Towards Inflation in String Theory

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Abstract. I will discuss the development of inflationary theory and its present status, including recent progress in describing de Sitter space and inflationary universe in string theory.

1. Brief history of inflation

The first model of inflationary type was proposed by Alexei Starobinsky [1]. It was based on investigation of conformal anomaly in quantum gravity.

A much simpler inflationary model with a very clear physical motivation was proposed by Alan Guth [2]. His model, which is now called "old inflation," was based on the theory of supercooling in the false vacuum state during the cosmological phase transitions [3]. Unlike the Starobinsky model, old inflation did not actually work, but nevertheless it played an important role in the development of inflationary cosmology since it contained a clear explanation how inflation may solve the major cosmological problems.

The problems of old inflation were resolved with the invention of the new inflationary theory [4]. In this theory, the inflaton field ϕ slowly rolls down from the maximum of the effective potential. The slow motion of the field away from the false vacuum is of crucial importance: density perturbations produced during the slow-roll inflation are inversely proportional to $\dot{\phi}$ [5, 6, 7]. The key difference between the new inflationary scenario and the old one is that the useful part of inflation in the new scenario, which is responsible for the homogeneity of our universe, does *not* occur in the false vacuum state, where $\dot{\phi} = 0$. Unfortunately, the new inflation scenario was plagued by its own problems [8]; no realistic versions of the new inflationary universe scenario have been proposed so far.

Old and new inflation represented a substantial but incomplete modification of the big bang theory. It was still assumed that the universe was in a state of thermal equilibrium from the very beginning, that it was relatively homogeneous and large enough to survive until the beginning of inflation, and that the stage of inflation was just an intermediate stage of the evolution of the universe. In the beginning of the 80's these assumptions seemed most natural and practically unavoidable. On the basis of all available observations (CMB, abundance of light elements) everybody believed that the universe was created in a hot big bang. That is why it was so difficult to overcome a certain psychological barrier and abandon all of these assumptions. This was done with the invention of the chaotic inflation scenario [9]. This scenario resolved all problems of old and new inflation. According to this scenario, inflation may occur even in the theories with simplest potentials such as $V(\phi) \sim \phi^n$. Inflation may begin even if there was no thermal equilibrium in the early universe, and it may start even at the Planckian density, in which case the problem of initial conditions for inflation can be easily resolved [8].

2. Chaotic Inflation

Consider the simplest model of a scalar field ϕ with a mass m and with the potential energy density $V(\phi) = \frac{m^2}{2}\phi^2$. Since this function has a minimum at $\phi = 0$, one may expect that the scalar field ϕ should oscillate near this minimum. This is indeed the case if the universe does not expand, in which case equation of motion for the scalar field coincides with equation for harmonic oscillator, $\ddot{\phi} = -m^2\phi$.

However, because of the expansion of the universe with Hubble constant $H = \dot{a}/a$, an additional term $3H\dot{\phi}$ appears in the harmonic oscillator equation:

$$\ddot{\phi} + 3H\dot{\phi} = -m^2\phi \ . \tag{1}$$

The term $3H\dot{\phi}$ can be interpreted as a friction term. The Einstein equation for a homogeneous universe containing scalar field ϕ looks as follows:

$$H^2 + \frac{k}{a^2} = \frac{1}{6}\left(\dot{\phi}^2 + m^2\phi^2)\right) \ . \tag{2}$$

Here $k = -1, 0, 1$ for an open, flat or closed universe respectively. We work in units $M_p^{-2} = 8\pi G = 1$.

If the scalar field ϕ initially was large, the Hubble parameter H was large too, according to the second equation. This means that the friction term $3H\dot{\phi}$ was very large, and therefore the scalar field was moving very slowly, as a ball in a viscous liquid. Therefore at this stage the energy density of the scalar field, unlike the density of ordinary matter, remained almost constant, and expansion of the universe continued with a much greater speed than in the old cosmological theory. Due to the rapid growth of the scale of the universe and a slow motion of the field ϕ, soon after the beginning of this regime one has $\ddot{\phi} \ll 3H\dot{\phi}$, $H^2 \gg \frac{k}{a^2}$, $\dot{\phi}^2 \ll m^2\phi^2$, so the system of equations can be simplified:

$$H = \frac{\dot{a}}{a} = \frac{m\phi}{\sqrt{6}} \ , \qquad \dot{\phi} = -m\ \sqrt{\frac{2}{3}}. \tag{3}$$

The first equation shows that if the field ϕ changes slowly, the size of the universe in this regime grows approximately as e^{Ht}, where $H = \frac{m\phi}{\sqrt{6}}$. This is the stage of inflation, which ends when the field ϕ becomes much smaller than $M_p = 1$. Solution of these equations shows that after a long stage of inflation the universe initially filled with the field $\phi = \phi_0 \gg 1$ grows exponentially [8], $a = a_0\ e^{\phi_0^2/4}$.

