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CP Symmetry Breaking, or the Lack of It, in the Strong Interactions *

Helen R. Quinn
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94309
E-mail: quinn@slac.stanford.edu

Abstract

I review the strong CP problem and its solutions

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1 Background: What causes CP Violation?

This is a review talk. I have nothing new to say on this subject. However I do think it is time that we began to think about it again, as it is intrinsically a problem for the Standard Model and all extensions of the theory, and should be included when we discuss the challenges for that theory.

I begin with some pedagogical background. I want to stress something that has been implicit in a number of previous talks. CP violation arises from complex *relative* phases of coupling constants in a physical theory. I begin by reminding you of how this works in a simple case, namely what is called direct CP violation. The reason I do this is simply to give a context in which to discuss how CP violation can occur in the QCD theory of strong interactions, and the lengths we seem to have to go to to prevent it. CP violation can occur in a decay process if two distinct amplitude terms contribute to the decay rate and the coupling constant coefficients in these terms have different phases ϕ_i [1]. In addition, we need some other phase in the problem before we are sensitive to this CP violating phase difference of couplings.

Now this seems a fairly simple statement, so the younger among you may wonder why it was that for so many years, from the time of Dirac's equation until CP violation was experimentally discovered in K decays [2], we only considered field theories in which there were no such phase differences of couplings and hence no possible CP violation. The clue is that in the Dirac equation and in pure QED CP symmetry is automatic. Gauge invariance requires that there is only one universal gauge coupling and Hermiticity of the Lagrangian automatically makes the gauge coupling real. In addition any fermion mass term can be made real by a chiral phase rotation of that fermion field, which has no impact on gauge coupling terms.

Indeed it turns out that, since all fields can be arbitrarily redefined by phase rotations, most simple field theories with only a few species of particles have no possible CP violation. If you write the most general Lagrangian with all couplings arbitrarily complex, you can use the freedom of phase redefinition of fields to make all couplings real. (If you calculate any rate using the complex form of the couplings you find the rates are all independent of these phases, as they must be if you can redefine the phases away.) However, except for gauge couplings, as you add new species of particles to your theory, you get more possible new independent couplings than the number of added fields, and so, with a sufficiently rich theory, CP violating coupling phases can appear.

Those who study CP violation in the quark sector will tell you that the two generation standard Model (with massless neutrinos) has no possible CP violation while with three generations there is one remaining phase in the CKM matrix that controls all CP violating effects. This is true, provided only that you have first gotten rid of a peculiar type of CP violation that is intrinsic to QCD, and that is the subject of my talk.

2 What about QCD?

The Lagrangian of QCD looks very similar to that for QED, so one might think that my argument about gauge invariance and Hermiticity applies there too. Indeed the gauge coupling must be real. However, there is an important difference. The set of possible gauge rotations for a non-Abelian gauge theory include a have a type of transformation that does not occur for Abelian (QED) case. These are called "large" gauge transformations. They have the property that they cannot be smoothly deformed to the identity. As was pointed out long ago [3] this means that there are a set of gauge-inequivalent zero energy states, or vacua. We can label them by an integer $|n\rangle$ that can take any negative or positive value.

Furthermore there are finite energy physical events, known as instantons [4], which correspond to a quantum tunnelling between these states $|n\rangle \rightarrow |n+1\rangle$. This being so, the $|n\rangle$ states, including the state $n = 0$ that corresponds to everywhere zero gauge field, are not acceptable physical ground states. To define a possible physical ground state one must choose a gauge invariant superposition of n-vacua

$$|\theta\rangle = \Sigma_n e^{in\theta}\ |n\rangle\ . \tag{1}$$

If one evaluates all physical amplitudes in such a state this is equivalent to having an additional effective term in the Lagrangian which takes the form $i\theta\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$. Notice that this is a CP violating $E.B$ type term with a complex coefficient. One more fact that you need to know about the instanton events is that they are chirality changing. The instanton gauge field configuration has fermionic zero modes (for massless fermions). An instanton event produces fermions of one chirality or absorbs those of the opposite chirality.

