A Comment on the Collective Excitation in $C^{12}(p, p')C^{12*}$ Reaction

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(Received September 2, 1968)

Differential cross sections and polarizations are calculated for the inelastic scattering of high energy nucleons from the collective excited states of C^{12}, $E_{exc}=4.43$ Mev 2^+ $T=0$, 9.63 Mev 3^- $T=0$ and 22.3 Mev 1^- $T=1$, using one particle-hole and one hole-particle amplitudes which are obtained by Goswami and Pal, and Gillet and Vinh Mau.

In the framework of plane wave impulse approximation (P. W. I. A.), polarizations are well reproduced, but cross sections are not.

§1. Introduction

Recently, according to the works of collective mode in the excited states of nuclei, the excited states of light nuclei have been described by the method of configuration mixing in terms of one particle-hole and one hole-particle amplitudes.

Fallieros and Ferrell[1] indicated that in order to explain the enhancement of electric multipole transition rate, one needs a correlation effect in the ground state. The treatment of the ground state correlation was investigated by Kobayasi and Marumori, Baranger and so on.[2]

Gillet and Vinh Mau[3] used a Gaussian potential for two body interaction in nucleus and succeed to fit a lot of theoretical collective levels of C^{12} and O^{16} to the experimental ones. They referred to the case of taking no account of the correlation effects in the ground state as approximation I and the case of taking account of the correlation effects in the ground state as approximation II.

Goswami and Pal[4] used the Yukawa type for the central part of potential with Soper exchange mixture for two body interaction. They determined the parameters of potential to fit the calculated energy level of the first excited state to the experimental one, $E_{exc}=4.43$ Mev 2^+ $T=0$. They investigated some other collective excited states of C^{12} and computed the energy levels corresponding to experimental energy levels, $E_{exc}=9.63$ Mev 3^- $T=0$ and $E_{exc}=22.3$ Mev 1^- $T=1$, using this potential. Their theoretical values are somewhat larger than the experimental values, viz., the value 11.58 Mev has been obtained for the level 3^- and the value 25.1 Mev for

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the level 1^-. The foregoing amplitudes and backgoing amplitudes of Goswami and Pal for these levels are considerably different from Gillet and Vinh Mau's.

Gillet and Melkanoff[5] computed the differential cross sections of the inelastic scatterings of $C^{12}(e, e')C^{12*}$, $O^{16}(e, e')O^{16*}$ and $Ca^{40}(e, e')Ca^{40*}$ using the amplitudes obtained by Gillet and vinh Mau. They have obtained results which are in fairly well agreement with experimental data.

Haybron and McManus[6], and Lee and McManus[7] exhaustively investigated the differential cross sections and polarizations in the high energy nucleon inelastic scatterings from many levels in $C^{12}(p, p')C^{12*}$, $O^{16}(p, p')O^{16*}$ and $Ca^{40}(p, p')Ca^{40*}$ using the amplitudes of Gillet and Vinh Mau as well as Gillet and Melkanoff. In their case, the energy of incident proton was 156 Mev and the distorted wave impulse approximation (D. W. I. A.) was adopted. According to their results, D. W. I. A. was better than the plane wave impulse approximation (P. W. I. A.) for the cross sections, but for the angular distribution of polarization, the results obtained by D. W. I. A. did not agree with the experimental ones rather than the results by P. W. I. A..

The main purpose of this paper is that assuming P. W. I. A. is a good approximation and using the foregoing and backgoing amplitudes of Gillet and Vinh Mau and of Goswami and Pal, we analyze the cross sections and the polarizations of the inelastic scattering $C^{12}(p, p')C^{12*}$ $E_p=156$ Mev from the levels with $E_{\rm exc}=4.43$ Mev 2^+ $T=0$, 9.63 Mev 3^- $T=0$, and 22.3 Mev 1^- $T=1$, and investigate what difference for the cross sections and polarization arises from the difference of Gillet and Vinh Mau's and Goswami and Pal's foregoing and backgoing amplitudes.

