150,000 Turns			200,000 Turns		
			EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}
1	0	.0001,.0001	0.95	0.97	0.99	0.96	0.97	0.99	0.94	0.95	0.95	0.93	0.93	0.94
2	0.050	.0001,.0032	1.53	5.66	9.80	1.52	3.28	5.04	3.14	4.02	4.90	3.54	4.08	4.63
3	0.045	.005,.0218	0.77	1.13	1.48	1.51	1.64	1.77	1.65	1.73	1.80	1.81	1.86	1.91
4	0.040	.010,.0300	1.86	2.03	2.21	1.56	1.64	1.71	1.60	1.65	1.69	1.67	1.70	1.73
5	0.035	.015,.0358	1.40	1.52	1.64	1.24	1.29	1.33	1.31	1.34	1.32	1.36	1.32	1.39
6	0.030	.020,.0401	1.38	1.46	1.55	1.35	1.38	1.42	1.37	1.39	1.41	1.25	1.26	1.28
7	0.025	.025,.0434	1.15	1.22	1.29	1.16	1.19	1.23	1.28	1.30	1.32	1.29	1.30	1.32
8	0.020	.030,.0459	1.23	1.30	1.37	1.10	1.12	1.15	1.04	1.06	1.07	1.14	1.16	1.17
9	0.015	.035,.0477	0.98	1.04	1.10	1.04	1.06	1.08	1.00	1.01	1.03	1.09	1.10	1.12
10	0.010	.040,.0490	0.83	0.86	0.89	0.96	0.98	0.99	1.07	1.08	1.09	1.08	1.09	1.10
11	0.005	.045,.0498	1.09	1.11	1.13	1.21	1.22	1.23	1.05	1.07	1.08	1.04	1.05	1.06
12	0	.05,.05	0.82	0.84	0.86	0.80	0.81	0.82	0.80	0.81	0.81	0.81	0.82	0.82

continued

TABLE 2 continued (single precision)

Fig. in Ref. 4	Δv	Tune Spread v v_0	50,000 Turns			100,000 Turns			150,000 Turns			200,000 Turns		
			EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}
13	0.100	.0001, .0045	7.87	14.3	21.6	5.30	7.56	9.81	4.40	5.60	6.79	5.24	6.02	6.80
14	0.095	.005, .0313	1.98	2.51	3.05	2.19	2.38	2.58	2.69	2.80	2.91	2.51	2.59	2.67
15	0.090	.010, .0437	1.32	1.62	1.92	2.02	2.15	2.28	2.07	2.14	2.21	2.29	2.34	2.39
16	0.085	.015, .0529	1.11	1.33	1.56	1.46	1.54	1.62	1.64	1.69	1.74	1.74	1.77	1.81
17	0.080	.020, .0603	1.74	1.91	2.09	1.59	1.65	1.72	1.67	1.71	1.75	1.72	1.75	1.78
18	0.075	.025, .0665	1.36	1.50	1.64	1.29	1.35	1.41	1.25	1.28	1.32	1.38	1.41	1.43
19	0.070	.030, .0718	1.47	1.57	1.68	1.64	1.69	1.74	1.44	1.48	1.51	1.35	1.37	1.39
20	0.065	.035, .0765	1.29	1.38	1.46	1.25	1.29	1.32	1.25	1.28	1.30	1.27	1.28	1.30
21	0.060	.040, .0805	1.59	1.68	1.77	1.59	1.62	1.66	1.51	1.54	1.56	1.30	1.32	1.35
22	0.055	.045, .0840	1.25	1.33	1.41	1.31	1.34	1.37	1.25	1.26	1.28	1.32	1.34	1.35
23	0.050	.050, .0871	1.38	1.45	1.52	1.22	1.25	1.28	1.15	1.17	1.19	1.17	1.18	1.19
24	0.045	.055, .0898	0.84	0.89	0.94	1.09	1.12	1.14	1.14	1.16	1.17	1.14	1.15	1.15
25	0.040	.060, .0921	0.92	0.97	0.95	0.99	1.01	1.04	1.03	1.04	1.06	1.11	1.12	1.13
26	0.035	.065, .0941	0.87	0.91	0.95	1.01	1.03	1.05	0.99	1.00	1.01	1.03	1.04	1.05
27	0.030	.070, .0957	0.92	0.98	1.03	1.19	1.21	1.23	1.23	1.24	1.25	1.17	1.18	1.19
28	0.025	.075, .0971	1.24	1.27	1.31	1.14	1.15	1.17	1.10	1.11	1.12	1.03	1.04	1.05
29	0.020	.080, .0982	1.47	1.51	1.55	1.35	1.36	1.38	1.35	1.36	1.37	1.40	1.40	1.41
30	0.015	.085, .0990	0.86	0.89	0.92	0.82	0.84	0.85	0.94	0.95	0.97	1.00	1.01	1.02
31	0.010	.090, .0996	0.85	0.87	0.89	0.95	0.96	0.97	0.94	0.94	0.95	1.02	1.03	1.04
32	0.005	.095, .0999	1.05	1.07	1.09	1.13	1.13	1.14	1.17	1.18	1.18	1.16	1.16	1.17
33	0	.1, .1	1.11	1.14	1.16	1.05	1.06	1.07	1.05	1.06	1.06	0.97	0.98	0.99

continued

TABLE 2 continued (single precision)

Fig. in Ref. 4	Δv	Tune Spread $v \quad v_z$	50,000 Turns			100,000 Turns			150,000 Turns			200,000 Turns		
			EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}
34	0.100	.050, .1134	1.04	1.25	1.46	1.27	1.33	1.39	1.38	1.41	1.44	1.45	1.47	1.49
35	0.095	.055, .1177	1.46	1.55	1.65	1.32	1.36	1.40	1.37	1.40	1.42	1.50	1.52	1.54
36	0.090	.060, .1217	1.26	1.35	1.44	1.08	1.12	1.25	1.01	1.03	1.05	1.15	1.17	1.19
37	0.085	0.065, .1253	1.11	1.17	1.24	1.14	1.17	1.21	1.36	1.38	1.41	1.44	1.46	1.48
38	0.080	.070, .1286	1.46	1.52	1.59	1.46	1.49	1.52	1.47	1.49	1.50	1.49	1.50	1.51
39	0.075	.075, .1316	1.21	1.29	1.37	1.41	1.45	1.48	1.56	1.58	1.61	1.57	1.58	1.60
40	0.070	.080, .1343	1.11	1.17	1.23	1.29	1.31	1.34	1.47	1.49	1.52	1.53	1.54	1.56
41	0.065	.085, .1368	1.14	1.18	1.22	1.21	1.23	1.24	1.12	1.13	1.15	1.15	1.16	1.17
42	0.060	.090, .1390	1.39	1.44	1.48	1.33	1.35	1.37	1.50	1.52	1.53	1.61	1.62	1.64
43	0.055	.095, .1409	0.92	0.97	1.03	1.27	1.30	1.32	1.33	1.34	1.36	1.27	1.28	1.29
44	0.050	.100, .1427	1.19	1.23	1.27	1.22	1.23	1.25	1.16	1.17	1.19	1.16	1.17	1.18
45	0.045	.105, .1442	1.16	1.19	1.22	0.98	0.98	1.00	0.88	0.89	0.91	0.99	1.00	1.01
46	0.040	.110, .1455	1.20	1.24	1.29	1.07	1.10	1.12	1.09	1.10	1.12	1.18	1.19	1.21
47	0.035	.115, .1467	0.98	1.02	1.05	1.22	1.24	1.26	1.35	1.37	1.38	1.36	1.37	1.38
48	0.030	.120, .1476	1.06	1.08	1.11	1.47	1.50	1.53	1.38	1.40	1.42	1.37	1.39	1.40
49	0.025	.125, .1484	1.03	1.06	1.09	0.97	0.99	1.00	0.94	0.94	0.95	0.91	0.92	0.93
50	0.020	.130, .1490	0.93	0.96	1.00	1.25	1.27	1.29	1.18	1.20	1.21	1.08	1.09	1.10
51	0.015	.135, .1494	0.83	0.87	0.91	0.88	0.89	0.90	0.93	0.94	0.94	1.00	1.00	1.01
52	0.010	.140, .1498	0.76	0.78	0.80	0.90	0.91	0.92	0.96	0.97	0.97	0.95	0.96	0.97
53	0.005	.145, .1499	1.21	1.23	1.25	1.00	1.02	1.03	0.86	0.87	0.89	0.87	0.88	0.88
54	0	.15, .15	1.18	1.21	1.23	1.15	1.16	1.18	1.24	1.25	1.26	1.25	1.25	1.26

continued

TABLE 2 continued (single precision)
50,000 Turns

Fig. in Ref. 4	Δv	Tune Spread v v_0	50,000 Turns			100,000 Turns			150,000 Turns			200,000 Turns		
			EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}
55	0.100	.100,.1775	2.25	2.32	2.39	1.87	1.91	1.94	1.67	1.70	1.72	1.79	1.81	1.83
56	0.095	.105,.1803	1.86	1.93	1.99	1.90	1.94	1.97	1.81	1.83	1.86	1.81	1.83	1.84
57	0.090	.110,.1828	1.87	1.94	2.01	1.78	1.81	1.84	1.92	1.93	1.95	1.89	1.91	1.92
58	0.085	.115,.1850	1.65	1.71	1.77	1.93	1.96	1.99	2.02	2.05	2.07	2.08	2.10	2.12
59	0.080	.120,.1871	1.44	1.49	1.55	1.42	1.44	1.46	1.45	1.46	1.47	1.63	1.65	1.68
60	0.075	.125,.1890	1.51	1.55	1.60	1.49	1.51	1.53	1.48	1.49	1.50	1.40	1.41	1.42
61	0.070	.130,.1907	1.73	1.81	1.89	1.30	1.33	1.37	1.32	1.34	1.36	1.35	1.37	1.38
62	0.065	.135,.1922	1.68	1.74	1.80	1.45	1.48	1.51	1.36	1.38	1.39	1.33	1.34	1.35
63	0.060	.140,.1936	1.91	1.97	2.04	1.67	1.70	1.74	1.54	1.56	1.58	1.53	1.54	1.56
64	0.055	.145,.1948	2.20	2.27	2.33	1.71	1.74	1.78	1.59	1.61	1.63	1.58	1.59	1.61
65	0.050	.150,.1959	1.92	2.01	2.10	2.08	2.11	2.15	2.11	2.13	2.15	1.96	1.98	2.00
66	0.045	.155,.1968	1.93	2.02	2.11	1.91	1.95	1.99	1.94	1.97	1.99	2.06	2.08	2.10
67	0.040	.160,.1975	1.89	2.04	2.18	1.80	1.87	1.93	2.04	2.08	2.11	2.02	2.04	2.07
68	0.035	.165,.1982	1.21	1.24	1.27	1.17	1.18	1.20	1.43	1.47	1.51	1.42	1.45	1.47
69	0.030	.170,.1987	0.97	1.02	1.07	0.89	0.90	0.92	1.06	1.08	1.10	1.16	1.18	1.19
70	0.025	.175,.1992	0.62	0.65	0.69	0.88	0.90	0.92	1.04	1.05	1.07	1.10	1.11	1.12
71	0.020	.180,.1995	1.08	1.14	1.20	1.30	1.32	1.35	1.10	1.12	1.14	1.03	1.05	1.06
72	0.015	.185,.1997	0.94	0.97	0.99	1.07	1.08	1.10	0.99	0.99	1.00	0.91	0.92	0.93
73	0.010	.190,.1999	0.82	0.85	0.88	0.91	0.92	0.93	0.93	0.95	0.96	1.00	1.01	1.02
74	0.005	.195,.2000	0.92	0.95	0.98	0.93	0.94	0.95	1.04	1.05	1.06	1.09	1.10	1.11
75	0	.2,.2	0.73	0.74	0.75	0.94	0.96	0.98	0.96	0.96	0.97	0.98	0.99	1.00