However I said that only relative phases give physical effects, so why do I worry about the phase of this term. What is the phase that it can compete against to give a physical consequence? The clue is in the statement that the instanton events produce fermions of a given chirality. I can thus change the phase of this term by redefining the chiral phase of the massless fermion fields, and thus, I can rotate away the phase θ by a chiral rotation of a fermion field. If this rotation does not make some other coupling constant complex then I have removed the possible CP violating effects.

This last statement is usually phrased as saying that if there is a single massless quark chiral rotation can set θ_{eff} to zero with no other consequences. We will revisit this in a short time, since it a bit slippery as a statement of the required condition. However if there are only massive quarks around then we see that there can be CP violations in strong interaction processes, proportional to the phase difference

$$\theta_{\text{eff}} = \theta\ - (\text{phase of quark mass matrix})\ . \tag{2}$$

I cannot change this difference by any chiral rotation. Such rotations simply move the phase between these two terms without altering their difference.

We know $\theta_{\rm eff}$ must be very small, since such a term would induce an electric dipole moment for the neutron. There are very stringent experimental limits on the size of any such effect [5]. With today's constraints this gives a limit $\theta_{\rm eff} \leq 10^{-10}$ [6]. The strong CP problem is thus posed as a question: why is this quantity so small?

There are three types of answer to this question. I will now discuss them all. Because I am biased, I will first tell you the history of the answer that Roberto Peccei and I suggested in 1977 [7], which is still viable today, though in somewhat altered form. I will then talk about the other options and give my opinionated evaluations of them. Finally I will return to the question of experimental consequences of what is now known as PQ symmetry. For a much more pedagogical review of the problem of strong CP violation see the TASI lectures on the subject by Michael Dine [8].

3 Peccei and Quinn 1977

The question that bothered Roberto and me in 1977 arose from the somewhat misleading statement that the θ parameter is irrelevant if quarks are massless. I knew that quarks are massless in the hot early Universe. So we puzzled about this: how can θ parameter be irrelevant in the early Universe but relevant later on when quarks get a mass? Hidden in the answer to this question was the key to our solution to the strong CP problem, so cosmology provided the guide to this particle physics idea.

The answer to the question is of course that a chiral rotation, even in the high temperature phase of the Standard Model is not innocuous, any such rotation changes the phases of some Higgs-fermion Yukawa couplings, since these involve different left and right-handed fermion multiplets. The fermion masses, when they appear, are a product of a Yukawa coupling times the Higgs field vacuum value and thus inherit some knowledge of these Yukawa coupling phases. Indeed, unless the Yukawa phases are exactly cancelled by the phase of the vacuum expectation value of the Higgs field in the fermion mass determinant we see that there can be a non-zero $\theta_{\rm eff}$ in the low temperature phase of the theory even if we began by rotating the initial θ to zero in the high temperature (massless) phase.

So then we asked what fixes the phase of the Higgs field vacuum value? Does this depend on θ in any way? If it does, can we devise a theory so that, for any θ, the Higgs field will pick as its vacuum value just the right phase so that $\theta_{\rm eff}$ is zero. The first answer is that, in the Standard Model, there is no θ dependence to the minimum of the Higgs potential, even when instanton effects are taken into account. However we soon saw that one can devise a situation where the minimum of the Higgs potential occurs for $\theta_{\rm eff} = 0$. This requires adding an additional Higgs doublet and an additional U(1) quasi-symmetry to the theory. This is now called PQ symmetry.

It is easiest to see how this works by considering the dilute gas approximation for instantons, and integrating out the fermions. One then gets instanton terms in the Higgs effective potential. In the Standard Model the Higgs doublet field is coupled to up-type quarks while the corresponding coupling for down-type quarks is to the complex conjugate of the Higgs doublet. The consequence is that, when all quarks are integrated out, one gets an instanton effect in the potential that is proportional to

$$\Delta V = G e^{i\theta} \phi^{N_f} (\text{up quarks}) \phi^{*N_f} (\text{down quarks}) + \text{hermitian conjugate} . \tag{3}$$

Clearly the θ dependence does not influence the phase of the Higgs field, the Higgs potential, even including the instanton effects has a perfect U(1) symmetry. At the end of the day θ_{eff} is arbitrary in the Standard Model.