We used the two body scattering amplitudes of Kerman, McManus and Thaler[8] for the matrix element of two body interaction between incident and target nucleons.

§2. Cross section and polarization

Using P. W. I. A., the scattering spin matrix is given by

$$T_{JM}=\langle\Psi_{JM}|\sum_{i=1}^{12}M_{i0}e^{-i\boldsymbol{q}\cdot\boldsymbol{r}_i}|\Psi_{00}\rangle, \tag{1}$$

where $|\Psi_{00}\rangle$ and $|\Psi_{JM}\rangle$ are the state vector of the ground state which includes the correlation effects and the state vector of excited state in C^{12}, respectively. M_{i0} is the spin matrix of two body scattering which is caused by the interaction between the i-th particle of C^{12} and the incident nucleon.

In either case of Gillet and Vinh Mau and Goswami and Pal, the excited state is created by operating a phonon creation operator on the ground state. This creation operator is represented by the linear combination of one particle-hole creation operator and it's destruction operator. Then, in order to use their method, it is very convenient to consider $\sum_{i=1}^{12} M_{i0}e^{-i\boldsymbol{q}\cdot\boldsymbol{r}_i}$ at Eq. (1) as an oprator and to represent it by the method of second quantization. We write it as follow,

$$\sum_{i=1}^{12} M_{i0}e^{-i\boldsymbol{q}\cdot\boldsymbol{r}_i} \rightarrow Q' = \sum_{\alpha,\beta} \langle\alpha|Me^{-i\boldsymbol{q}\cdot\boldsymbol{r}}|\beta\rangle a_\alpha^+ a_\beta, \tag{2}$$

where α and β are the quantum numbers, $n(ls)jm\tau m_\tau$, which represent the j-j coupled single particle state. M is the spin matrix of two body scattering which is defined by Kerman, McManus and Thaler. One can represent this M as the following:

$$\begin{aligned}
&M = M(\gamma) + M(\delta)(\boldsymbol{\tau}_0 \cdot \boldsymbol{\tau}_i), \\
&M(\gamma) = M_0(\gamma) + \sum_\mu M_\mu^{1+}(\gamma)\sigma_\mu^1, \quad (\mu = 1, 0, -1) \\
&M(\delta) = M_0(\delta) + \sum_\mu M_\mu^1(\delta)\sigma_\mu^1,
\end{aligned} \tag{3}$$

where

$$\sigma_1^1 = -\frac{1}{\sqrt{2}}(\sigma_n + i\sigma_p),$$

$$\sigma_0^1 = \sigma_q,$$

$$\sigma_{-1}^1 = +\frac{1}{\sqrt{2}}(\sigma_n - i\sigma_p),$$

$$M_0(\eta) = A_\eta + C_\eta\sigma_{0n},$$

$$M_1^{1+}(\eta) = -\frac{1}{\sqrt{2}}(C_\eta + B_\eta\sigma_{0n} - iF_\eta\sigma_{0p}), \quad (\eta = \gamma, \delta)$$

$$M_0^{1+}(\eta) = E_\eta\sigma_0 q,$$

$$M_{-1}^{1+}(\eta) = +\frac{1}{\sqrt{2}}(C_\eta + B_\eta\sigma_{0n} + iF_\eta\sigma_{0p}),$$

$$A_\gamma = \frac{1}{4}(3A_1 + A_0), \quad A_\delta = \frac{1}{4}(A_1 - A_0), \quad \text{etc.}$$

A_1 etc. and A_0 etc. are the scattering amplitudes of spin triplet and of spin singlet respectively. These are defined in reference 8).