continued

TABLE 2 continued (single precision)

Fig. in Ref. 4	$\Delta\checkmark$	Tune Spread $\checkmark$ $\%$	50,000 Turns			100,000 Turns			150,000 Turns			200,000 Turns		
			EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}
76	0.100	.150, .2373	2.07	2.18	2.29	2.87	2.94	3.01	2.74	2.78	2.81	2.77	2.80	2.83
77	0.095	.155, .2391	2.33	2.49	2.66	2.05	2.11	2.18	2.09	2.13	2.18	2.34	2.38	2.41
78	0.090	.160, .2407	1.23	1.29	1.34	1.54	1.59	1.63	1.94	1.99	2.04	2.07	2.11	2.14
79	0.085	.165, .2421	1.18	1.24	1.29	1.25	1.27	1.29	1.30	1.32	1.33	1.39	1.40	1.42
80	0.080	.170, .2434	0.99	1.03	1.08	1.14	1.17	1.19	1.16	1.17	1.18	1.18	1.19	1.19
81	0.075	.175, .2446	0.90	0.93	0.97	1.02	1.04	1.05	1.13	1.14	1.15	1.15	1.16	1.17
82	0.070	.180, .2456	0.65	0.68	0.70	0.86	0.88	0.90	0.88	0.89	0.90	0.93	0.93	0.94
83	0.065	.185, .2464	0.99	1.01	1.04	0.82	0.84	0.85	0.97	0.99	1.00	1.07	1.08	1.09
84	0.060	.190, .2472	1.04	1.08	1.12	1.20	1.22	1.24	1.16	1.17	1.18	1.09	1.10	1.11
85	0.055	.195, .2478	1.23	1.27	1.31	1.34	1.35	1.37	1.26	1.27	1.28	1.17	1.18	1.19
86	0.050	.200, .2484	0.86	0.92	0.96	1.15	1.17	1.19	1.11	1.13	1.14	1.12	1.13	1.13
87	0.045	.205, .2488	0.83	0.86	0.90	0.93	0.94	0.96	0.91	0.92	0.92	0.95	0.96	0.97
88	0.040	.210, .2492	1.10	1.14	1.17	1.06	1.07	1.09	1.06	1.07	1.07	1.10	1.10	1.11
89	0.035	.215, .2494	1.02	1.06	1.11	0.90	0.92	0.94	0.86	0.88	0.89	0.91	0.92	0.92
90	0.030	.220, .2496	1.06	1.10	1.14	1.06	1.07	1.09	1.07	1.08	1.09	1.14	1.14	1.15
91	0.025	.225, .2498	0.91	0.94	0.97	0.80	0.82	0.83	0.90	0.91	0.92	0.98	0.99	1.00
92	0.020	.230, .2499	1.46	1.49	1.52	1.29	1.31	1.32	1.13	1.15	1.17	1.07	1.08	1.09
93	0.015	.235, .2500	0.88	0.91	0.94	0.98	0.99	1.01	1.16	1.18	1.20	1.19	1.20	1.21
94	0.010	.240, .2500	0.95	0.98	1.01	0.99	1.00	1.02	1.08	1.09	1.11	1.09	1.09	1.10
95	0.005	.245, .2500	1.02	1.05	1.08	1.08	1.10	1.11	1.05	1.06	1.07	1.05	1.06	1.06
96	0	.25, .25	1.00	1.02	1.04	1.00	1.01	1.02	1.11	1.12	1.14	1.19	1.20	1.21

continued

TABLE 2 continued (single precision)

Fig. in Ref. 4	Δv	Tune Spread v v_0	50,000 Turns			100,000 Turns			150,000 Turns			200,000 Turns		
			EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}
97	0.100	.200,.2966	3.44	3.60	3.76	3.09	3.17	3.24	2.63	2.68	2.74	2.48	2.52	2.55
98	0.095	.205,.2975	3.11	3.32	3.52	2.89	2.96	3.03	2.66	2.70	2.75	2.67	2.70	2.73
99	0.090	.210,.2983	3.10	3.28	3.46	3.16	3.23	3.30	3.04	3.08	3.12	2.75	2.79	2.83
100	0.085	.215,.2990	4.35	4.56	4.78	3.46	3.57	3.67	3.13	3.19	3.25	2.89	2.94	2.98
101	0.080	.220,.2996	4.20	4.46	4.71	4.40	4.48	4.57	3.79	3.86	3.94	3.49	3.55	3.60
102	0.075	.225,.3000	4.08	4.40	4.72	3.84	3.94	4.03	3.84	3.90	3.96	3.69	3.73	3.77
103	0.070	.230,.3003	5.01	5.35	5.69	4.44	4.56	4.69	4.12	4.19	4.26	3.74	3.80	3.86
104	0.065	.235,.3006	4.44	4.79	5.15	4.82	4.97	5.12	4.83	4.92	5.02	4.90	4.97	5.04
105	0.060	.240,.3007	5.30	5.71	6.13	5.37	5.53	5.70	5.93	6.07	6.21	6.80	6.92	7.03
106	0.055	.245,.3008	6.54	7.36	8.19	6.36	6.66	6.97	6.64	6.83	7.01	6.55	6.68	6.81
107	0.050	.250,.3009	1.20	1.27	1.33	1.11	1.14	1.16	1.27	1.29	1.31	1.32	1.33	1.35
108	0.045	.255,.3008	0.82	0.86	0.91	0.86	0.88	0.90	0.94	0.96	0.97	0.92	0.93	0.94
109	0.040	.260,.3008	0.94	0.97	1.01	0.78	0.80	0.82	0.92	0.94	0.95	0.96	0.97	0.98
110	0.035	.265,.3007	0.85	0.89	0.92	0.85	0.86	0.88	0.87	0.87	0.88	0.91	0.91	0.92
111	0.030	.270,.3006	1.03	1.06	1.09	0.92	0.93	0.95	0.94	0.95	0.95	0.94	0.94	0.95
112	0.025	.275,.3004	0.97	1.00	1.03	0.94	0.95	0.97	0.87	0.88	0.89	0.80	0.81	0.82
113	0.020	.280,.3003	0.94	0.97	0.99	0.86	0.87	0.88	0.92	0.93	0.94	0.96	0.96	0.97
114	0.015	.285,.3002	1.26	1.28	1.30	1.17	1.18	1.19	1.23	1.24	1.25	1.21	1.22	1.23
115	0.010	.290,.3001	0.85	0.89	0.93	1.02	1.03	1.04	1.14	1.16	1.17	1.11	1.12	1.13
116	0.005	.295,.3000	0.90	0.92	0.94	1.00	1.02	1.03	0.92	0.93	0.94	0.92	0.92	0.92
117	0	.3,.3	0.66	0.68	0.70	0.88	0.90	0.93	1.03	1.04	1.06	1.04	1.05	1.05

continued

TABLE 2 continued (single precision)

Fig. in Ref. 4	Δv	Tune Spread v v_0	50,000 Turns			100,000 Turns			150,000 Turns			200,000 Turns		
			EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}
118	0.100	.250,.3581	1.04	1.10	1.17	1.00	1.02	1.05	0.94	0.95	0.97	0.99	1.00	1.02
119	0.095	.255,.3581	1.11	1.17	1.22	0.94	0.97	0.99	0.93	0.94	0.95	0.91	0.92	0.92
120	0.090	.260,.3579	0.84	0.88	0.92	0.83	0.84	0.86	0.86	0.87	0.89	0.89	0.90	0.91
121	0.085	.265,.3576	0.89	0.92	0.96	1.11	1.13	1.15	1.06	1.07	1.08	1.05	1.05	1.06
122	0.080	.270,.3573	0.75	0.78	0.81	0.82	0.83	0.84	0.91	0.93	0.94	0.97	0.97	0.98
123	0.075	.275,.3568	1.05	1.09	1.13	1.05	1.06	1.08	0.97	0.99	1.00	1.01	1.02	1.03
124	0.070	.280,.3563	0.98	1.01	1.05	0.99	1.01	1.02	0.93	0.93	0.94	1.00	1.01	1.02
125	0.065	.285,.3558	0.79	0.82	0.86	0.85	0.86	0.87	0.89	0.89	0.90	0.98	0.99	1.00
126	0.060	.290,.3552	0.91	0.93	0.96	0.88	0.89	0.90	0.75	0.77	0.78	0.73	0.74	0.75
127	0.055	.295,.3546	0.83	0.86	0.88	1.02	1.03	1.04	1.03	1.04	1.05	1.06	1.07	1.08
128	0.050	.300,.3540	1.25	1.28	1.31	0.94	0.97	0.99	0.86	0.88	0.89	0.89	0.90	0.91
129	0.045	.305,.3534	1.07	1.11	1.14	1.03	1.04	1.06	1.15	1.16	1.17	1.20	1.21	1.21
130	0.040	.310,.3528	0.95	0.99	1.02	1.22	1.25	1.28	1.30	1.32	1.33	1.20	1.21	1.23
131	0.035	.315,.3522	1.28	1.31	1.35	1.28	1.31	1.33	1.22	1.23	1.25	1.22	1.22	1.23
132	0.030	.320,.3517	1.88	1.93	1.98	1.51	1.55	1.58	1.39	1.40	1.42	1.39	1.41	1.42
133	0.025	.325,.3512	1.35	1.46	1.56	1.53	1.57	1.61	1.49	1.51	1.53	1.48	1.49	1.50
134	0.020	.330,.3508	1.27	1.34	1.41	1.44	1.48	1.51	1.39	1.42	1.44	1.57	1.59	1.62
135	0.015	.335,.3505	0.97	1.00	1.04	0.94	0.96	0.97	0.95	0.95	0.96	0.97	0.98	0.98
136	0.010	.340,.3502	0.65	0.66	0.68	0.76	0.77	0.79	0.87	0.88	0.90	0.89	0.90	0.91
137	0.005	.345,.3501	1.01	1.04	1.06	0.95	0.96	0.97	0.96	0.97	0.97	1.01	1.01	1.02
138	0	.35,.35	1.15	1.18	1.21	1.03	1.05	1.06	1.01	1.02	1.03	0.97	0.98	0.98

continued

TABLE 2 continued (single precision)