Thus, inflation does not require supercooling and tunnelling from the false vacuum [2], or rolling from an artificially flat top of the effective potential [4]. It appears in the theories that can be as simple as a theory of a harmonic oscillator [9].

In realistic versions of inflationary theory the duration of inflation could be as short as 10^{-35} seconds. When inflation ends, the scalar field ϕ begins to oscillate near the minimum of $V(\phi)$. As any rapidly oscillating classical field, it looses its energy by creating pairs of elementary particles. These particles interact with each other and come to a state of thermal equilibrium with some temperature T [10, 11, 12]. ¿From this time on, the universe can be described by the usual big bang theory.

The main difference between inflationary theory and the old cosmology becomes clear when one calculates the size of a typical inflationary domain at the end of inflation. Investigation of this question shows that even if the initial size of inflationary universe was as small as the Planck size $l_P \sim 10^{-33}$ cm, after 10^{-35} seconds of inflation the universe acquires a huge size of $l \sim 10^{10^{12}}$ cm! This number is model-dependent, but in all realistic models the size of the universe after inflation appears to be many orders of magnitude greater than the size of the

part of the universe which we can see now, $l \sim 10^{28}$ cm. This immediately solves most of the problems of the old cosmological theory [9, 8].

Our universe is almost exactly homogeneous on large scale because all inhomogeneities were exponentially stretched during inflation. The density of primordial monopoles and other undesirable "defects" becomes exponentially diluted by inflation. The universe becomes enormously large. Even if it was a closed universe of a size $\sim 10^{-33}$ cm, after inflation the distance between its "South" and "North" poles becomes many orders of magnitude greater than 10^{28} cm. We see only a tiny part of the huge cosmic balloon. That is why nobody has ever seen how parallel lines cross. That is why the universe looks so flat.

The first models of chaotic inflation were based on the theories with polynomial potentials, such as $V(\phi) = \pm \frac{m^2}{2} \phi^2 + \frac{\lambda}{4} \phi^4$. But the main idea of this scenario is quite generic. One should consider any particular potential $V(\phi)$, polynomial or not, with or without spontaneous symmetry breaking, and study all possible initial conditions without assuming that the universe was in a state of thermal equilibrium, and that the field ϕ was in the minimum of its effective potential from the very beginning.

3. Quantum fluctuations and density perturbations

The vacuum structure in the exponentially expanding universe is much more complicated than in ordinary Minkowski space. The wavelengths of all vacuum fluctuations of the scalar field ϕ grow exponentially during inflation. When the wavelength of any particular fluctuation becomes greater than H^{-1}, this fluctuation stops oscillating, and its amplitude freezes at some nonzero value $\delta\phi(x)$ because of the large friction term $3H\dot{\phi}$ in the equation of motion of the field ϕ. The amplitude of this fluctuation then remains almost unchanged for a very long time, whereas its wavelength grows exponentially. Therefore, the appearance of such a frozen fluctuation is equivalent to the appearance of a classical field $\delta\phi(x)$ that does not vanish after averaging over macroscopic intervals of space and time.

Because the vacuum contains fluctuations of all wavelengths, inflation leads to the creation of more and more new perturbations of the classical field with wavelengths greater than H^{-1}. The average amplitude of such perturbations generated during a typical time interval H^{-1} is given by [13, 14]

$$|\delta\phi(x)| \approx \frac{H}{2\pi} . \tag{4}$$

These fluctuations lead to density perturbations that later produce galaxies. The theory of this effect is very complicated [5, 6], and it was fully understood only in the second part of the 80's [7]. The main idea can be described as follows:

Fluctuations of the field ϕ lead to a local delay of the time of the end of inflation, $\delta t = \frac{\delta\phi}{\dot{\phi}} \sim \frac{H}{2\pi\dot{\phi}}$. Once the usual post-inflationary stage begins, the density of the universe starts to decrease as $\rho = 3H^2$, where $H \sim t^{-1}$. Therefore a local delay of expansion leads to a local density increase δ_H such that $\delta_H \sim \delta\rho/\rho \sim \delta t/t$. Combining these estimates together yields the famous result [5, 6, 7]

$$\delta_H \sim \frac{\delta\rho}{\rho} \sim \frac{H^2}{2\pi\dot{\phi}} . \tag{5}$$

This derivation is oversimplified; it does not tell, in particular, whether H should be calculated during inflation or after it. This issue is of crucial importance for chaotic inflation.