However this story obviously leads to our solution (because I am telling you the thought pattern that got us there). All we have to do is add an additional Higgs multiplet and devise the quantum numbers of the fermions and the two Higgs fields under the new U(1) so that the up and down quark types couple to different combinations of the two Higgs multiplets. (In our first, simplest, version we chose to couple one Higgs multiplet to only up-type right-handed quarks and the other only to down-type right handed quarks.) The consequence is that the vacuum values of both Higgs fields then do depend on θ_{eff}, and in just such a way that the minimum of the Higgs effective potential occurs exactly at $\theta_{\text{eff}} = 0$. This tells us that the θ-dependent terms do not respect the additional U(1) symmetry that we imposed on the Lagrangian, which is why I called it a "quasi-symmetry".

There are two other ideas discussed in the literature as ways to achieve a very small θ parameter. One is to chose the Yukawa coupling parameters so that, even in the low temperature phase, one quark (presumably the up quark) is massless. This solution seems a bit hypothetical, as we know from current algebra that the masses of the pseudoscalar mesons are inconsistent with zero quark masses. However perhaps one has to be more careful; current algebra tells us about the quark masses at some low scale, whereas the (renormalized) Yukawa couplings define the quark masses at some high scale. Lagrangian quark masses are not physical quantities. Could it be that the physical up quark mass is all due to renormalization effects between the two scales? It seems now that this answer is strongly disfavored by lattice calculations [9].

Furthermore, once again, just as in the high temperature phase, we have to be careful about what we mean by zero quark mass. If we adjust the θ parameter to zero at a scale where the quark mass is zero, but renormalization effects then reintroduce a quark mass they probably also reintroduce an effective θ parameter. Indeed recent papers have questioned that there is any physical meaning to the concept of a single massless quark in the Standard Model theory because of the renormalization prescription dependence of this concept [10]. I conclude from all this that this answer is not an acceptable solution to the problem.

The second answer is to say that we impose CP symmetry on the Lagrangian, which includes imposing the choice $\theta = 0$ for the QCD θ-vacuum. However we know that CP is not an exact symmetry, so in this approach the weak CP violation arises via spontaneous breaking of this symmetry. This will then induce a non-zero effective θ parameter. If the theory is carefully constructed (and indeed with Standard Model weak interaction structure) this is a multi-loop effect, and thus does not violate the experimental bound on θ. Recent results on weak CP violation in B decays tell us that the weak phase in the CKM matrix is not small. Some models of this type predicted small phases and are thus excluded. However there are viable extensions of the Standard Model which use this method to avoid the strong CP problem [11].

Personally I find this type of answer quite unsatisfactory. (This is a statement of taste, not a solid physics argument so you should take it with whatever grains of salt you wish.) It seems to me that although we may specify the symmetries of the Lagrangian, for any Lagrangian with QCD all possible QCD θ-vacua exist. Requiring $\theta = 0$, that is saying we can only be in in a particular one of these ground state, seems to me to be somehow different from fixing other Lagrangian parameters by symmetry arguments. Thus, quite aside from my taste about the complexity of models needed to achieve this solution given current constraints, my taste about what it means to be in a given choice of θ makes me personally not enthusiastic about this answer to the problem. However I believe it is technically a viable answer.

Now let me return to discuss the current status of the $U(1)_{PQ}$ solution. We added an additional U(1) quasi-symmetry to the Lagrangian, which is broken by the Higgs vacuum expectation values. This means that there is an additional quasi-Goldstone boson in this theory, the particle known as the axion. Roberto and I missed this; Weinberg [12] and Wilczek [13] quickly pointed it out.