Fig. in Ref. 4	$\Delta \nu$	Tune Spread $\nu \quad \nu_0$	50,000 Turns			100,000 Turns			150,000 Turns			200,000 Turns		
			EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}
139	0.100	.300,.4307	0.49	0.56	0.62	0.57	0.60	0.64	0.60	0.62	0.63	0.59	0.60	0.61
140	0.095	.305,.4283	0.80	0.87	0.94	0.76	0.78	0.80	0.70	0.71	0.73	0.73	0.74	0.75
141	0.090	.310,.4260	0.74	0.81	0.87	0.77	0.79	0.82	0.79	0.80	0.82	0.80	0.81	0.82
142	0.085	.315,.4237	0.55	0.62	0.69	0.71	0.74	0.77	0.76	0.78	0.80	0.75	0.76	0.77
143	0.080	.320,.4214	0.76	0.80	0.84	0.68	0.70	0.72	0.64	0.65	0.67	0.69	0.70	0.71
144	0.075	.325,.4192	0.78	0.82	0.87	0.81	0.83	0.85	0.81	0.82	0.82	0.74	0.75	0.76
145	0.070	.330,.4170	0.72	0.76	0.81	0.80	0.82	0.84	0.86	0.87	0.86	0.82	0.83	0.84
146	0.065	.335,.4149	0.60	0.64	0.69	0.57	0.59	0.61	0.65	0.66	0.68	0.71	0.72	0.73
147	0.060	.340,.4130	0.78	0.83	0.88	0.76	0.77	0.79	0.77	0.78	0.79	0.75	0.76	0.76
148	0.055	.345,.4111	0.56	0.60	0.65	0.65	0.66	0.68	0.66	0.67	0.68	0.74	0.75	0.76
149	0.050	.350,.4093	0.97	1.01	1.06	0.91	0.93	0.95	0.92	0.93	0.94	0.95	0.96	0.96
150	0.045	.355,.4076	0.90	0.94	0.97	0.95	0.97	0.98	0.92	0.93	0.94	0.90	0.91	0.92
151	0.040	.360,.4061	0.81	0.85	0.89	0.99	1.01	1.03	0.97	0.98	0.99	0.95	0.96	0.97
152	0.035	.365,.4048	0.73	0.76	0.80	0.89	0.90	0.92	0.89	0.90	0.91	0.92	0.92	0.93
153	0.030	.370,.4036	0.77	0.81	0.84	0.99	1.01	1.03	0.98	0.99	1.00	1.00	1.01	1.01
154	0.025	.375,.4025	0.83	0.86	0.88	0.95	0.96	0.98	1.02	1.02	1.03	0.95	0.96	0.97
155	0.020	.380,.4016	0.78	0.81	0.85	0.90	0.91	0.93	0.90	0.91	0.92	0.93	0.94	0.95
156	0.015	.385,.4009	1.02	1.05	1.08	0.85	0.86	0.88	0.82	0.83	0.84	0.84	0.85	0.85
157	0.010	.390,.4004	0.85	0.87	0.90	1.01	1.02	1.03	1.04	1.04	1.05	1.00	1.01	1.01
158	0.005	.395,.4001	0.81	0.84	0.87	0.96	0.97	0.98	1.01	1.02	1.02	1.00	1.00	1.01
159	0	.4,.4	0.90	0.92	0.94	0.85	0.87	0.88	0.85	0.85	0.86	0.81	0.81	0.82

continued

TABLE 2 continued (single precision)

Fig. in Ref. 4	$\Delta\nu$	Tune Spread ν ν_0	50,000 Turns			100,000 Turns			150,000 Turns			200,000 Turns		
			EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}
160	0.050	.400, .4821	0.70	1.09	1.48	0.43	0.55	0.66	0.41	0.47	0.52	0.39	0.42	0.45
161	0.045	.405, .4733	1.00	1.09	1.18	0.61	0.65	0.70	0.54	0.56	0.59	0.54	0.56	0.57
162	0.040	.410, .4673	0.53	0.61	0.69	0.65	0.68	0.71	0.59	0.61	0.63	0.54	0.56	0.57
163	0.035	.415, .4627	0.68	0.75	0.82	0.84	0.87	0.90	0.83	0.85	0.87	0.76	0.78	0.79
164	0.030	.420, .4591	0.56	0.64	0.71	0.66	0.68	0.71	0.59	0.60	0.62	0.68	0.70	0.71
165	0.025	.425, .4562	0.68	0.72	0.75	0.75	0.77	0.79	0.72	0.73	0.74	0.67	0.68	0.69
166	0.020	.430, .4539	0.70	0.74	0.78	0.82	0.83	0.85	0.82	0.83	0.84	0.80	0.81	0.81
167	0.015	.435, .4522	0.89	0.92	0.95	0.85	0.86	0.88	0.91	0.92	0.93	0.88	0.89	0.90
168	0.010	.440, .4510	0.85	0.88	0.91	0.89	0.89	0.90	0.81	0.82	0.83	0.80	0.80	0.81
169	0.005	.445, .4502	0.94	0.97	1.01	1.03	1.04	1.05	0.97	0.98	0.99	0.87	0.88	0.89
170	0	.45, .45	1.14	1.18	1.21	0.87	0.89	0.91	0.89	0.90	0.92	0.97	0.98	1.00

Concluded

The data were obtained during a test run of experiment E623 at the M6W beam of the Meson Laboratory. A schematic plan view of the apparatus is shown in Figure 1. The main elements of the trigger are 3 PWC hodoscopes H_1 , H_2 , and H_3 each consisting of 32 elements of width 1.6cm, 3.2cm, and 6.4cm respectively and a 30 element multicell cherenkov counter C_2 segmented roughly into equal cell spacing in pseudorapidity.

Detector locations were chosen to optimize the detection efficiency for centrally produced η_0 . The choice of N_2 for the C_2 gas was also dictated by this requirement. A Monte Carlo program was written to produce the η_0 using a phenomenological model⁴, let them decay, and track the charged kaons through the apparatus. The efficiency to pass through H_1 , H_2 , and H_3 and to be above pion threshold but below kaon threshold in C_2 ($6 < p_K < 22$ GeV/c) was found to be $\epsilon_K = 0.71$. Thus $\epsilon_\phi = 0.42$ and $\epsilon_{\phi\phi} = 0.15$. The detection efficiency peaks at $x_{\eta_0} = 0.1$ and has a FWHM in $x_{\eta_0} = 0.15$. The efficiency is insensitive to $p_{\perp\eta_0}$ for $p_{\perp\eta_0} < 2.0$ GeV/c.

The Monte Carlo program also generated lists of allowed coincidences for the hodoscopes, $H_1^i \cdot H_2^j \cdot H_3^k \cdot C_2^l$. All listed coincidences were wired in a trigger processor whose details are given elsewhere⁵. The hodoscope multiplicity in H_3 was required to be $4 \leq N_{H_3} \leq 10$. In addition the number of valid

coincidence candidates (called K^+) was required to be $N_{K^+} \geq 2 \cdot N_{K^-} \geq 2$. The processor then used the hodoscope addresses for valid coincidences to roughly calculate the K^+K^- mass and transverse momentum in the bend plane. The mass was required to be small for both the left most and right most K^+K^- combinations. The final trigger rate was ~ 10 μ b. The option to bias the data toward large $\phi\phi$ masses by requiring large ϕ transverse momentum was not imposed during the test run, although it will certainly be used during the main data taking.

During interruptions in data taking calibration runs using light flashers were taken to find the photoelectron equivalents of both C_1 and C_2 . The data was then passed through pattern recognition and track fitting programs⁶. The resulting tracks were propagated to C_2 and kaon candidates were required to have no light in the appropriate mirror. The typical spectrometer ($x > 0.0$) multiplicity was 10. Of all triggers about 10% have at least 2 K^+ and at least 2 K^- which were reconstructed successfully.

The K^+K^- mass spectrum for these events is shown in Figure 2. In this figure, and in subsequent figures, the smooth curves represent combinatorial background. These curves are generated by combining K^+ from one event with K^- from 10 other distinct events. The normalization is set by

TABLE 3 Double precision enhancement values after 200,000 turns; 200 particles.