The result of a more detailed investigation [7] shows that H and $\dot{\phi}$ should be calculated during inflation, at different times for perturbations with different momenta k. For each of these perturbations the value of H should be taken at the time when the wavelength of the perturbation becomes of the order of H^{-1}. However, the field ϕ during inflation changes very slowly, so the

quantity $\frac{H^2}{2\pi\phi}$ remains almost constant over exponentially large range of wavelengths. This means that the spectrum of perturbations of metric is flat.

A detailed calculation in our simplest chaotic inflation model the amplitude of perturbations gives

$$\delta_H \sim \frac{m\phi^2}{5\pi\sqrt{6}} \ . \tag{6}$$

The perturbations on scale of the horizon were produced at $\phi_H \sim 15$ [8]. This, together with COBE normalization $\delta_H \sim 2 \times 10^{-5}$ gives $m \sim 3 \times 10^{-6}$, in Planck units, which is approximately equivalent to 7×10^{12} GeV. Exact numbers depend on ϕ_H, which in its turn depends slightly on the subsequent thermal history of the universe.

The magnitude of density perturbations δ_H in our model depends on the scale l only logarithmically. Since the observations provide us with an information about a rather limited range of l, it is possible to parametrize the scale dependence of density perturbations by a simple power law, $\delta_H \sim l^{(1-n)/2}$. An exactly flat spectrum would correspond to $n = 1$.

4. Eternal inflation

A significant step in the development of inflationary theory was the discovery of the process of self-reproduction of inflationary universe. This process was known to exist in old inflationary theory [2] and in the new one [15], but its significance was fully realized only after the discovery of the regime of eternal inflation in the simplest versions of the chaotic inflation scenario [16, 17]. It appears that in many models large quantum fluctuations produced during inflation which may locally increase the value of the energy density in some parts of the universe. These regions expand at a greater rate than their parent domains, and quantum fluctuations inside them lead to production of new inflationary domains which expand even faster. This leads to an eternal process of self-reproduction of the universe.

To understand the mechanism of self-reproduction one should remember that the processes separated by distances l greater than H^{-1} proceed independently of one another. This is so because during exponential expansion the distance between any two objects separated by more than H^{-1} is growing with a speed exceeding the speed of light. As a result, an observer in the inflationary universe can see only the processes occurring inside the horizon of the radius H^{-1}. An important consequence of this general result is that the process of inflation in any spatial domain of radius H^{-1} occurs independently of any events outside it. In this sense any inflationary domain of initial radius exceeding H^{-1} can be considered as a separate mini-universe.

To investigate the behavior of such a mini-universe, with an account taken of quantum fluctuations, let us consider an inflationary domain of initial radius H^{-1} containing sufficiently homogeneous field with initial value $\phi \gg M_p$. Equation (3) implies that during a typical time interval $\Delta t = H^{-1}$ the field inside this domain will be reduced by $\Delta\phi = \frac{2}{\phi}$. By comparison this expression with $|\delta\phi(x)| \approx \frac{H}{2\pi} = \frac{m\phi}{2\pi\sqrt{6}}$ one can easily see that if ϕ is much less than $\phi^* \sim \frac{5}{\sqrt{m}}$, then the decrease of the field ϕ due to its classical motion is much greater than the average amplitude of the quantum fluctuations $\delta\phi$ generated during the same time. But for $\phi \gg \phi^*$ one has $\delta\phi(x) \gg \Delta\phi$. Because the typical wavelength of the fluctuations $\delta\phi(x)$ generated during the time is H^{-1}, the whole domain after $\Delta t = H^{-1}$ effectively becomes divided into $e^3 \sim 20$ separate domains (mini-universes) of radius H^{-1}, each containing almost homogeneous field $\phi - \Delta\phi + \delta\phi$. In almost a half of these domains the field ϕ grows by $|\delta\phi(x)| - \Delta\phi \approx |\delta\phi(x)| = H/2\pi$, rather than decreases. This means that the total volume of the universe containing *growing* field ϕ increases 10 times. During the next time interval $\Delta t = H^{-1}$ the situates repeats. Thus, after the two time intervals H^{-1} the total volume of the universe containing the growing scalar field increases 100 times, etc. The universe enters eternal process of self-reproduction and becomes immortal.