Using Eq. (3) for the two body scattering spin matrix, expanding $e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}$ in Legendre polynomials and substituting them into Eq. (2), we obtain the following form for Q';

$$\begin{aligned}
Q' = &\sum_L \sqrt{4\pi[L]}\,(-i)^L [M_0(\gamma) \sum_{\alpha,\beta} \langle\alpha|j_L(qr)Y_0^L|\beta\rangle a_\alpha^+ a_\beta \\
&+ \sum_\mu M_\mu^{1+}(\gamma) \sum_{\alpha,\beta} \langle\alpha|j_L(qr)Y_0^L\sigma_\mu^1|\beta\rangle a_\alpha^+ a_\beta] \\
&+ \sum_L \sqrt{4\pi[L]}\,(-i)^L [M_0(\delta) \sum_{\alpha,\beta} \langle\alpha|j_L(qr)Y_0^L \sum_{\mu'} \tau_{\mu'}^{1+}(0)\tau_{\mu'}^1(i)|\beta\rangle a_\alpha^+ a_\beta
\end{aligned}$$

$$+\sum_{\mu} M_{\mu}^{1+}(\delta)\sum_{\alpha,\beta}\langle\alpha|j_L(qr)\,Y_{0\sigma_\mu}^{L\,1}\sum_{\mu'}\tau_{\mu'}^{1+}(0)\tau_{\mu'}^{1}(i)\,|\beta\rangle a_\alpha^+ a_\beta],$$

where $\alpha=n_1(l_1s_1)j_1m_1\tau_1m_{\tau_1}$, $\beta=n_2(l_2s_2)j_2m_2\tau_2m_{\tau_2}$ and $[L]=2L+1$.

Using Wigner-Eckart theorem for the j-j coupled single particle matrix elements, we obtain more explicit form for Q';

$$Q'=\sum_L\sqrt{4\pi[L]}\,(-i)^L[M_0(\gamma)\sum_{\alpha,\beta}(n_1(l_1s_1)j_1\|j_L(qr)\,Y^L\|n_2(l_2s_2)j_2)(\tau_1\|1\|\tau_2)$$
$$\times(-)^{j_1-m_1+\tau_1-m_{\tau_1}}\begin{pmatrix} j_1 & L & j_2\\ -m_1 & 0 & m_2\end{pmatrix}\begin{pmatrix} \tau_1 & 0 & \tau_2\\ -m_{\tau_1} & 0 & -m_{\tau_2}\end{pmatrix}a_\alpha^+a_\beta$$
$$+\sum_\mu M_\mu^{1+}(\gamma)\sum_\lambda(1\mu L0|1L\lambda\mu)\sum_{\alpha,\beta}(n_1(l_1s_1)j_1\|j_L(qr)y^\lambda\|n_2(l_2s_2)j_2)(\tau_1\|1\|\tau_2)$$
$$\times(-)^{L+1-\lambda+j_1-m_1+\tau_1-m_{\tau_1}}\begin{pmatrix} j_1 & \lambda & j_2\\ -m_1 & \mu & m_2\end{pmatrix}\begin{pmatrix} \tau_1 & 0 & \tau_2\\ -m_{\tau_1} & 0 & m_{\tau_2}\end{pmatrix}a_\alpha^+a_\beta]$$
$$+\sum_L\sqrt{4\pi[L]}\,(-i)^L[M_0(\delta)\sum_{\mu'}\tau_{\mu'}^{1+}(0)\sum_{\alpha;\beta}(n_1(l_1(l_1s_1)j_2\|j_L(qr)\,Y^L\|n_2(l_2s_2)j_2)$$
$$\times(\tau_1\|\tau'\|\tau_2)(-)^{j_1-m_1+\tau_1-m_{\tau_1}}\begin{pmatrix} j_1 & L & j_2\\ -m_1 & 0 & m_2\end{pmatrix}\begin{pmatrix} \tau_1 & 1 & \tau_2\\ -m_{\tau_1} & \mu' & m_{\tau_2}\end{pmatrix}a_\alpha^+a_\beta$$
$$+\sum_\mu M_\mu^{1+}(\delta)\sum_{\mu'}\tau_{\mu'}^{1+}(0)\sum_\lambda(1\mu L0|1L\lambda\mu)\sum_{\alpha,\beta}(n_1(l_1s_1)j_1\|j_L(qr)y^\lambda\|n_2(l_2s_2)j_2)$$
$$\times(\tau_1\|\tau^1\|\tau_2)(-)^{L+1-\lambda+j_1-m_1+\tau_1-m_{\tau_1}}\begin{pmatrix} j_1 & \lambda & j_2\\ -m_1 & \mu & m_2\end{pmatrix}\begin{pmatrix} \tau_1 & 1 & \tau_2\\ -m_{\tau_1} & \mu' & m_{\tau_2}\end{pmatrix}a_\alpha^+a_\beta]. \tag{4}$$