Δv										
0.100	10.3	1.35	1.65	2.95	2.33	1.11	0.69			
0.095	2.12	1.48	1.81	2.88	2.28	1.01	0.66			
0.090	1.86	1.72	1.69	1.83	3.23	0.89	0.58			
0.085	1.88	1.67	1.28	1.24	2.88	1.04	0.80			
0.080	1.55	1.55	1.41	1.02	4.44	0.98	0.73			
0.075	1.55	1.61	1.42	1.09	3.77	0.97	0.86			
0.070	1.66	1.51	1.28	1.33	4.61	1.20	0.63			
0.065	1.60	1.35	1.56	1.23	4.45	0.78	0.88			
0.060	1.08	1.09	1.12	1.24	6.39	1.16	0.66			
0.055	1.50	1.07	1.53	1.14	4.87	0.99	1.01			
0.050	4.58	1.37	1.25	2.10	0.95	0.98	1.07	0.85	0.31	
0.045	2.17	1.50	1.41	1.85	1.07	0.91	1.05	0.97	0.51	
0.040	1.90	1.03	1.15	2.01	0.86	0.86	1.36	0.99	0.67	
0.035	1.30	1.03	1.00	1.13	1.09	0.99	1.12	0.84	0.61	
0.030	1.18	0.98	0.96	1.00	0.57	1.17	1.56	0.95	0.73	
0.025	1.06	1.17	1.00	0.88	1.07	0.75	1.78	0.93	0.87	
0.020	1.02	1.29	0.80	1.10	0.78	1.17	1.66	0.91	0.71	
0.015	1.29	1.25	1.17	0.95	0.66	0.84	1.01	1.01	0.94	
0.010	1.07	0.73	0.93	1.02	0.97	1.04	0.85	0.96	0.89	
0.005	1.21	0.86	0.85	0.81	1.30	0.96	1.09	0.95	0.77	
0	0.76	0.89	0.93	1.10	1.08	0.96	0.97	0.81	0.87	0.98
$v_0^* = 0.0001$		0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45

TABLE 4 Double precision enhancement values; 200 particles.

Fig. in Ref. 4	Δv	Tune Spread v_v v_o	50,000 Turns			100,000 Turns			150,000 Turns			200,000 Turns		
			EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}
1	0	.0001,.0001	1.02	1.10	1.18	0.71	0.74	0.77	0.73	0.75	0.76	0.75	0.76	0.77
2	0.050	.0001,.0032	5.28	11.1	16.9	0.70	3.01	5.33	3.08	4.31	5.54	3.82	4.58	5.34
3	0.045	.005,.0218	2.82	3.30	3.77	2.15	1.33	1.50	2.14	2.24	2.34	2.10	2.17	2.24
4	0.040	.010,.0300	1.11	1.44	1.77	2.07	2.18	2.30	1.87	1.94	2.00	1.86	1.90	1.94
5	0.035	.015,.0358	0.77	1.01	1.24	0.99	1.07	1.15	1.12	1.16	1.21	1.27	1.30	1.34
6	0.030	.020,.0401	0.99	1.10	1.21	1.15	1.20	1.26	1.27	1.30	1.33	1.15	1.18	1.20
7	0.025	.025,.0434	1.34	1.47	1.60	1.28	1.33	1.38	1.14	1.17	1.20	1.04	1.06	1.08
8	0.020	.030,.0459	1.31	1.41	1.51	0.90	0.94	0.99	0.82	0.85	0.87	1.00	1.02	1.05
9	0.015	.035,.0477	0.91	0.96	1.01	1.29	1.33	1.36	1.24	1.26	1.28	1.28	1.29	1.31
10	0.010	.040,.0490	1.28	1.33	1.38	1.31	1.33	1.35	1.22	1.24	1.26	1.06	1.07	1.09
11	0.005	.045,.0498	1.02	1.06	1.11	1.07	1.08	1.10	1.12	1.13	1.14	1.20	1.21	1.22
12	0	.05,.05	1.08	1.13	1.18	0.92	0.94	0.96	0.86	0.87	0.89	0.88	0.89	0.90

continued

TABLE 4 continued (double precision)

Fig. in Ref. 4	Δv	Tune Spread v v_s	50,000 Turns			100,000 Turns			150,000 Turns			200,000 Turns		
			EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}
13	0.100	.0001, .0045	-2.72	5.27	13.3	6.62	9.84	13.1	0.80	10.5	12.2	9.17	10.3	11.5
14	0.095	.005, .0313	1.67	2.48	3.29	1.68	1.99	2.30	1.84	2.01	2.17	2.01	2.12	2.24
15	0.090	.010, .0437	1.68	2.19	2.68	2.12	2.30	2.48	1.73	1.84	1.95	1.78	1.86	1.94
16	0.085	.015, .0529	0.30	0.63	1.06	1.73	1.89	2.05	1.99	2.08	2.17	1.82	1.88	1.94
17	0.080	.020, .0603	0.88	1.15	1.42	0.75	0.86	0.96	1.35	1.42	1.49	1.50	1.55	1.60
18	0.075	.025, .0665	1.27	1.50	1.73	1.78	1.88	1.97	1.43	1.49	1.56	1.51	1.55	1.59
19	0.070	.030, .0718	0.89	1.07	1.25	1.02	1.10	1.17	1.31	1.36	1.40	1.62	1.66	1.71
20	0.065	.035, .0765	1.26	1.40	1.54	1.51	1.56	1.62	1.56	1.59	1.62	1.58	1.60	1.62
21	0.060	.040, .0805	1.10	1.21	1.32	1.08	1.12	1.17	1.04	1.07	1.10	1.06	1.08	1.10
22	0.055	.045, .0840	1.68	1.80	1.91	1.61	1.66	1.70	1.52	1.55	1.58	1.48	1.50	1.52
23	0.050	.050, .0871	1.02	1.11	1.20	1.58	1.63	1.69	1.38	1.42	1.45	1.34	1.37	1.39
24	0.045	.055, .0898	0.64	0.75	0.86	1.16	1.21	1.26	1.29	1.32	1.35	1.48	1.50	1.53
25	0.040	.060, .0921	0.68	0.77	0.86	1.12	1.15	1.19	1.18	1.20	1.23	1.06	1.08	1.09
26	0.035	.065, .0941	0.83	0.90	0.97	0.97	1.00	1.03	1.04	1.06	1.08	1.06	1.08	1.09
27	0.030	.070, .0957	0.42	0.43	0.53	0.64	0.66	0.69	0.84	0.86	0.88	0.96	0.98	1.00
28	0.025	.075, .0971	1.22	1.23	1.33	1.10	1.12	1.14	1.07	1.08	1.10	1.15	1.17	1.18
29	0.020	.080, .0982	1.43	1.48	1.53	1.42	1.44	1.46	1.33	1.35	1.36	1.28	1.29	1.30
30	0.015	.085, .0990	1.15	1.20	1.26	1.20	1.22	1.23	1.15	1.16	1.17	1.24	1.25	1.26
31	0.010	.090, .0996	0.44	0.53	0.62	0.52	0.55	0.58	0.68	0.70	0.72	0.72	0.73	0.74
32	0.005	.095, .0999	1.13	1.18	1.22	0.93	0.95	0.97	0.83	0.84	0.86	0.85	0.86	0.87
33	0	.1, .1	1.29	1.31	1.34	0.98	1.01	1.04	0.87	0.89	0.90	0.92	0.93	0.94

continued

TABLE 4 continued (double precision)

Fig. no. Ref. 4	Δv	Tune Spread v v_0	50,000 Turns			100,000 Turns			150,000 Turns			200,000 Turns		
			EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}
34	0.100	.050,.1134	0.81	0.97	1.14	0.80	0.86	0.93	1.15	1.19	1.24	1.32	1.35	1.39
35	0.095	.055,.1177	1.10	1.23	1.35	1.39	1.44	1.50	1.40	1.45	1.47	1.46	1.48	1.50
36	0.090	.060,.1217	1.22	1.33	1.45	1.57	1.62	1.67	1.56	1.58	1.61	1.70	1.72	1.75
37	0.085	.065,.1253	0.89	0.93	1.06	1.86	1.93	2.00	1.81	1.85	1.88	1.64	1.67	1.70
38	0.080	.070,.1286	1.48	1.60	1.72	1.33	1.38	1.43	1.28	1.31	1.33	1.52	1.55	1.58
39	0.075	.075,.1316	1.45	1.54	1.63	1.51	1.55	1.58	1.57	1.59	1.61	1.59	1.61	1.63
40	0.070	.080,.1343	0.77	0.87	0.97	1.19	1.23	1.28	1.46	1.49	1.52	1.50	1.51	1.53
41	0.065	.085,.1368	0.89	0.96	1.04	1.24	1.28	1.32	1.39	1.42	1.44	1.34	1.35	1.37
42	0.060	.090,.1390	0.84	0.92	0.99	0.97	1.00	1.03	1.13	1.15	1.18	1.07	1.09	1.10
43	0.055	.095,.1409	0.81	0.89	0.96	0.49	0.52	0.55	0.85	0.89	0.92	1.05	1.07	1.10
44	0.050	.100,.1427	1.40	1.47	1.53	1.46	1.49	1.51	1.46	1.48	1.50	1.23	1.25	1.27
45	0.045	.105,.1442	0.97	1.03	1.10	1.27	1.30	1.33	1.35	1.37	1.39	1.40	1.41	1.42
46	0.040	.110,.1455	0.92	0.97	1.03	0.67	0.70	0.73	0.93	0.96	0.98	1.13	1.15	1.17
47	0.035	.115,.1467	0.99	1.06	1.13	0.91	0.94	0.96	1.03	1.05	1.06	0.99	1.00	1.01
48	0.030	.120,.1476	1.15	1.22	1.30	0.87	0.91	0.94	0.93	0.95	0.96	0.95	0.96	0.98
49	0.025	.125,.1484	0.87	0.90	0.94	1.02	1.05	1.07	1.10	1.11	1.13	0.99	1.00	1.02
50	0.020	.130,.1490	1.18	1.22	1.26	0.88	0.91	0.94	0.81	0.81	0.84	0.79	0.80	0.81
51	0.015	.135,.1494	1.04	1.19	1.14	1.30	1.32	1.35	1.21	1.23	1.25	1.15	1.17	1.18
52	0.010	.140,.1498	0.93	0.97	1.01	0.67	0.69	0.71	0.77	0.79	0.81	0.91	0.93	0.94
53	0.005	.145,.1499	1.01	1.04	1.07	0.92	0.94	0.95	0.81	0.83	0.84	0.84	0.85	0.86
54	0	.15,.15	0.62	0.65	0.69	0.86	0.88	0.90	0.97	0.98	1.00	1.08	1.10	1.11

continued

TABLE 4 continued (double precision)