During the process of eternal inflation in the simplest versions of the chaotic inflation scenario, some parts of the universe spend indefinitely long time at the nearly Planckian density, expanding with the Hubble constant $H = O(1)$, in Planck mass units. In this regime, all scalar fields persistently experience quantum jumps of the magnitude comparable to the Planck mass. This forces the fields to browse between all possible vacuum states. As a result, the universe becomes divided into infinitely many exponentially large domains that have different laws of low-energy physics [16, 17]. This result may have especially interesting implications in the context of string theory, which allows exponentially large number of different vacuum states [8, 18, 19].

5. Inflation and observations
Inflation is not just an interesting theory that can resolve many difficult problems of the standard Big Bang cosmology. This theory made several predictions which can be tested by cosmological observations. Here are the most important predictions:

1) The universe must be flat. In most models $\Omega_{total} = 1 \pm 10^{-4}$.

2) Perturbations of metric produced during inflation are adiabatic.

3) Inflationary perturbations have flat spectrum. In most inflationary models the spectral index $n = 1 \pm 0.2$ ($n = 1$ means totally flat.)

4) These perturbations are gaussian.

5) Perturbations of metric could be scalar, vector or tensor. Inflation mostly produces scalar perturbations, but it also produces tensor perturbations with nearly flat spectrum, and it does *not* produce vector perturbations. There are certain relations between the properties of scalar and tensor perturbations produced by inflation.

6) Inflationary perturbations produce specific peaks in the spectrum of CMB radiation.

It is possible to violate each of these predictions if one makes this theory sufficiently complicated. For example, it is possible to produce vector perturbations of metric in the models where cosmic strings are produced at the end of inflation. It is possible to have an open or closed inflationary universe, or even a small periodic inflationary universe, it is possible to have models with nongaussian isocurvature fluctuations with a non-flat spectrum. However, it is very difficult to do so, and most of the inflationary models satisfy the simple rules given above.

It is not easy to test all of these predictions. The major breakthrough in this direction was achieved due to the recent measurements of the CMB anisotropy. The latest results based on the WMAP experiment, in combination with the Sloan Digital Sky Survey, are consistent with predictions of the simplest inflationary models with adiabatic gaussian perturbations, with $\Omega = 1.01 \pm 0.02$, and $n = 0.98 \pm 0.03$ [20, 21].

There are still some question marks to be examined, such as the unexpectedly small anisotropy of CMB at large angles [20]. It is not quite clear whether we deal with a real anomaly here or with a manifestation of cosmic variance [22], but in any case, it is quite significant that all proposed resolutions of this problem are based on inflationary cosmology, see e.g. [23].

6. Shift symmetry and chaotic inflation in supergravity
It would be most important to construct realistic inflationary models based on supergravity and string theory. The effective potential of the complex scalar field Φ in supergravity is given by the well-known expression (in units $M_p = 1$):

$$V = e^K \left[K_{\Phi\bar{\Phi}}^{-1} |D_\Phi W|^2 - 3|W|^2 \right].$$ (7)

Here $W(\Phi)$ is the superpotential, Φ denotes the scalar component of the superfield Φ; $D_\Phi W = \frac{\partial W}{\partial \Phi} + \frac{\partial K}{\partial \Phi} W$. The kinetic term of the scalar field is given by $K_{\Phi\bar{\Phi}} \partial_\mu \Phi \partial_\mu \bar{\Phi}$. The standard textbook choice of the Kähler potential corresponding to the canonically normalized fields Φ and $\bar{\Phi}$ is $K = \Phi\bar{\Phi}$, so that $K_{\Phi\bar{\Phi}} = 1$.

This immediately reveals a problem: At $\Phi > 1$ the potential is extremely steep. It blows up as $e^{|\Phi|^2}$, which makes it very difficult to realize chaotic inflation in supergravity at $\phi \equiv \sqrt{2}|\Phi| > 1$. Moreover, the problem persists even at small ϕ. If, for example, one considers the simplest case when there are many other scalar fields in the theory and the superpotential does not depend on the inflaton field ϕ, then Eq. (7) implies that at $\phi \ll 1$ the effective mass of the inflaton field is $m_\phi^2 = 3H^2$. This violates the condition $m_\phi^2 \ll H^2$ required for a successful slow-roll inflation.

