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A new bound on the anomalous magnetic moment of the W-boson

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Abstract

The effect of an anomalous magnetic moment $\Delta\kappa$ of the W-boson on the photon structure is calculated. The result depends quadratically on the cut-off Λ. Comparison with PETRA-data gives a limit $|\Delta\kappa(\Lambda/M_W)| \lesssim 33$. This bound is compared with the constraint from the anomalous magnetic moment of the muon.

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Now that their existence is established , the next subject in vector boson physics is the study of their self interactions. Within the standard model these are completely determined by the gauge structure of the theory. Alternative models exist however in which the vectorbosons are not fundamental particles , but are composite objects. If the vector bosons are really composite , their self couplings will in general be different from the standard model predictions. However they are not completely arbitrary, since present experiments are sometimes sensitive to deviations from a gauge structure and can therefore constrain the magnitude of the coupling constants .

The particular coupling to be studied in this note is the magnetic moment μ_W of the charged W-boson. For a gauge theory one has $\mu_W = e/M_W$. A deviation from this relation is described by adding to the standard model an interaction :

$$\mathcal{L}_{int} = ie\Delta\kappa F^{\mu\nu} W_\mu^+ W_\nu^- \tag{1}$$

where $F_{\mu\nu}$ is the electromagnetic field tensor .
The possibility of $\Delta\kappa \neq 0$ has been studied before in refs.[1-4] , where the correction to the (g-2)-factor of the muon was calculated. Using the experimental constraints the following bound was found :

$$|\Delta\kappa \, ln(\Lambda/M_W)| \lesssim 2.2 \tag{2}$$

In this expression Λ is a cut-off that is needed to regularize a divergent loop-integral. It presumably corresponds to the energy scale where the structure of the W-boson becomes manifest.

However the magnetic moment of the muon is not the only quantity in present-day physics that is sensitive to the W-boson magnetic moment. Also the photon propagator is affected by the interaction Lagrangian (1) through the intermediate vector boson loop of fig.(1) . Since this graph is naively quartically divergent one cannot use dimensional regularization as in ref.[2,3] , since dimensional regularization automatically puts quadratic and higher divergencies equal to zero. It is therefore necessary to introduce a structure in the vectorboson propagator . A simple way to do this is to write the W-boson propagator as :

$$W_\mu^+(k)W_\nu^-(-k) = \frac{\delta_{\mu\nu}}{f(k^2)k^2 + M_W^2} + \frac{k_\mu k_\nu}{k^2}\left(\frac{1}{g(k^2)k^2 + M_W^2} - \frac{1}{f(k^2)k^2 + M_W^2}\right) \quad (3)$$

where $f(k^2)$ and $g(k^2)$ are structure functions that depend on the cut-off Λ . For a pointlike W-boson one has $f(k^2) = 1$ and $g(k^2) = 0$. Deviations of these values are assumed to be of order M_W^2/Λ^2 . Keeping only quadratically and more divergent terms we find the following contribution to the photon two-point function :

$$\begin{aligned} A_\mu(k)A_\nu(-k) : &\frac{e^2(\Delta\kappa)^2}{4(2\pi)^4 i} k^2(k^2\delta_{\mu\nu} - k_\mu k_\nu) \int \frac{d^4p}{p^2(g(p^2)p^2 + M_W^2)^2} \\ &+ (k^2\delta_{\mu\nu} - k_\mu k_\nu).O(\Lambda^2) + O(ln(\Lambda)) \end{aligned} \quad (4)$$

We notice that the naively present quartic divergence disappears upon integration and contraction with the vertices. The second part of this correction (quadratic in the momentum) is just a wave function renormalization of the photon and therefore has no experimental consequences. The first part however modifies the form of the photon propagator and will lead to a deviation from QED predictions in experiments . For the simplest case of $g(p^2) = M_W^2/\Lambda^2$ it is given by :

$$A_\mu(k)A_\nu(-k) = \frac{e^2(\Delta\kappa)^2\Lambda^2}{64\pi^2 M_W^4} k^2(k^2\delta_{\mu\nu} - k_\mu k_\nu) \quad (5)$$

With the contribution (5) the photon propagator $W_{\mu\nu}(k)$ becomes in the Feynman gauge:

$$W_{\mu\nu}(k) = \frac{1}{k^2}\left[\frac{\Lambda_{exp}^2 \delta_{\mu\nu} - k_\mu k_\nu}{\Lambda_{exp}^2 - k^2}\right] \tag{6}$$

where:

$$\Lambda_{exp} = \frac{8\pi M_W^2}{e\Delta\kappa\,\Lambda} \tag{7}$$

Because of current conservation the $k_\mu k_\nu$ piece of the propagator does not contribute to cross-sections and we are left with an overall form factor $\Lambda_{exp}^2/(\Lambda_{exp}^2 - k^2)$. This formfactor can be determined by a precise measurement of the cross-sections for $e^+e^- \rightarrow \ell^+\ell^-$ or $\bar{q}q$, with ℓ any lepton and q any quark . In the experiments at PETRA these measurements have been done[5-7] and a bound $\Lambda_{exp} \gtrsim 200 GeV$ has been found. Combining this limit with (7) one has :