Fig. in Ref. 4	Δv	Tune Spread $v \quad v_0$	50,000 Turns			100,000 Turns			150,000 Turns			200,000 Turns		
			EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}
55	0.100	.100,.1775	1.72	1.83	1.93	1.66	1.70	1.75	1.53	1.56	1.58	1.63	1.65	1.67
56	0.095	.105,.1803	1.55	1.62	1.69	1.53	1.56	1.59	1.72	1.74	1.77	1.79	1.81	1.83
57	0.090	.110,.1828	1.18	1.29	1.39	1.43	1.48	1.53	1.59	1.62	1.65	1.67	1.69	1.71
58	0.085	.115,.1850	0.91	1.00	1.10	0.86	0.89	0.92	1.15	1.18	1.21	1.26	1.28	1.31
59	0.080	.120,.1871	1.42	1.52	1.63	1.20	1.24	1.28	1.38	1.40	1.43	1.39	1.41	1.43
60	0.075	.125,.1890	1.32	1.40	1.48	1.26	1.29	1.33	1.26	1.28	1.29	1.41	1.42	1.44
61	0.070	.130,.1907	1.66	1.72	1.77	1.63	1.67	1.70	1.26	1.30	1.33	1.26	1.28	1.30
62	0.065	.135,.1922	1.63	1.70	1.77	1.75	1.78	1.81	1.52	1.55	1.58	1.54	1.56	1.58
63	0.060	.140,.1936	1.59	1.68	1.76	1.50	1.54	1.58	1.10	1.13	1.17	1.10	1.12	1.15
64	0.055	.145,.1948	1.14	1.23	1.33	1.48	1.52	1.57	1.48	1.51	1.54	1.51	1.53	1.55
65	0.050	.150,.1959	2.10	2.22	2.35	1.83	1.89	1.95	1.89	1.92	1.96	2.07	2.10	2.12
66	0.045	.155,.1968	1.35	1.49	1.63	1.75	1.80	1.86	1.85	1.89	1.92	1.83	1.85	1.88
67	0.040	.160,.1975	1.15	1.22	1.29	1.85	1.92	1.98	1.84	1.88	1.92	1.98	2.01	2.05
68	0.035	.165,.1982	1.15	1.18	1.22	1.05	1.07	1.09	1.03	1.05	1.06	1.12	1.13	1.15
69	0.030	.170,.1987	0.64	0.70	0.75	1.09	1.13	1.16	0.93	0.96	0.98	0.98	1.00	1.01
70	0.025	.175,.1992	0.72	0.78	0.83	0.93	0.96	0.98	1.00	1.02	1.03	0.86	0.88	0.89
71	0.020	.180,.1995	1.11	1.14	1.18	0.95	0.97	1.00	0.91	0.92	0.94	1.08	1.10	1.12
72	0.015	.185,.1997	1.47	1.52	1.57	1.20	1.22	1.25	1.01	1.03	1.05	0.94	0.95	0.97
73	0.010	.190,.1999	1.30	1.34	1.37	0.80	0.83	0.87	0.82	0.84	0.86	0.99	1.02	1.04
74	0.005	.195,.2000	1.22	1.24	1.26	0.93	0.97	1.01	0.75	0.77	0.80	0.80	0.81	0.83
75	0	.2,.2	0.74	0.77	0.78	0.93	0.95	0.96	1.05	1.07	1.08	1.07	1.08	1.09

continued

TABLE 4 continued (double precision)

Fig. in Ref. 4	Δv	Tune Spread v v_0	50,000 Turns			100,000 Turns			150,000 Turns			200,000 Turns		
			EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}
76	0.100	.150,.2373	1.79	1.94	2.10	2.16	1.23	1.30	2.72	2.78	2.84	2.91	2.95	3.00
77	0.095	.155,.2391	1.91	2.04	2.18	1.96	2.03	2.10	2.20	2.25	2.31	2.81	2.88	2.95
78	0.090	.160,.2407	1.56	1.62	1.68	1.65	1.71	1.77	1.90	1.95	2.01	1.78	1.83	1.87
79	0.085	.165,.2421	0.64	0.72	0.81	0.86	0.89	0.93	1.17	1.20	1.23	1.22	1.24	1.26
80	0.080	.170,.2434	1.18	1.28	1.38	1.30	1.34	1.37	1.08	1.11	1.13	1.00	1.02	1.03
81	0.075	.175,.2446	0.68	0.73	0.78	0.69	0.71	0.73	0.85	0.87	0.89	1.06	1.09	1.11
82	0.070	.180,.2456	1.26	1.31	1.36	1.35	1.37	1.39	1.24	1.26	1.27	1.31	1.33	1.34
83	0.065	.185,.2464	1.13	1.17	1.22	1.10	1.12	1.14	1.07	1.09	1.10	1.21	1.23	1.24
84	0.060	.190,.2472	0.82	0.88	0.94	1.00	1.02	1.05	1.16	1.18	1.20	1.23	1.24	1.26
85	0.055	.195,.2478	0.97	1.01	1.05	1.07	1.09	1.11	1.07	1.08	1.10	1.12	1.14	1.15
86	0.050	.200,.2484	0.88	0.93	0.98	1.01	1.03	1.05	1.04	1.05	1.07	0.94	0.95	0.97
87	0.045	.205,.2488	1.08	1.15	1.22	1.22	1.25	1.28	1.02	1.04	1.06	1.06	1.07	1.09
88	0.040	.210,.2492	1.14	1.19	1.25	1.04	1.07	1.09	0.87	0.89	0.90	0.85	0.86	0.87
89	0.035	.215,.2494	0.87	0.91	0.95	1.18	1.21	1.24	1.09	1.11	1.13	1.08	1.09	1.11
90	0.030	.220,.2496	0.87	0.93	0.99	0.69	0.71	0.73	0.59	0.61	0.62	0.56	0.57	0.58
91	0.025	.225,.2498	1.06	1.11	1.17	1.22	1.24	1.27	1.08	1.10	1.12	1.06	1.07	1.08
92	0.020	.230,.2499	0.39	0.44	0.49	0.45	0.47	0.49	0.70	0.73	0.75	0.76	0.78	0.79
93	0.015	.235,.2500	0.93	0.98	1.02	0.60	0.63	0.66	0.66	0.68	0.69	0.65	0.66	0.67
94	0.010	.240,.2500	0.89	0.95	1.01	0.98	1.00	1.02	0.98	1.00	1.01	0.96	0.97	0.98
95	0.005	.245,.2500	1.18	1.23	1.28	1.48	1.50	1.53	1.41	1.43	1.44	1.29	1.30	1.32
96	0	.25,.25	0.74	0.79	0.84	0.90	0.93	0.95	0.95	0.96	0.98	0.96	0.96	0.97

continued

TABLE 4 continued (double precision)

Fig. in Ref. 4	Δv	Tune Spread v v_0	50,000 Turns			100,000 Turns			150,000 Turns			200,000 Turns		
			EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}
97	0.100	.200, .2966	3.24	3.56	3.87	2.71	2.82	2.93	2.52	2.59	2.65	2.29	2.33	2.38
98	0.095	.205, .2975	4.20	4.48	4.75	3.99	4.10	4.22	3.07	3.17	3.27	2.18	2.28	2.38
99	0.090	.210, .2983	2.76	3.11	3.47	3.09	3.21	3.33	3.21	3.27	3.34	3.19	3.23	3.27
100	0.085	.215, .2990	4.78	5.09	5.40	3.68	3.84	3.99	3.12	3.21	3.31	2.81	2.88	2.95
101	0.080	.220, .2996	3.92	4.32	4.73	5.56	5.74	5.91	5.12	5.23	5.34	4.34	4.44	4.54
102	0.075	.225, .3000	4.32	4.77	5.21	4.26	4.43	4.59	4.11	4.21	4.31	3.69	3.77	3.84
103	0.070	.230, .3003	3.68	4.19	4.71	4.41	4.61	4.82	4.67	4.78	4.90	4.53	4.61	4.68
104	0.065	.235, .3006	5.10	5.81	6.52	4.48	4.74	5.00	4.73	4.88	5.02	4.35	4.45	4.56
105	0.060	.240, .3007	4.48	5.22	5.97	5.71	6.05	6.39	5.97	6.18	6.38	6.24	6.39	6.54
106	0.055	.245, .3008	3.84	4.61	5.38	4.32	4.66	4.99	4.91	5.12	5.33	4.69	4.87	5.05
107	0.050	.250, .3009	0.80	0.86	0.92	0.98	1.01	1.04	0.95	0.98	1.00	0.96	0.98	1.00
108	0.045	.255, .3008	0.47	0.53	0.58	0.65	0.68	0.70	0.77	0.79	0.81	0.89	0.91	0.92
109	0.040	.260, .3008	1.08	1.14	1.20	0.99	1.01	1.03	0.99	1.00	1.02	0.85	0.86	0.88
110	0.035	.265, .3007	1.09	1.15	1.20	1.13	1.15	1.18	1.03	1.04	1.05	0.98	0.99	1.00
111	0.030	.270, .3006	1.12	1.18	1.24	1.17	1.20	1.22	1.15	1.16	1.17	1.16	1.17	1.18
112	0.025	.275, .3004	0.89	0.93	0.98	0.85	0.87	0.89	0.78	0.79	0.80	0.74	0.75	0.76
113	0.020	.280, .3003	1.29	1.33	1.36	1.26	1.28	1.29	1.28	1.29	1.30	1.16	1.17	1.19
114	0.015	.285, .3002	0.66	0.70	0.73	0.96	0.98	1.00	0.93	0.94	0.95	0.82	0.84	0.85
115	0.010	.290, .3001	0.40	0.44	0.48	0.83	0.87	0.90	0.96	0.98	1.00	1.03	1.04	1.06
116	0.005	.295, .3000	1.20	1.27	1.34	1.10	1.12	1.15	1.03	1.04	1.05	0.95	0.96	0.97
117	0	.3, .3	0.68	0.72	0.76	0.72	0.74	0.75	0.87	0.88	0.89	0.95	0.97	0.98

continued

TABLE 4 continued (double precision)