It took almost 20 years to find a natural realization of chaotic inflation model in supergravity. Kawasaki, Yamaguchi and Yanagida suggested to take the Kähler potential $K = \frac{1}{2}(\Phi+\bar{\Phi})^2 + X\bar{X}$ of the fields Φ and X, with the superpotential $m\Phi X$ [24].

The new Kähler potential, just as the old one, leads to canonical kinetic terms for the fields Φ and X, so it is as simple and legitimate as the standard textbook Kähler potential. However, instead of the U(1) symmetry with respect to rotation of the field Φ in the complex plane, the new Kähler potential has a *shift symmetry*; it does not depend on the imaginary part of the field Φ. The shift symmetry is broken only by the superpotential.

This leads to a profound change of the potential (7): the dangerous term e^K continues growing exponentially in the direction $(\Phi+\bar{\Phi})$, but it remains constant in the direction $(\Phi-\bar{\Phi})$. Decomposing the complex field Φ into two real scalar fields, $\Phi = \frac{1}{\sqrt{2}}(\eta + i\phi)$, one can find the resulting potential $V(\phi, \eta, X)$ for $\eta, |X| \ll 1$:

$$V = \frac{m^2}{2}\phi^2(1+\eta^2) + m^2|X|^2. \qquad (8)$$

This potential has a deep valley, with a minimum at $\eta = X = 0$. Therefore the fields η and X rapidly fall down towards $\eta = X = 0$, after which the potential for the field ϕ becomes $V = \frac{m^2}{2}\phi^2$. This provides a very simple realization of eternal chaotic inflation scenario in supergravity [24].

It is amazing that for almost 20 years nothing but inertia was keeping us from using the version of the supergravity which was free from the famous η problem. As we will see shortly, the situation with inflation in string theory is very similar, and may have a similar resolution.

7. Towards Inflation in String Theory

7.1. de Sitter vacua in string theory

For a long time, it seemed rather difficult to obtain inflation in M/string theory. The main problem here was the stability of compactification of internal dimensions. For example, ignoring non-perturbative effects to be discussed below, a typical effective potential of the effective 4d theory obtained by compactification in string theory of type IIB can be represented in the following form:

$$V(\sigma, \rho, \phi) \sim e^{\sqrt{2}\sigma - \sqrt{6}\rho} \; \tilde{V}(\phi) \qquad (9)$$

Here σ and ρ are canonically normalized fields representing the dilaton field and the volume of the compactified space; ϕ stays for all other fields.

If σ and ρ were constant, then the potential $\tilde{V}(\phi)$ could drive inflation. However, this does not happen because of the steep exponent $e^{\sqrt{2}\sigma - \sqrt{6}\rho}$, which rapidly pushes the dilaton field σ to $-\infty$, and the volume modulus ρ to $+\infty$. As a result, the radius of compactification becomes infinite; instead of inflating, 4d space decompactifies and becomes 10d.

Thus in order to describe inflation one should first learn how to stabilize the dilaton and the volume modulus. The dilaton stabilization was achieved in [25]. The most difficult problem was to stabilize the volume. The solution of this problem was found in [26] (KKLT construction). It consists of two steps.

First of all, due to a combination of effects related to warped geometry of the compactified space and nonperturbative effects calculated directly in 4d (instead of being obtained by compactification), it was possible to obtain a supersymmetric AdS minimum of the effective

potential for ρ. This fixed the volume modulus, but in a state with a negative vacuum energy. Then we added an anti-$D3$ brane with the positive energy $\sim \rho^{-2}$. This addition uplifted the minimum of the potential to the state with a positive vacuum energy.

Instead of adding an anti-$D3$ brane, which explicitly breaks supersymmetry, one can add a D7 brane with fluxes. This results in the appearance of a D-term which has a similar dependence on ρ, but leads to spontaneous supersymmetry breaking [27]. In either case, one ends up with a metastable dS state which can decay by tunnelling and formation of bubbles of 10d space with vanishing vacuum energy density. The decay rate is extremely small [26], so for all practical purposes, one obtains an exponentially expanding de Sitter space with the stabilized volume of the internal space.