According to Goswami and Pal, the state vector for the excited state is given by operating a phonon creation operator Q^+ on the state vector of the true ground state. Q^+ is defined by the following equation:

$$Q^+=\sum_k(\chi_kA_k^+-S_k\bar{\chi}_kA_{\bar{k}}^-), \tag{5}$$

where $S_k=(-)^{J-M+T-M_T}$, χ_k and $\bar{\chi}_k$ are a foregoing amplitude and a backgoing amplitude, respectively, which are given in reference 4). A_k^+ and A_k represent one particle-hole creation operator and its destruction operator, respectively. $\bar{k}$ means changing the sign of M in k, where M is a quantum number of z-component of total angular momentum.

The state vector for excited state is defined as follow

$$|\Psi_{JM}\rangle=Q^+|\Psi_{00}\rangle, \tag{6}$$

where $|\Psi_{00}\rangle$ is the state vector of the true ground state and satisfies a subsidary condition $Q|\Psi_{00}\rangle=0$.

One can obtain a transition spin matrix;

$$T_{JM}=\langle\Psi_{JM}|Q'|\Psi_{00}\rangle$$
$$=\sum_L\sqrt{4\pi[L]}\,(-i)^L\Big[M_0(\gamma)\sum_{\substack{n(ls)j\\ \bar{n}(\bar{l}\bar{s})\bar{j}}}\frac{(-)^{\bar{j}-j+1}}{\sqrt{2L+1}}(n(ls)j\|j_L(qr)\,Y^L\|\bar{n}(\bar{l}\bar{s})\bar{j})$$

$$\times(\tfrac{1}{2}\|1\|\tfrac{1}{2})((-)^{L}\chi+\bar{\chi})\delta_{JL}\delta_{M0}\delta_{T0}\delta_{M_T0}$$

$$+\sum_{\mu}M_{\mu}^{1+}(\gamma)\sum_{\lambda}(1\mu L0|1L\lambda\mu)\sum_{\substack{n(ls)j\\ \bar{n}(\bar{l}\bar{s})\bar{j}}}\frac{(-)^{\bar{j}-j+L}}{\sqrt{2\lambda+1}}(n(ls)j\|j_L(qr)y^{\lambda}\|\bar{n}(\bar{l}\bar{s})\bar{j})$$

$$\left.\times(\tfrac{1}{2}\|1\|\tfrac{1}{2})(\chi+(-)^{\lambda+1}\bar{\chi})\delta_{J\lambda}\delta_{M\mu}\delta_{T0}\delta_{M_T0}\right]$$

$$+\sum_{L}\sqrt{4\pi[L]}(-i)^{L}\left[M_0(\delta)\sum_{\substack{n(ls)j\\ \bar{n}(\bar{l}\bar{s})\bar{j}}}\frac{(-)^{\bar{j}-j+1}}{\sqrt{2L+1}}(n(ls)j\|j_L(qr)Y^{L}\|\bar{n}(\bar{l}\bar{s})\bar{j})\right.$$

$$\times\frac{(\tfrac{1}{2}\|\tau^1\|\tfrac{1}{2})}{\sqrt{3}}((-)^{L+1}\chi+\bar{\chi})\delta_{JL}\delta_{M0}\delta_{T1}\delta_{M_T0}$$

$$+\sum_{\mu}M_{\mu}^{1+}(\delta)\sum_{\lambda}(1\mu L0|1L\lambda\mu)\sum_{\substack{n(ls)j\\ \bar{n}(\bar{l}\bar{s})\bar{j}}}\frac{(-)^{\bar{j}-j+L}}{\sqrt{2\lambda+1}}(n(ls)j\|j_L(qr)y^{\lambda}\|\bar{n}(\bar{l}\bar{s})\bar{j})$$

$$\left.\times\frac{(\tfrac{1}{2}\|\tau^1\|\tfrac{1}{2})}{\sqrt{3}}((-)^{1}\chi+(-)^{\lambda+1}\bar{\chi})\delta_{J\lambda}\delta_{M\mu}\delta_{T1}\delta_{M_T0}\right], \qquad (7)$$

where unbarred quantum numbers mean the particle state ones and barred ones the hole state ones.