It is interesting to look back and see what we wrote about this, in our 1977 paper; talking about the fact that the θ-dependent terms break the U(1) symmetry, we said: [7]

"This same lack of U(1) symmetry was noted by 't Hooft and explains the lack of a Goldstone boson in this theory even though one might naively have expected to find one. In a more physical model it explains the absence of a ninth light pseudoscalar meson."

Of course we should immediately have realized that the "naively expected" particle did not just vanish, that there was still an "almost goldstone" particle corresponding to this almost symmetry. I can only partially reconstruct our thinking. One thing I do remember is that neither Roberto nor I took the particular model that we wrote down very seriously, we thought of it as an illustration of the general idea, not a unique realization of it, and hence we did not care much about its phenomenology. We expected it would take some more careful model building to get an experimentally viable realization of this symmetry idea. We knew that a true goldstone boson would be a disaster, but we were not too concerned about anything else the model predicted. Of course, the axion is not model specific, any realization of our symmetry idea will have one, but somehow we did not go the second step to see that. The second sentence I quote above may also be a partial explanation, somehow we have our new U(1) problem—the axion, mixed up in our minds with the old U(1) problem—the lack of a ninth light pseudoscalar meson. Indeed in our theory there are three U(1) symmetries, the usual one whose goldstone boson is eaten by the Z, our additional one that gives the axion, and the last, in the quark sector, broken both by the quark masses and instanton effects, to give a massive ninth pseudoscalar meson.

The axion of our initial model was quickly ruled out because it was not seen in accelerator searches [14]. This led to the formulation of "invisible axion" versions of $U(1)_{PQ}$, where the accelerator limits are avoided by contriving a very weakly-coupled axion. There are two invisible axion theories that are commonly studied, that of Kim, Shifman, Vainshtein and Zakharov [15] and that of Dine, Fischler, Srednicki, and Zhitnitskii [16].

Such a particle has both astrophysical and cosmological implications [17]. For example any light weakly-coupled particle provides a mechanism for cooling the interior of stars, and hence long-lived stars, such as red giants, put limits on its parameters. Cosmologically it is an interesting (and I would say somewhat neglected) candidate for dark matter. Indeed the constraint that the axion density is less than or equal to the observed dark matter density puts another, and complementary, limit on axion parameters. Between these limits there is a restricted region in which the axion is still viable, so it is reasonable to search for axions in this range. The actual values of mass and coupling and their relationships are model dependent.

If axions are indeed the major constituent of dark matter, then our own galactic halo is composed of axions. Pierre Sikivie [18] suggested an ingenious experiment to be sensitive to these low-energy and weakly-coupled particles. In both surviving models the axion couples to two photons. The decay of an axion can thus be stimulated by providing a finely tuned cavity with a strong magnetic field in it, the tuning of the cavity picks a particular axion mass. This experiment [19] has been going on for some time now at Livermore (LLNL) with recent cleaver improvements to increase its sensitivity. It is now beginning to probe the interesting region, indeed its latest results rule out the presence of KSVZ axions at halo density, and limit the DFSZ density to less than about ten times halo density. This work has had limited support, and I am always disappointed when reviews of dark matter searches do not mention it, but concentrate only on searches for neutralinos or other WIMP (weakly interacting massive particle) candidates. It is a beautiful experiment. Axions as a dark matter candidate are still viable; this experiment may either find them or eliminate these models as a possibility before too many more years go by.

Having expressed my frustration at the neglect of this experiment in my talk, I was embarrassed, but pleased, to be told in the discussion time that there is another interesting axion search experiment—the CAST experiment at CERN [20]. This search is based on the astrophysical production of axions in the core of a star, in this case the sun. Using a large magnet at CERN as an axion telescope this experiment has devised detectors sensitive to x-ray photons from the stimulated decay of the axions from the sun in the magnet. The sensitivity of this experiment is also beginning to probe interesting parameter regions.

This story is not over. We know that the CP violation in strong interactions is very small, and in the context of the Standard Model this remains a puzzle. It is a puzzle that connects the weak interaction sector (in which I include all Higgs-fermion couplings) and the strong interaction sector and thus is usually ignored when thinking about either, but every so often we need to remind ourselves to step back and think about the full story.

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