7.2. Inflation in string theory and shift symmetry

During the last few years there were many suggestions how to obtain hybrid inflation in string theory by considering motion of branes in the compactified space, see [28, 29] and references therein. The main problem of all of these models was the absence of stabilization of the compactified space. Once this problem was solved for dS space [26], one could try to revisit these models and develop models of brane inflation compatible with the volume stabilization.

The first idea [30] was to consider a pair of D3 and anti-D3 branes in the warped geometry studied in [26]. The role of the inflaton field could be played by the interbrane separation. A description of this situation in terms of the effective 4d supergravity involved Kähler potential

$$K = -3\log(\rho + \bar{\rho} - k(\phi, \bar{\phi})), \tag{10}$$

where the function $k(\phi, \bar{\phi})$ for the inflaton field ϕ, at small ϕ, was taken in the simplest form $k(\phi, \bar{\phi}) = \phi\bar{\phi}$. If one makes the simplest assumption that the superpotential does not depend on ϕ, then the ϕ dependence of the potential (7) comes from the term $e^K = (\rho + \bar{\rho} - \phi\bar{\phi})^{-3}$. Expanding this term near the stabilization point $\rho = \rho_0$, one finds that the inflaton field has a mass $m_\phi^2 = 2H^2$. Just like the similar relation $m_\phi^2 = 3H^2$ in the simplest models of supergravity, this is not what we want for inflation.

One way to solve this problem is to consider ϕ-dependent superpotentials. By doing so, one may fine-tune m_ϕ^2 to be $O(10^{-2})H^2$ in a vicinity of the point where inflation occurs [30]. Whereas fine-tuning is certainly undesirable, in the context of string cosmology it may not be a serious drawback. Indeed, if there exist many realizations of string theory [19], then one might argue that all realizations not leading to inflation can be discarded, because they do not describe a universe in which we could live. Meanwhile, those non-generic realizations, which lead to eternal inflation, describe inflationary universes with an indefinitely large and ever-growing volume of inflationary domains. This makes the issue of fine-tuning less problematic.

Can we avoid fine-tuning altogether? One of the possible ideas is to find theories with some kind of shift symmetry. Another possibility is to construct something like D-term inflation, where the flatness of the potential is not spoiled by the term e^K. Both of these ideas were explored in a recent paper [31] based on the model of D3/D7 inflation in string theory [32]. In this model the Kähler potential is given by

$$K = -3\log(\rho + \bar{\rho}) - \frac{1}{2}(\phi - \bar{\phi})^2, \tag{11}$$

and superpotential depends only on ρ. The shift symmetry $\phi \to \phi + c$ in this model is related to the requirement of unbroken supersymmetry of branes in a BPS state.

The effective potential with respect to the field ρ in this model coincides with the KKLT potential [26, 27]. In the direction of the real part of the field ϕ, which can be considered an inflaton, the potential is exactly flat, until one adds other fields which break this flatness due to quantum corrections and produce a potential similar to the potential of D-term inflation [31].

Shift symmetry may help to obtain inflation in other models as well. For example, one may explore the possibility of using the Kähler potential $K = -3\log(\rho + \bar{\rho} - \frac{1}{2}(\phi - \bar{\phi})^2))$ instead of the potential used in [30]. The modified Kähler potential does not depend on the real part of the field ϕ, which can be considered an inflaton. Therefore the dangerous term $m_\phi^2 = 2H^2$ vanishes, i.e. the main obstacle to the consistent brane inflation in the model of Ref. [30] disappears!

However, for a while it still remained unclear whether shift symmetry is just a condition which we want to impose on the theory in order to get inflation, or an unavoidable property of the theory, which remains valid even after the KKLT volume stabilization. The answer to this question was found only very recently, and it appears to be model-dependent. It was shown in [33] that in a certain class of models, including D3/D7 models [32, 31], the shift symmetry of the effective 4d theory is not an assumption but an unambiguous consequence of the underlying mathematical structure of the theory. This may allow us to obtain a natural realization of inflation in string theory.

Note, however, that in the inflationary models based on the simplest version of the KKLT mechanism the Hubble constant during inflation is always smaller than the gravitino mass. This is a rather strong constraint, which can be avoided by considering models with slightly more complicated potentials of the racetrack type [34].

8. Eternal inflation and stringy landscape

Even though we are still at the very first stages of implementing inflation in string theory, it is very tempting to speculate about possible generic features and consequences of such a construction.

