Developments in numerical relativity and gravitational wave observations

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This paper provides a summary of the fourteen talks that were presented in the session GW4 on various aspects of numerical relativity and computation concerning gravitational waves.

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1. Introduction

A total of fourteen talks were presented over two days, covering a number of different topics within numerical relativity and computation concerning gravitational waves (GWs). Since the authors of each talk are publishing their work in these Proceedings or elsewhere, the purpose of this article is not to present any of the technical details of the talks. Rather, this session summary is aimed at presenting an overview of, and background information about, the various topics covered. As such, it is intended to be accessible to those working on other aspects of general realativty and astrophysics.

2. Inclusion of null infinity

Numerical relativity simulations of events such as compact object mergers, are normally carried out in a finite region, and the GW emission is estimated from the geometry near the outer boundary. However, because of the nonlinear nature of the Einstein equations, GWs are unambiguously defined only at future null infinity. Now the Einstein equations have the property that they can be compactified in a radial null direction and remain regular, so there are numerical approaches that compute GWs at future null infinity. The method commonly utilized is known as characteristic extraction, in which geometric data near the outer boundary of a merger simulation is used to provide boundary data to a characteristic code, i.e. one in which the spacetime is foliated by outgoing null cones. An alternative approach uses hyperboloidal slicing, in which the spacetime slices are spacelike in the central region, and become asymptotically null in the far region; however, this approach is less developed and has not yet produced gravitational waveforms.

Giannakopoulos *et al.*[1,2] presented results concerning the well-posedness of the Einstein equations in characteristic coordinates. In partial differential equation (PDE) theory, well-posedness in the standard L_2 norm is characterized by a system that is strongly (or symmetric) hyperbolic. However, it was shown that the characteristic Einstein equations form a PDE system that is only weakly hyperbolic. Further, it was demonstrated, using numerical examples, that weak hyperbolicity can have the effect of reducing the order of convergence of a code from that expected from considerations of the truncation error.

Gautam *et al.*[3] discussed a dual foliation formulation on hyperboloidal slices. There has been much effort over many years to implement a successful hyperboloidal scheme, but there remain problems of instability and of the need to regularize quantities that can become unbounded at null infinity. The essential idea presented is that these problems can be resolved by making a nonlinear change of variables, and the approach has been tested and shown to work in the special case of spherical symmetry. The approach is currently being extended to the general case, and if successful, this would be a significant achievement.

3. Surrogate models

Surrogate models have become an essential tool in searches in detector data for GWs from binary black hole mergers. Numerical relativity provides the most accurate waveforms, but they are computationally expensive and can take over a month for a single waveform. Since the merger phase space has, in principle, up to 7 dimensions, it is not feasible to populate it with numerical relativity waveforms. A surrogate waveform can be regarded as a mapping from (a restricted part of) the merger phase space to give a waveform. The quality of the model depends on how the mapping function is parametrized, and then fixing the parameter values so as to be consistent with known numerical relativity waveforms.

Islam *et al.*[4] reported a surrogate model for waveforms produced when the orbit is eccentric rather than circular. Eccentricity has long been neglected in waveform calculations, because it is well known that the reaction to GW emission damps out eccentricity at a rate that is much faster than that of the inspiral. However, there are possible astrophysical scenarios which could lead to the orbit being eccentric near merger, and the identification of such an event would be significant in terms of the understanding of formation channels of binary black hole systems. The model reported is restricted, at this stage, to non-spinning equal-mass black hole binary systems.

4. Effect of matter on GW propagation

As GWs pass through matter, some energy will be transferred to the matter causing the GW to be attenuated, but in astrophysical scenarios this effect is so small that it has been completely negelcted. However, recent work has shown that matter can affect GW propagation in other ways, i.e. as a phase shift or as an echo. The possibility of a GW echo is interesting because there are (controversial) reports that echoes have been observed in GW data, caused by the remnant being some form of exotic compact object (ECO).

Naidoo *et al.*[5] applied a model of a GW source inside a shell of matter to GW events for which there are claims of echo observations. It was shown that any echo could not have been produced by the effect of a matter shell, since if it were the shell would have to be so massive as to constitute a black hole. Thus, confirmation of an echo in GW data would provide evidence in favour of the ECO hypothesis. However, it was also shown that there are astrophysical events, such as a binary neutron star merger and especially a core collapse supernova, for which the echo and phase-shift effects would alter the GW signal sufficiently for the difference to be measurable.

5. Hadron-quark phase transition in a binary neutron star merger

During the merger of two neutron stars, the matter becomes denser than nuclear matter and with a temperature of order $10^{12}\,^{\circ}$K, and it is feasible that a phase transition from hadronic matter to quark matter may occur. Such a phase transition would change the effective equation of state, and therefore would affect the motion of the matter and thus the emitted GW signal.

Hanauske *et al.*[6] considers whether a hadron-quark phase transition could lead to an observational signature of the event in the GW signal.[7–10] Computational models for a number of possible post-merger scenarios are constructed, and results obtained for the GW signal in runs to about 20ms. The results show that an increase in the frequency of the post-merger GW signal would indicate the occurrence of a phase transition.