Fig. in Ref. 4	Δv	Tune Spread v v_0	50,000 Turns			100,000 Turns			150,000 Turns			200,000 Turns		
			EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}
118	0.100	.250,.3581	1.31	1.37	1.43	1.38	1.41	1.45	1.17	1.19	1.22	1.09	1.11	1.13
119	0.095	.255,.3581	0.93	1.00	1.08	0.85	0.88	0.91	0.93	0.95	0.96	1.00	1.01	1.02
120	0.090	.260,.3579	0.81	0.86	0.90	0.78	0.80	0.82	0.85	0.87	0.88	0.88	0.89	0.90
121	0.085	.265,.3576	0.78	0.83	0.88	1.06	1.10	1.13	1.10	1.02	1.04	1.03	1.04	1.05
122	0.080	.270,.3573	1.03	1.08	1.12	0.67	0.69	0.72	0.93	0.96	0.98	0.97	0.98	1.00
123	0.075	.275,.3568	0.83	0.87	0.92	0.78	0.81	0.83	0.89	0.91	0.93	0.96	0.97	0.98
124	0.070	.280,.3563	0.78	0.82	0.86	0.97	0.99	1.01	1.09	1.11	1.12	1.19	1.20	1.22
125	0.065	.285,.3558	0.58	0.63	0.68	0.72	0.75	0.77	0.76	0.77	0.79	0.77	0.78	0.79
126	0.060	.290,.3552	0.73	0.80	0.82	1.04	1.07	1.09	1.15	1.17	1.19	1.15	1.16	1.17
127	0.055	.295,.3546	0.92	0.99	1.07	0.80	0.83	0.85	0.90	0.91	0.93	0.98	0.99	1.00
128	0.050	.300,.3540	1.30	1.74	1.39	1.13	1.15	1.18	1.06	1.07	1.09	1.06	1.07	1.08
129	0.045	.305,.3534	1.05	1.10	1.14	1.09	1.12	1.14	1.05	1.06	1.07	1.04	1.05	1.06
130	0.040	.310,.3528	1.06	1.13	1.20	1.29	1.32	1.35	1.34	1.35	1.37	1.35	1.36	1.37
131	0.035	.315,.3522	1.45	1.52	1.59	1.11	1.15	1.18	1.17	1.19	1.21	1.11	1.12	1.14
132	0.030	.320,.3517	1.09	1.18	1.26	1.38	1.41	1.44	1.47	1.49	1.51	1.54	1.56	1.58
133	0.025	.325,.3512	1.46	1.54	1.62	1.78	1.81	1.85	1.60	1.63	1.65	1.76	1.78	1.80
134	0.020	.330,.3508	0.65	0.69	0.72	0.87	0.91	0.94	1.34	1.40	1.45	1.62	1.66	1.71
135	0.015	.335,.3505	1.23	1.28	1.32	0.85	0.89	0.92	0.93	0.95	0.97	1.00	1.01	1.03
136	0.010	.340,.3502	1.11	1.14	1.18	0.75	0.78	0.80	0.75	0.77	0.78	0.84	0.85	0.86
137	0.005	.345,.3501	0.74	0.77	0.81	1.05	1.07	1.09	1.12	1.13	1.15	1.08	1.09	1.10
138	0	.35,.35	0.65	0.69	0.73	0.94	0.96	0.99	0.85	0.87	0.88	0.79	0.81	0.82

continued

TABLE 4 continued (double precision)

Fig. in Ref. 4	Δv	Tune Spread v v_c	50,000 Turns			100,000 Turns			150,000 Turns			200,000 Turns		
			EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}
139	0.100	.300, .4307	0.99	1.08	1.18	0.54	0.58	0.63	0.59	0.62	0.64	0.67	0.69	0.71
140	0.095	.305, .4283	0.38	0.46	0.53	0.53	0.56	0.60	0.55	0.57	0.58	0.64	0.66	0.67
141	0.090	.310, .4260	0.42	0.50	0.59	0.49	0.52	0.56	0.59	0.60	0.62	0.57	0.58	0.59
142	0.085	.315, .4237	1.12	1.20	1.27	0.95	0.99	1.02	0.88	0.90	0.92	0.78	0.80	0.82
143	0.080	.320, .4214	0.61	0.70	0.79	0.57	0.60	0.63	0.73	0.75	0.77	0.72	0.73	0.75
144	0.075	.325, .4192	0.75	0.85	0.95	0.95	0.98	1.01	0.88	0.90	0.92	0.85	0.86	0.87
145	0.070	.330, .4170	0.74	0.81	0.89	0.55	0.59	0.62	0.55	0.57	0.59	0.61	0.63	0.64
146	0.065	.335, .4149	0.94	1.01	1.08	0.82	0.84	0.87	0.83	0.85	0.86	0.87	0.88	0.89
147	0.060	.340, .4130	0.58	0.64	0.70	0.63	0.66	0.68	0.55	0.56	0.58	0.64	0.66	0.67
148	0.055	.345, .4111	0.99	1.07	1.16	1.01	1.04	1.08	1.10	1.12	1.13	0.99	1.01	1.02
149	0.050	.350, .4093	0.83	0.89	0.96	0.89	0.92	0.94	0.90	0.91	0.93	0.84	0.85	0.86
150	0.045	.355, .4076	0.92	0.99	1.07	1.11	1.14	1.17	0.95	0.97	0.99	0.95	0.97	0.98
151	0.040	.360, .4061	0.82	0.89	0.96	0.90	0.92	0.94	0.96	0.97	0.99	0.98	0.99	1.00
152	0.035	.365, .4048	1.18	1.24	1.30	0.95	0.98	1.01	0.91	0.92	0.94	0.83	0.84	0.85
153	0.030	.370, .4036	1.02	1.10	1.17	0.72	0.75	0.78	0.90	0.92	0.94	0.94	0.95	0.97
154	0.025	.375, .4025	0.75	0.79	0.83	0.80	0.82	0.84	0.89	0.90	0.91	0.93	0.93	0.94
155	0.020	.380, .4016	0.82	0.87	0.93	0.73	0.74	0.76	0.81	0.82	0.83	0.90	0.91	0.92
156	0.015	.385, .4009	0.84	0.87	0.90	0.97	0.98	1.00	0.96	0.97	0.98	1.01	1.01	1.02
157	0.010	.390, .4004	0.94	0.98	1.02	0.91	0.94	0.96	1.03	1.04	1.06	0.95	0.96	0.97
158	0.005	.395, .4001	0.83	0.87	0.90	0.98	0.99	1.01	0.97	0.98	0.99	0.94	0.95	0.95
159	0	.4, .4	1.20	1.22	1.25	0.95	0.97	0.99	0.88	0.89	0.90	0.87	0.87	0.88

continued

TABLE 4 continued (double precision)

Fig. in Ref. 4	$\Delta\nu$	Tune Spread ν ν_0	50,000 Turns			100,000 Turns			150,000 Turns			200,000 Turns		
			EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}	EN _{min}	EN	EN _{max}
160	0.050	.400, .4821	-0.19	0.05	0.29	0.03	0.12	0.21	0.19	0.23	0.28	0.28	0.31	0.34
161	0.045	.405, .4733	0.85	0.98	1.11	0.58	0.64	0.69	0.53	0.56	0.60	0.48	0.51	0.53
162	0.040	.410, .4673	0.11	0.23	0.34	0.44	0.48	0.52	0.60	0.63	0.66	0.65	0.67	0.69
163	0.035	.415, .4627	0.55	0.66	0.78	0.85	0.89	0.94	0.74	0.76	0.79	0.59	0.61	0.63
164	0.030	.420, .4591	0.92	1.01	1.10	0.70	0.74	0.78	0.71	0.73	0.75	0.71	0.73	0.74
165	0.025	.425, .4562	0.77	0.86	0.95	0.90	0.93	0.96	0.86	0.88	0.90	0.86	0.87	0.88
166	0.020	.430, .4539	0.68	0.75	0.82	0.67	0.70	0.72	0.70	0.71	0.72	0.70	0.71	0.71
167	0.015	.435, .4522	0.99	1.03	1.08	0.83	0.85	0.87	0.87	0.88	0.90	0.93	0.94	0.95
168	0.010	.440, .4510	1.09	1.15	1.22	0.85	0.88	0.91	0.96	0.98	0.99	0.87	0.89	0.90
169	0.005	.445, .4502	0.75	0.80	0.86	0.64	0.66	0.68	0.71	0.72	0.74	0.76	0.77	0.78
170	0	.45, .45	0.60	0.63	0.67	0.75	0.77	0.80	0.83	0.85	0.87	0.97	0.98	1.00