In the case of Gillet and Vinh Mau, a transition spin matrix is given by

$$T_{JM}=\sum_{L}\sqrt{4\pi[L]}(-i)^{L}\left[M_0(\gamma)\sum_{\substack{n(ls)j\\ \bar{n}(\bar{l}\bar{s})\bar{j}}}\frac{(-)^{\bar{j}+(3/2)}}{\sqrt{2L+1}}(n(ls)j\|j_L(qr)Y^{L}\|\bar{n}(\bar{l}\bar{s})\bar{j})\right.$$

$$\times(\tfrac{1}{2}\|1\|\tfrac{1}{2})(X_{\bar{j}j}+Y_{\bar{j}j})\delta_{JL}\delta_{M0}\delta_{T0}\delta_{M_T0}$$

$$+\sum_{\mu}M_{\mu}^{1+}(\gamma)\sum_{\lambda}(L01\mu|L1\lambda\mu)\sum_{\substack{n(ls)j\\ \bar{n}(\bar{l}\bar{s})\bar{j}}}\frac{(-)^{\bar{j}+(3/2)}}{\sqrt{2\lambda+1}}(n(ls)j\|j_L(qr)y^{\lambda}\|\bar{n}(\bar{l}\bar{s})\bar{j})$$

$$\left.\times(\tfrac{1}{2}\|1\|\tfrac{1}{2})(X_{\bar{j}j}-Y_{\bar{j}j})\delta_{J\lambda}\delta_{M\mu}\delta_{T0}\delta_{M_T0}\right]$$

$$+\sum_{L}\sqrt{4\pi[L]}(-i)^{L}\left[M_0(\delta)\sum_{\substack{n(ls)j\\ \bar{n}(\bar{l}\bar{s})\bar{j}}}\frac{(-)^{\bar{j}+(3/2)}}{\sqrt{2L+1}}(n(ls)j\|j_L(qr)Y^{L}\|\bar{n}(\bar{l}\bar{s})\bar{j}\right.$$

$$\times\frac{(\tfrac{1}{2}\|\tau^1\|\tfrac{1}{2})}{\sqrt{3}}(X_{\bar{j}j}+Y_{\bar{j}j})\delta_{JL}\delta_{M0}\delta_{T1}\delta_{M_T0}$$

$$+\sum_{\mu}M_{\mu}^{1+}(\delta)\sum_{\lambda}(L01\mu|L1\lambda\mu)\sum_{\substack{n(ls)j\\ \bar{n}(\bar{l}\bar{s})\bar{j}}}\frac{(-)^{\bar{j}+(3/2)}}{\sqrt{2\lambda+1}}(n(ls)j\|j_L(qr)y^{\lambda}\|\bar{n}(\bar{l}\bar{s})\bar{j})$$

$$\left.\times\frac{(\tfrac{1}{2}\|\tau^1\|\tfrac{1}{2})}{\sqrt{3}}(X_{\bar{j}j}-Y_{\bar{j}j})\delta_{J\lambda}\delta_{M\mu}\delta_{T1}\delta_{M_T0}\right], \qquad (8)$$

where the meaning of quantum numbers are the same as those in Eq. (7). $X_{\bar{j}j}$ and $Y_{\bar{j}j}$ are the amplitudes corresponding to a foregoing and a backgoing amplitudes in the case of Goswami and Pal, and are tabulated in reference 3).