6. Alternative theories of gravity

General relativity is consistent with every observation and experiment to date. However, there are reasons for investigating alternative theories. General relativity is not consistent with quantum mechanics, so it needs to be modified in a way that would affect its properties at a very small scale. On the other hand, at the very large scale of cosmology, the issues of the unknown nature of dark matter and dark energy may be resolved by using an alternative theory. Clearly, any alternative theory would need to be consistent with observations including those by LIGO/Virgo, and that motivates the need to calculate GW emission from compact object coalescence in the theory.

Lim *et al.*[11] reported the construction of a numerical code for an effective field theory with a Lagrangian that is that of general relativity plus terms that are quadratic in the Ricci tensor. The resulting field equations involve up to 4^{th} order derivatives. The code uses the harmonic gauge, and is restricted to the case of spherical symmetry. It has been tested with initital data that is Minkowski or Schwarzschild plus noise, and has been shown to be stable and convergent.

Khlopunov *et al.*[12] calculates the GW emission due to a system of two point masses on a 3-brane embedded in a 5-dimensional spacetime, using analytical methods. In the non-relativistic limit, a generalization of the quadrupole formula is obtained.

7. GW memory

The memory effect in GW theory is well-known. Consider a system of free particles, each particle being at rest relative to the others, in a region of spacetime that can be regarded as initially Minkowskian. Supose that the particles are now perturbed by GWs, and that afterwards the spacetime region is again Minkowskian. Then the particles will be at rest relative to each other, but in general the positions of the particles relative to each other will have changed.

Grant *et al.*[13,14] generalizes the memory concept to the case that the particles have initial relative velocity and acceleration (due to forces other than gravity), and obtains formulas in terms of the GWs and non-radiative quantities for the memory-like changes to these quantities. In the case of zero relative velocity and acceleration, the formulas reduce to what is already known.

8. Searches for intermediate mass black hole (IMBH) mergers in GW data

IMBHs are usually defined as being in the mass range $10^2 M_\odot \leq M \leq 10^5 M_\odot$. They are less massive than the supermassive black holes at galactic centres, and too massive to have been formed by stellar collapse, because the pair instability gap means that stellar collapse cannot produce a black hole in the mass range $50M_\odot \lessapprox M \lessapprox 130M_\odot$. The event GW190521 is estimated to have been caused by the merger of black holes with approximate masses $85M_\odot$ and $66M_\odot$ yielding a remnant with mass about $142M_\odot$, providing conclusive evidence for the existence of IMBHs. This observation has motivated searches that focus on identifying IMBH events in LIGO/Virgo data.

PyCBC is a free and open software package used to search for GW events in detector data. Chandra *et al.*[15] reported a PyCBC-based search that is optimized to identify IMBH events. The search re-identified GW190521 with a much better false alarm rate than that of the original search.

Gayathri *et al.*[16] presented, on behalf of the LIGO, Virgo and KAGRA collaborations, the results of their searches for IMBH events in the observing period O3.

9. Searches for a GW signal of a spinning neutron star

An object spinning with frequency f, unless it is exactly axisymmetric, emits GWs at frequency $2f$. Pulsars are spinning neutron stars that emit regular radio pulses, and thus have rotational frequencies that are accurately determined. However, searches for GW signals from a number of pulsars have not yielded any positive result, and have only been able to place upper bounds on the magnitude of such signals. It is well-known that glitches occur in the radio pulses, indicating that some form of deformation is occurring in the neutron star. Thus it can be expected that there are periods of time during which the neutron star would be emitting GWs, but when such periods occur and their duration is not known.

Fesik *et al.*[17,18] presented a method for localising a long-duration signal in GW data, that is, to estimate the start time and duration of the signal. The method applies when the signal duration is much longer than the wave period, so more than a few hours. This knowledge can then be used to improve the signal-to-noise ratio of the recovered signal.

10. Machine learning

In recent years, the use of machine learning has become more and more commonplace throughout science, and this certainly includes applications to GW calculations and GW data analysis. The key advantages of machine learning are that: (a) in many circumstances it has proven to be remarkably accurate; and (b) there are now a number of software packages available, so from a user perspective, implementation can be straightforward. A common application of machine learning is the construction of a function that maps an input vector $\mathbf{x}$ to an output vector $\mathbf{y}$, i.e $\mathbf{y} = \mathcal{F}(\mathbf{x})$. We need to know the expected output $\mathbf{z}_i$ in a number of cases, and then construct $\mathcal{F}$ by minimizing the error $||\mathbf{z}_i - \mathbf{y}_i||$; this process is called *training* the network, and the quality of the network is described by the magnitude of the error, particularly for data points that are not used in the training.

Mishra[19] applied machine learning to search for a coherent WaveBurst in GW detector data. The network is trained on calculated waveforms of binary black hole mergers; the complete waveform is not used, but rather certain statistics representing the waveform are calculated, and these are used for the training. The resulting network is reported to improve the detection of binary black hole merger events by between 15% and 25%.