First of all, KKLT construction shows that the vacuum energy after the volume stabilization is a function of many different parameters in the theory. One may wonder how many different choices do we actually have. Counting different flux vacua [35, 19] gives the numbers in the range of 10^{100} to 10^{1000}. Some of these vacuum states with positive vacuum energy can be stabilized using the KKLT approach. Each of such states will correspond to a metastable vacuum state. It decays within a cosmologically large time, which is, however, smaller than the 'recurrence time' $e^{S(\phi)}$, where $S(\phi) = \frac{24\pi^2}{V(\phi)}$ is the entropy of dS space with the vacuum energy density $V(\phi)$ [26].

But old inflation does not describe our world. In addition to these metastable vacuum states, there should exist various slow-roll inflationary solutions, where the properties of the system practically do not change during the cosmological time H^{-1}. It might happen that such states, corresponding to flat directions in the string theory landscape, exist not only during inflation in the very early universe, but also at the present stage of the accelerated expansion of the universe. This would simplify obtaining an anthropic solution of the cosmological constant problem along the lines of [36, 35].

If the slow-roll condition $V'' \ll V$ is satisfied all the way from one dS minimum of the effective potential to another, then one can show, using stochastic approach to inflation, that the probability to find the field ϕ at any of these minima, or at any given point between them, is proportional to $e^{S(\phi)}$. In other words, the relative probability to find the field taking some value ϕ_1 as compared to some other value ϕ_0, is proportional to $e^{\Delta S} = e^{S(\phi_1) - S(\phi_0)}$ [37, 26]. One may argue, using Euclidean approach, that this simple thermodynamic relation should remain valid for the relative probability to find a given point in any of the metastable dS vacua, even if the trajectory between them does not satisfy the slow-roll condition $m^2 \ll H^2$ [38, 39, 40, 41].

The resulting picture resembles eternal inflation in the old inflation scenario. However, now we have an incredibly large number of false vacuum states, plus some states which may allow slow-roll inflation. Once inflation begins, different parts of the universe start wondering from one of these vacuum states to another, so that the universe becomes divided into indefinitely many regions with all possible laws of low-energy physics corresponding to different 4d vacua of string theory [8].

As we already argued, the best inflationary scenario would describe a slow-roll eternal inflation starting at the maximal possible energy density (minimal dS entropy). It would be almost as good to have a low-energy slow-roll eternal inflation. Under certain conditions, such regimes may exist in string theory [30]. However, whereas any of these regimes would make us happy, we already have something that can make us smile. Multi-level eternal inflation of the old inflation type, which appears in string theory in the context of the KKLT construction, may be very useful being combined with the slow-roll inflation, even if the slow-roll inflation by itself is not eternal. We will give a particular example, which is very similar to the one considered in [42].

Suppose we have two noninteracting scalar fields: field ϕ with the potential of the old inflation type, and field χ with the potential which may lead to a slow-roll inflation. Let us assume that the slow-roll inflation occurs only on low energy scale, and it is not eternal. How can we provide initial conditions for such a low-scale inflation?

Let us assume that the Hubble constant at the stage of old inflation is much greater than the curvature of the potential which drives the slow-roll inflation. (This is a natural assumption, considering huge number of possible dS states, and the presumed smallness of energy scale of the slow-roll inflation.) In this case large inflationary fluctuations of the field χ will be generated during eternal old inflation. These fluctuations will give the field χ different values in different exponentially large parts of the universe. When old inflation ends, there will be many practically homogeneous parts of the universe where the field χ will take values corresponding to good initial conditions for a slow-roll inflation. Then the relative fraction of the volume of such parts will grow exponentially.

Moreover, as it was argued in [17], the probability (per unit time and unit volume) to jump back to the eternally inflating regime is always *finite*, even after the field enters the regime where, naively, one would not expect eternal inflation. Each bubble of a new phase which appears during the decay of the eternally inflating dS space is an open universe of an *infinite* volume. Therefore during the slow-roll inflation there always will be some inflationary domains jumping back to the original dS space, so some kind of stationary equilibrium will always exist between various parts of the inflationary universe.

Thus, the existence of many different dS vacua in string theory leads to the regime of eternal inflation. This regime may help us to solve the problem of initial conditions for the slow-roll inflation even in the models where the slow-roll inflation by itself is not eternal and would occur only on a small energy scale.

8.1. Acknowledgments

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