In either case, Gillet and Vinh Mau and Goswami and Pal, the differential cross section is calculated from the following equation:

$$\sigma(\theta)=\left(\frac{2A}{A+1}\right)^2\frac{k_f}{k_i}\frac{1}{2}\sum_M \mathrm{Tr}(T_{JM}\cdot T_{JM}^{+}), \tag{9}$$

where A is a mass number of target nucleus, k_i and k_f are the wave numbers of the incident and the scattered nucleon in center of mass system, respectively.

The angular distribution of polarization is given dy

$$P(\theta)=\frac{(1/2)\sum_M \mathrm{Tr}(T_{JM}\sigma_n T_{JM}^{+})}{(1/2)\sum_M \mathrm{Tr}(T_{JM}\cdot T_{JM}^{+})}. \tag{10}$$

The cross sections of nucleon inelastic scattering are calculated for the excited energy levels $E_{\mathrm{exc}}=4.43$ Mev 2^+ $T=0$, 9.63 Mev 3^- $T=0$, and 22.3 Mev 1^- $T=1$ and shown in Figs. 1, 2 and 3, respectively. The polarizations for these levels are shown in Figs. 4, 5 and 6, respectively. We used $b=1.64$ fm for a range parameter of harmonic oscillator in accordance with Gillet and Vinh Mau and also Goswami and Pal.

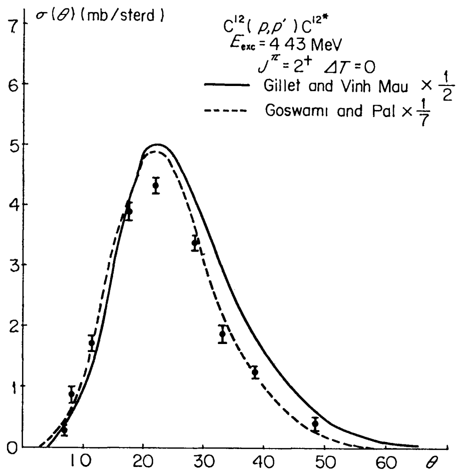

Fig. 1. Differential cross section for $E_p=156$ Mev proton from $E_{\mathrm{exc}}=4.43$ Mev 2^+ $T=0$ state. The solid line is the result obtained by use of Gillet and Vinh Mau's amplitudes. The result has been reduced by 1/2. The dashed line is the result obtained by use of Goswami and Pal's amplitudes. The result has been reduced by 1/7. Data are taken from Jacmart, Garron, Rion and Ruhla[9].

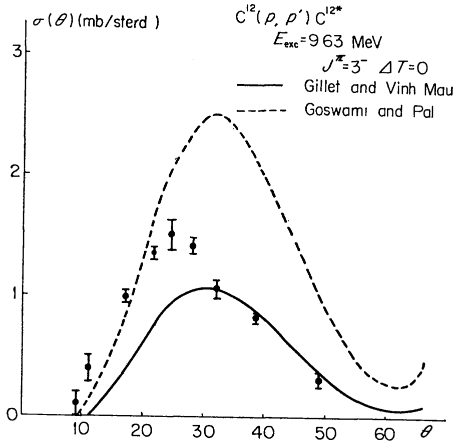

Fig. 2. Cross section for $E_p=156$ Mev proton from $E_{\mathrm{exc}}=9.63$ Mev 3^- $T=0$ state. The solid line is the result obtained by use of Gillet and Vinh Mau's amplitudes. The dashed line is the result obtained by use of Goswami and Pal's amplitudes. These result have not been reduced. Data are taken from reference 9).

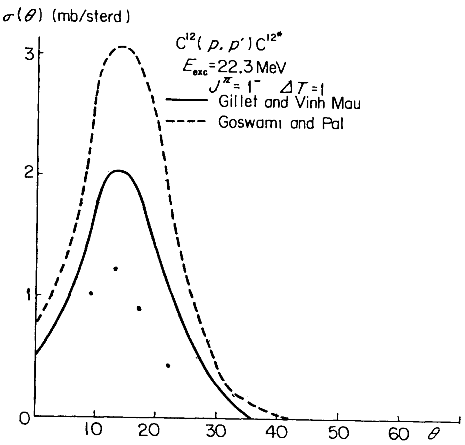

Fig. 3. Cross section for E_p=156 Mev proton from E_{exc}=22.3 Mev 1^- T=1 state. For the others, see Fig. 2.