CONCLUDED

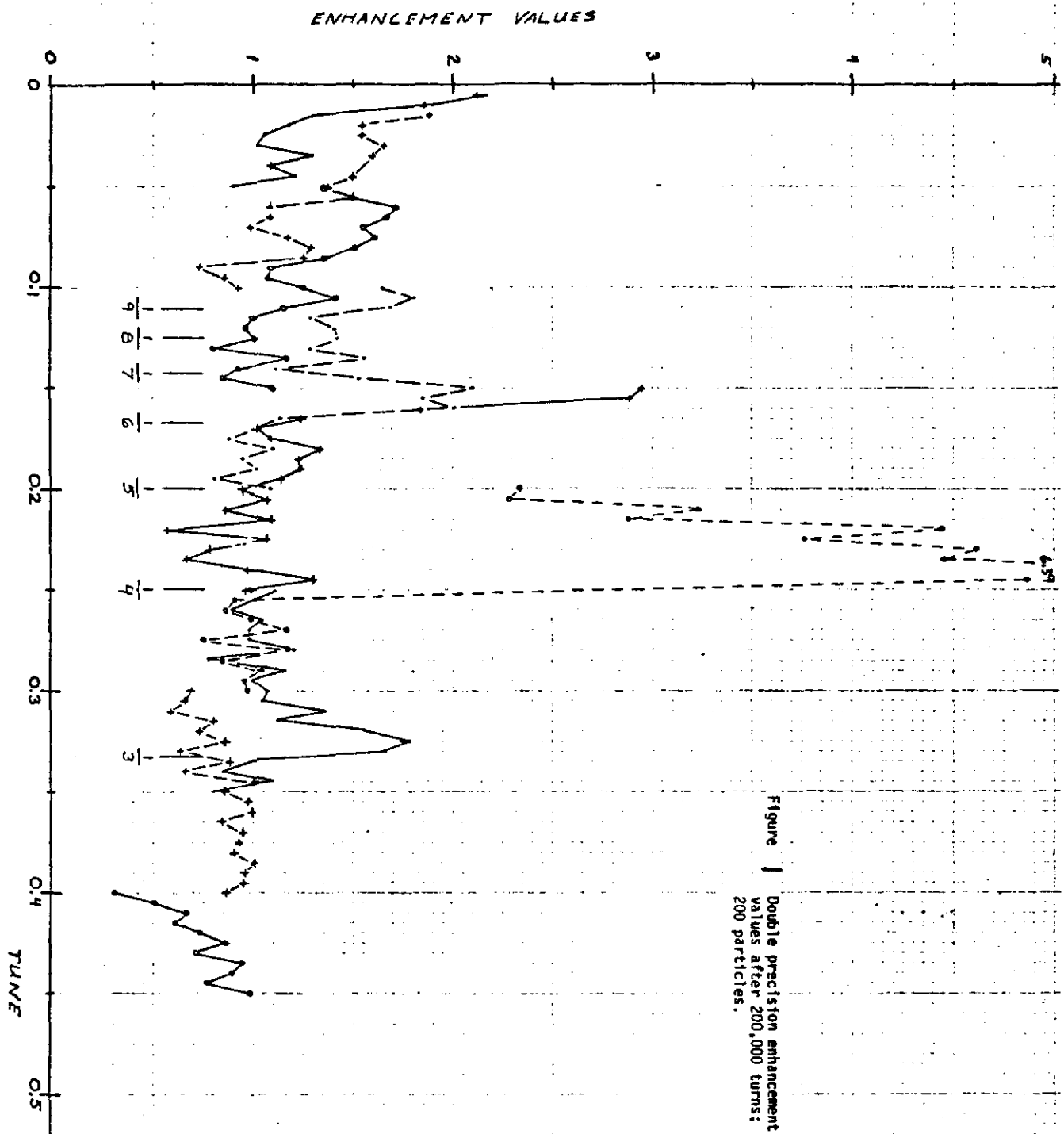
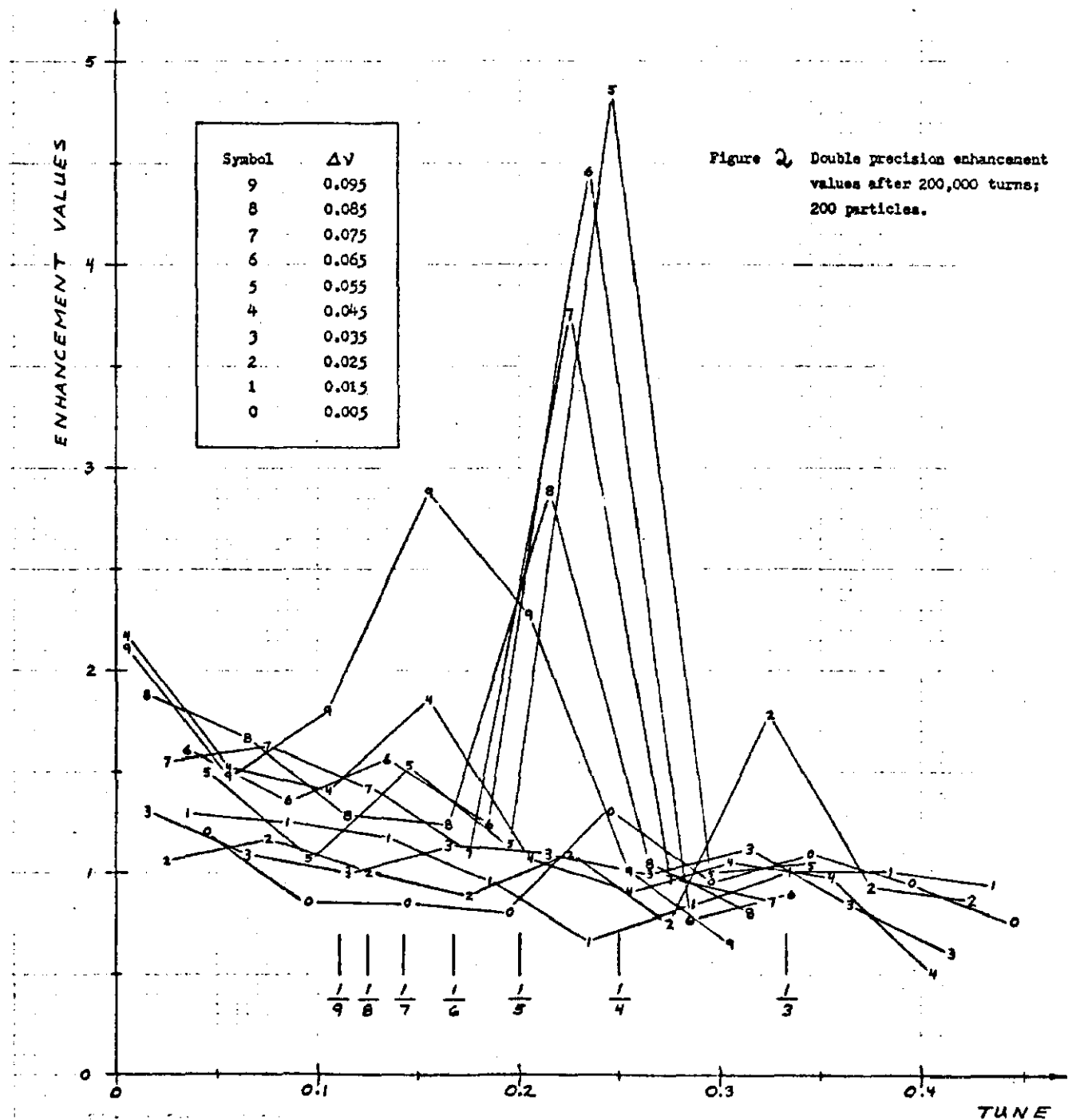


Figure 1 Double precision enhancement values after 200,000 turns: 200 particles.



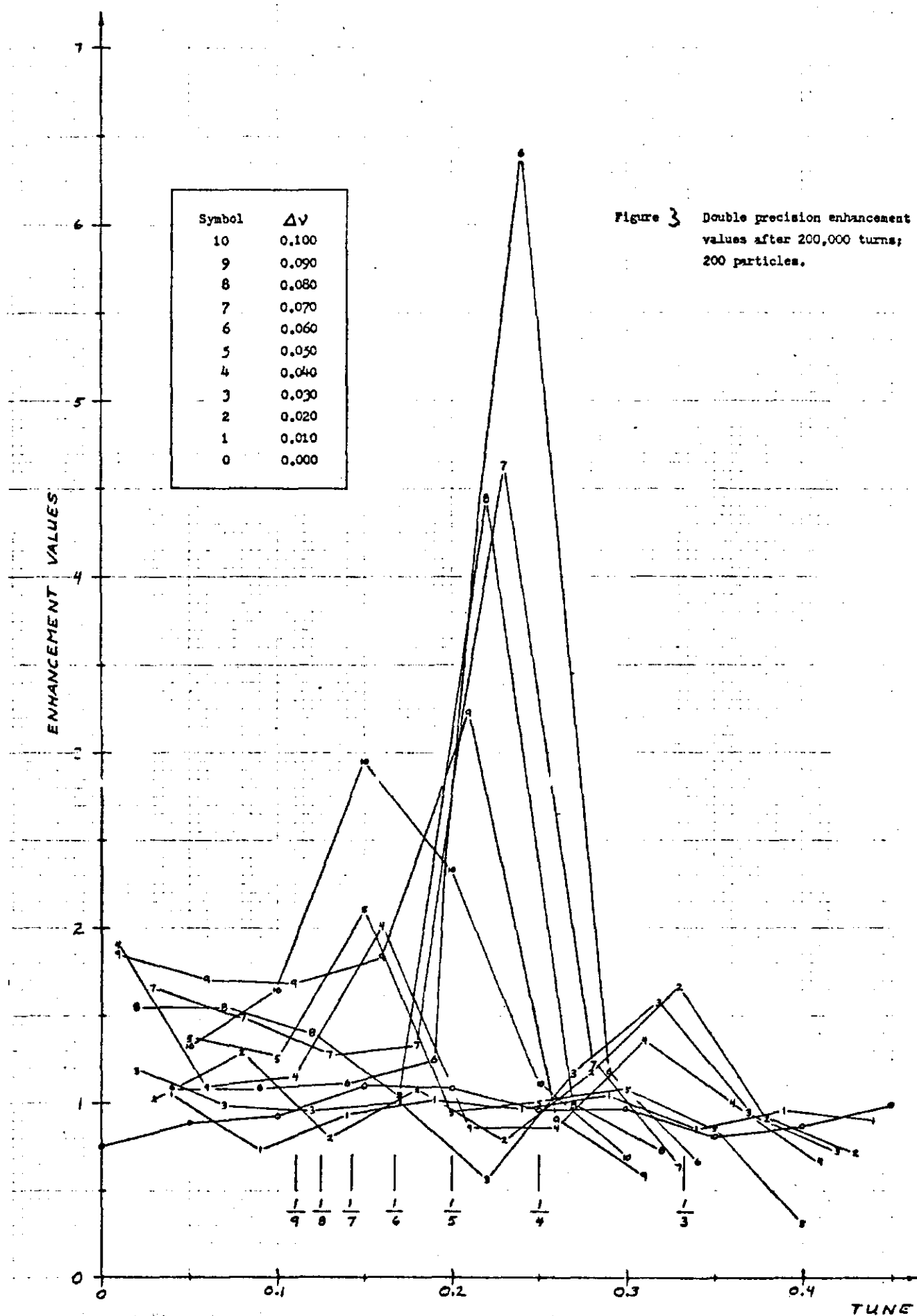
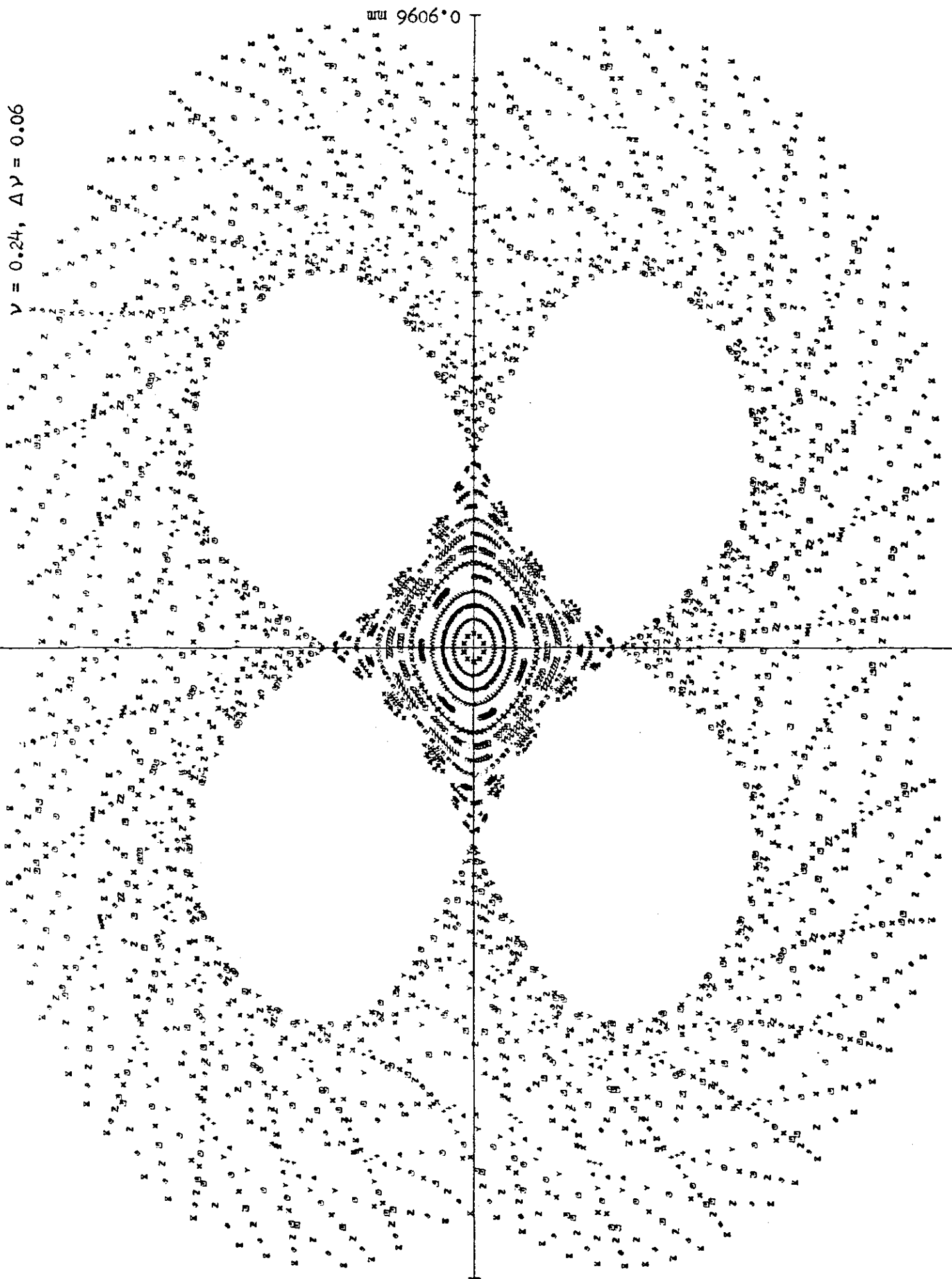