$$|\Delta\kappa(\Lambda/M_W)| \lesssim 33 \tag{8}$$

Implicit in the derivation of the bound (8) is the assumption that it is sensible to introduce an intrinsic structure to the W-boson , but not to the photon . This assumption is justified in composite models where the W-boson , being massive , is composite but the photon is still a fundamental gauge particle . Without this assumption there is no reason to exclude a "bare" term in the Lagrangian , due to the compositeness of the photon, that can partially or completely cancel the contribution of formula (4) . In that case the bound (8) is not rigorous and one can only say that large deviations of (8) are unlikely , because they imply an unnatural cancellation of terms . Similar assumptions also have to be made in order that formula (2) be rigorously valid .

With the previous caveat formula (8) provides a usefull limit on $\Delta\kappa$ as a function of the compositeness-scale Λ . For large cut-off scales $\Delta\kappa \rightarrow 0$ and essentially no deviation of a gauge-structure is allowed . This is in keeping with the so-called

Veltman theorem[8] , which states that the effective low-energy theory of composite states with a mass much smaller than their binding scale has to be renormalizable, if they are to be described by perturbation theory . Because of the stronger cut-off dependence the new bound (8) becomes stronger than the bound from $(g-2)_\mu$ for a scale $\Lambda \gtrsim 5TeV$ (see table 1.) . Finally one could argue from the fact that no substructure of the W-boson has been seen so far, that $\Lambda \gtrsim 100GeV$, giving a rather useless bound of $|\Delta\kappa| \lesssim 26$. This argument is fallacious however, because formula (4) shows that the cut-off dependence appears only in the longitudinal structure $g(k^2)$, for which present experiments are insensitive .

References

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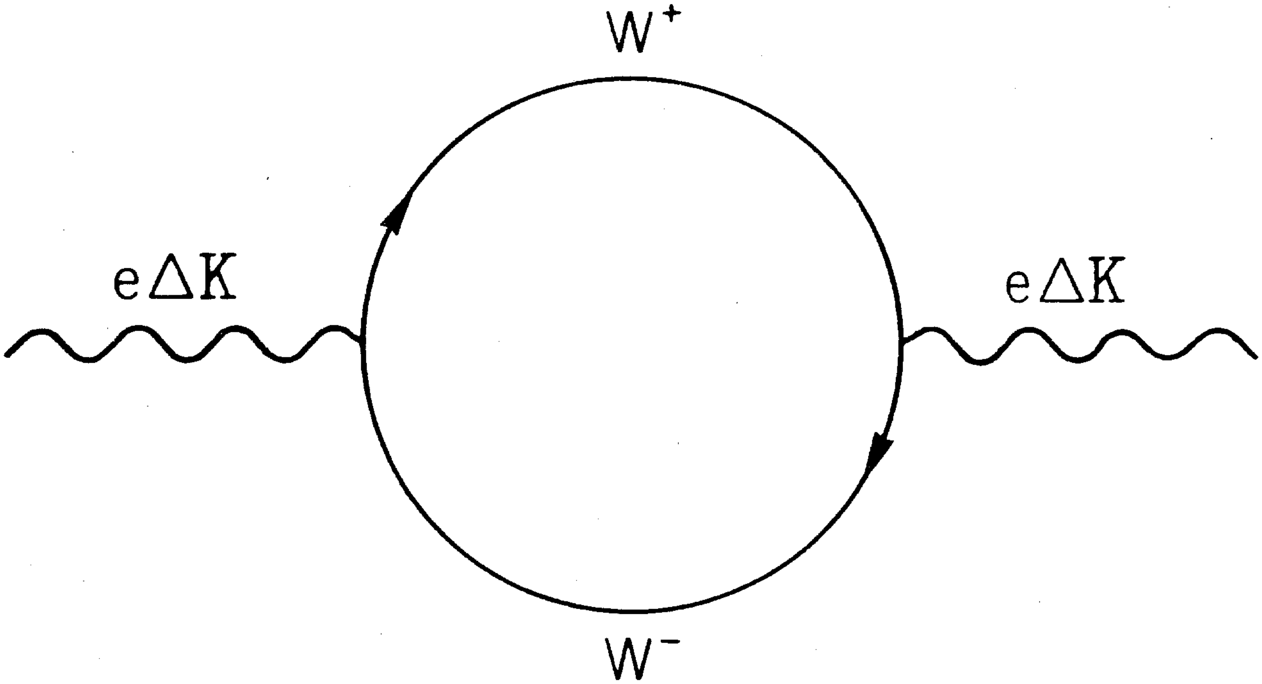
$\Lambda(TeV)$	$(g\text{-}2)_\mu$	γ-structure
1	0.87	2.6
3	0.61	0.88
5	0.53	0.53
10	0.46	0.26
20	0.40	0.13

Table 1: Limits on $|\Delta\kappa|$ from different experimental quantities.

Figure Captions

Fig. 1: Contribution to photon structure due to the magnetic moment of the W-boson.



W+
eΔK
eΔK
W−