Field *et al.*[20,21] use machine learning to determine orbital dynamics of binary black hole systems from GW measurements. The system is written in the form of a universal differential equation, $\dot{\mathbf{x}} = \mathcal{F}(\mathbf{x})$, where $\mathbf{x}$ is a vector with components representing various aspects of the system dynamics. Once a choice of $\mathcal{F}$ is made, $\mathbf{x}$ is determined and then the quadrupole formula is applied to find the emitted GWs. Given observed (or numerically calculated) GW data from a binary black hole system, the next step is network training, i.e. to find $\mathcal{F}$ by minimizing the mismatch between this data and that associated with the choice made for $\mathcal{F}$. It was found that this model makes quite accurate waveform predictions outside the time interval over which the model is trained.

References

1. Thanasis Giannakopoulos, Nigel T. Bishop, David Hilditch, Denis Pollney and Miguel Zilhao, *Hyperbolicity of General Relativity in null foliations*, Book of Abstracts, p. 325 (2021).
2. Thanasis Giannakopoulos, David Hilditch and Miguel Zilhao, *Hyperbolicity of General Relativity in Bondi-like gauges*, Phys. Rev. D **102**, 064035 (2020).
3. Shalabh Gautam, Alex Vañó-Viñuales, Edgar Gasperin, David Hilditch and Sukanta Bose, *Einstein Field Equations in Spherical Symmetry on Hyperboloidal Slices: Dual Foliation Formulation*, Book of Abstracts, p. 324 (2021).
4. Tousif Islam, Vijay Varma, Jackie Lodman, Scott Field, Gaurav Khanna, Mark Scheel, Harald Pfeiffer, David Gerosa and Lawrence Kidder, *Eccentric binary black hole surrogate models for the gravitational waveform andremnant properties: comparable mass, nonspinning case*, Book of Abstracts, p. 328 (2021).
5. M Naidoo, Nigel T. Bishop and Petrus van der Walt, *Matter shells modifying gravitational wave signals*, Book of Abstracts, p. 328, and in these Proceedings (2021).
6. Matthias Hanauske, Gloria Montana, Lukas Weih and Luciano Rezzolla, *Gravitational-wave signatures of the hadron-quark phase transition in binary compact star mergers*, Book of Abstracts, p. 326 (2021).
7. M. Hanauske and L. R. Weih, *Neutron star collisions and gravitational waves*, Astronomische Nachrichten **342**, 788 (June 2021).
8. L. R. Weih, M. Hanauske and L. Rezzolla, *Postmerger Gravitational-Wave Signatures of Phase Transitions in Binary Mergers*, Physical review letters **124**, p. 171103 (May 2020).
9. E. R. Most, L. J. Papenfort, V. Dexheimer, M. Hanauske, S. Schramm, H. Stöcker and L. Rezzolla, *Signatures of Quark-Hadron Phase Transitions in General-Relativistic Neutron-Star Mergers*, Physical review letters **122**, p. 061101 (February 2019).
10. G. Montaña, L. Tolós, M. Hanauske and L. Rezzolla, *Constraining twin stars with GW170817*, Physical Review D **99**, p. 103009 (May 2019).
11. Hyun Lim and Aaron Held, *Dynamics of Quadratic Gravity in Spherical Symmetry*, Book of Abstracts, p. 323 (2021).
12. Mikhail Khlopunov and Dmitry Galtsov, *Odd-dimensional gravitational waves from the binary system on three-brane*, Book of Abstracts, p. 329, and in these Proceedings (2021).
13. Alexander Grant and David Nichols, *Memory-like effects due to relative velocity and acceleration*, Book of Abstracts, p. 327 (2021).
14. Alexander Grant and David Nichols, *Persistent gravitational wave observables: Curve deviation in asymptotically flat spacetimes*, `https://arxiv.org/abs/2109.03832` (2021).
15. Koustav Chandra, Veronica Villa-Ortega, Thomas Dent, Connor McIsaac, Archana Pai, Ian Harry, Gareth Davies and Kanchan Soni, *An optimised PyCBC search for gravitational waves from intermediate-mass black hole mergers*, Book of Abstracts, p. 324, and in these Proceedings (2021).
16. V. Gayathri, The LIGO Scientific Collaboration, The Virgo Collaboration and The KAGRA Collaboration, *Intermediate mass black hole search in LIGO-Virgo's third observing period*, Book of Abstracts, p. 325 (2021).
17. Liudmila Fesik and Maria Alessandra Papa, *Localisation of a long-duration gravitational wave signal in time*, Book of Abstracts, p. 327 (2021).
18. Liudmila Fesik and Maria Alessandra Papa, *The detectability of long-duration gravitational wave signals*, Book of Abstracts, p. 327 (2021).

19. Tanmaya Mishra, *Optimization of model independent gravitational wave search using machine learning*, Book of Abstracts, p. 326 (2021).
20. Scott Field, Brendan Keith and Akshay Khadse, *Learning orbital dynamics of binary black hole systems from gravitational wave measurements*, Book of Abstracts, p. 323 (2021).
21. Brendan Keith, Akshay Khadse and Scott E. Field, *Learning orbital dynamics of binary black hole systems from gravitational wave measurements*, Phys. Rev. Research **3**, 043101 (2021).