Figure 4 (X,X') Phase Space

0.4913 mrad



9606.0

Figure 3 (x,x') Phase Plane
 $\nu = 0.32$; $\Delta\nu = 0.08$
 100 turns

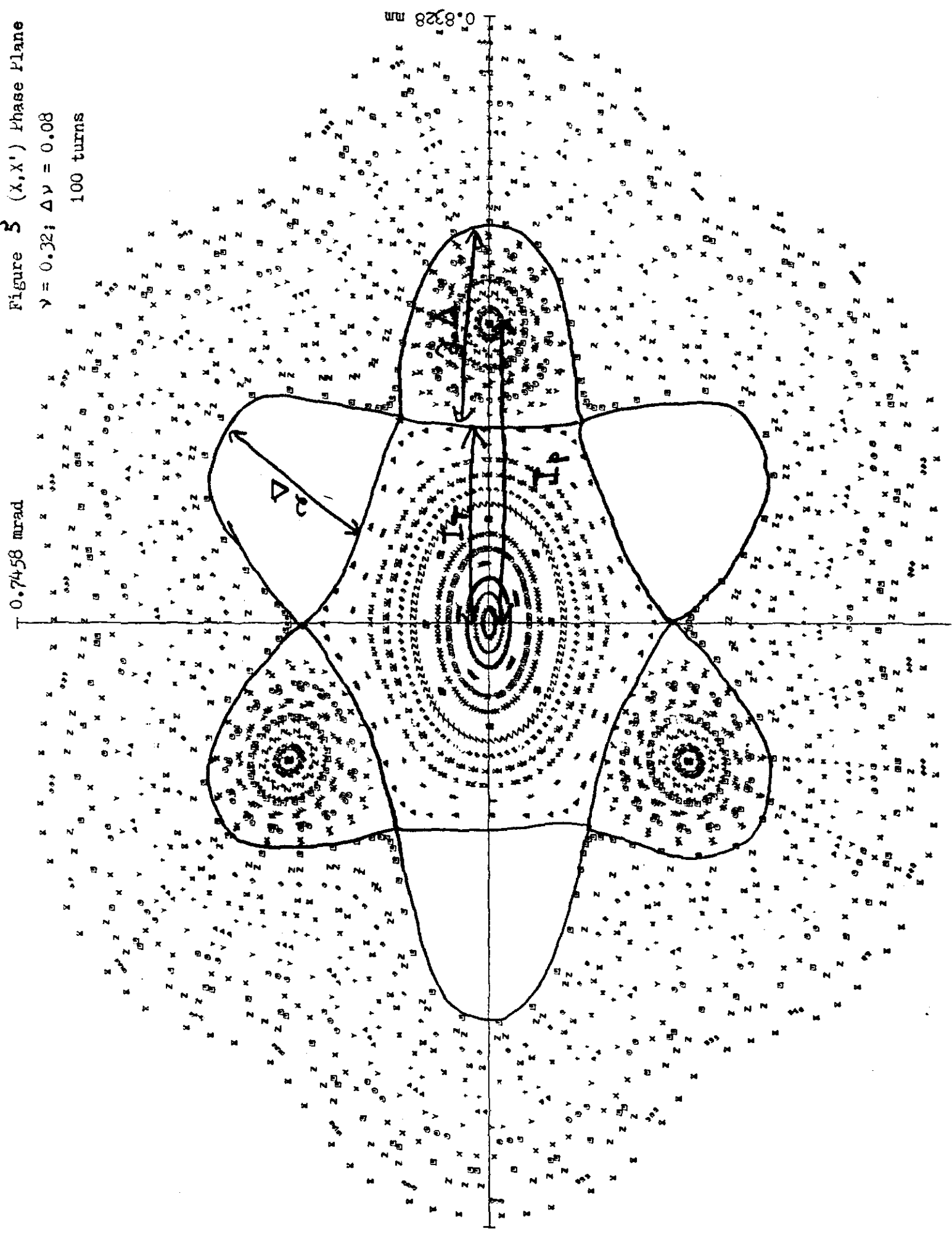
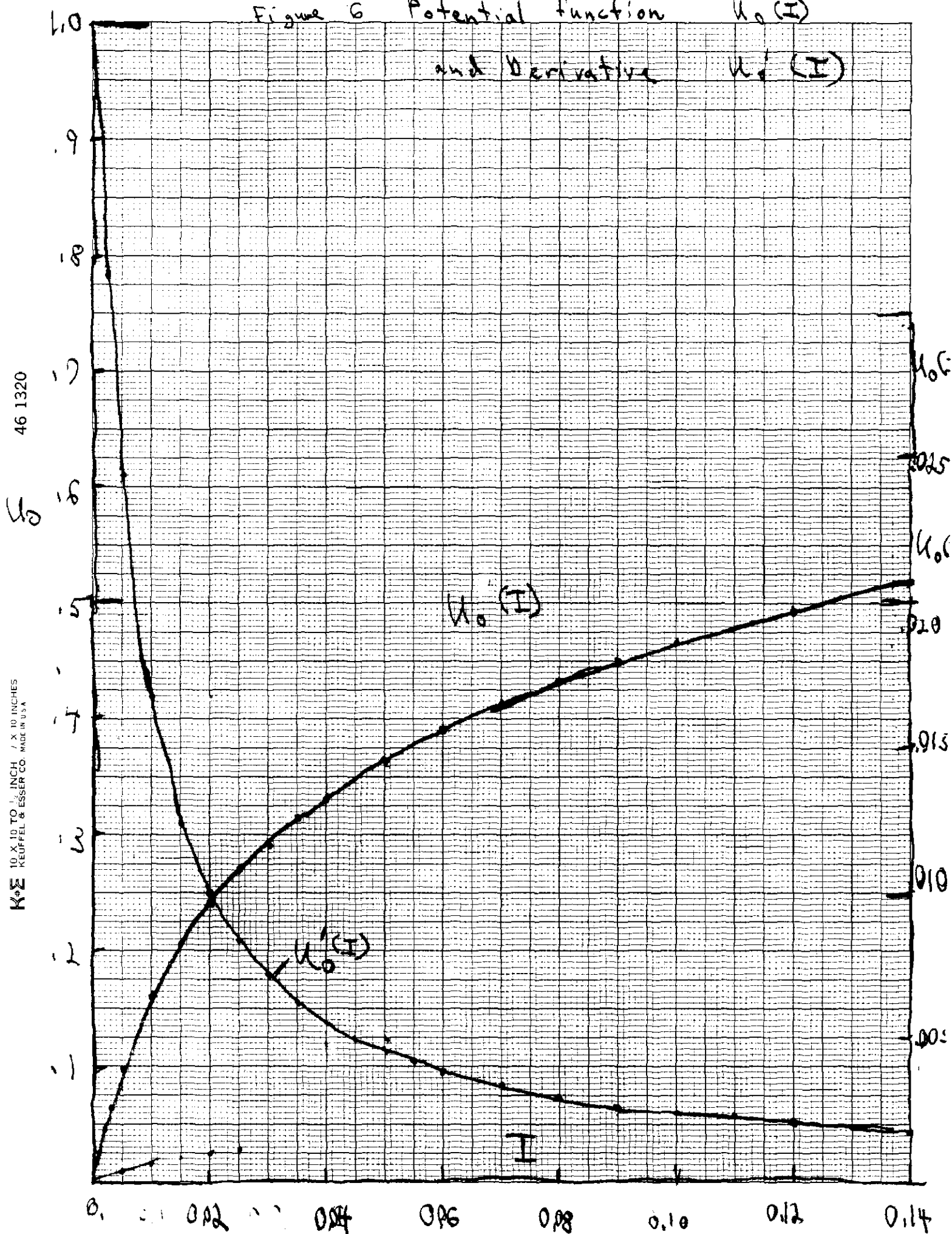


Figure 6 Potential function $U_0(I)$
and Derivative $U'_0(I)$

46 1320

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$u_n(I)$

Figure 7
Fourier Integrals $u_n(I)$