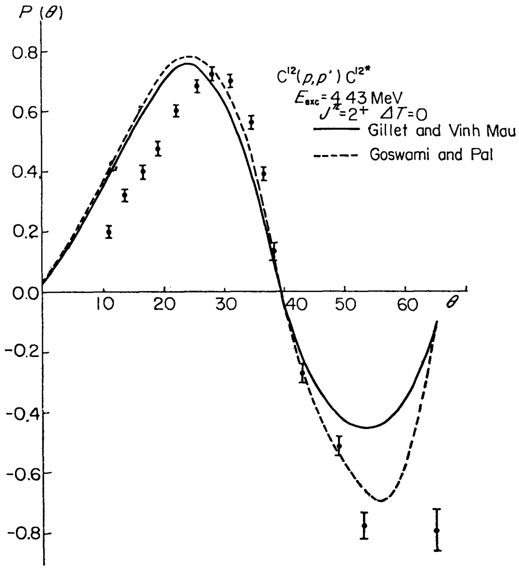

Fig. 4. Polarization for E_p=156 Mev proton from E_{xte}=4.43 Mev 2^+ T=0 state. Data are taken from reference 7). For the others, see Fig. 2.

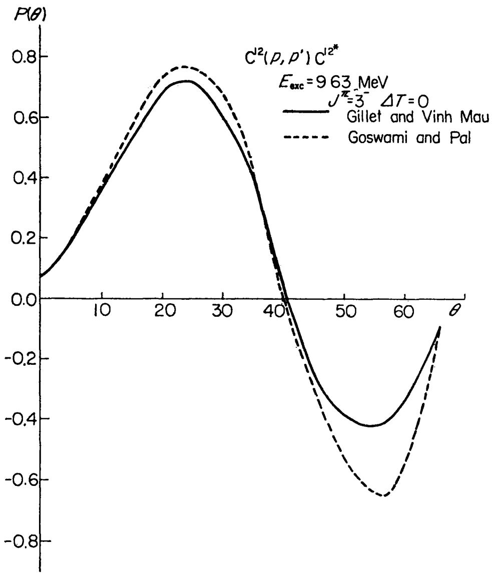

Fig. 5. Polarization for E_p=156 Mev proton from E_{exc}=9.63 Mev 3^- T=0 state. For the others, see Fig. 2.

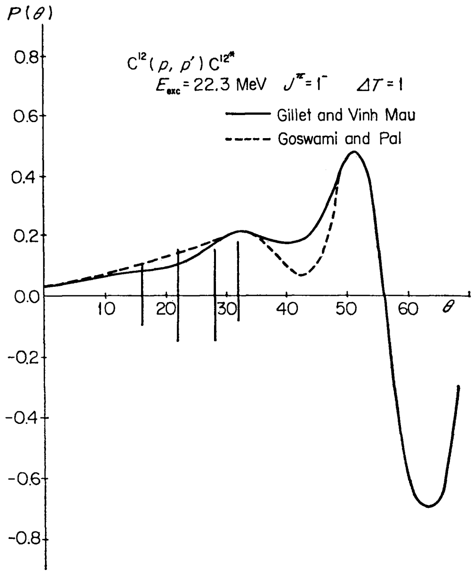

Fig. 6. Polarization for E_p=156 Mev proton from E_{exc}=22.3 Mev 1^- T=1 state. Data are taken from reference 7).

§3. Conclusion

In the case of Goswami and Pal, the sum of terms $\chi(\bar{j}jL0)+(-)^{L+T}\times\bar{\chi}(\bar{j}jL0)$ contribute to the spin nonflip term in transition spin matrix, but the sum of terms $\chi(\bar{j}jL\mu)-(-)^{L+T}\bar{\chi}(\bar{j}jL-\mu)$ to the spin flip term in transition spin matrix for each excited state. For the first excited state, that is $E_{exc}=4.43$ Mev 2^+ $T=0$, the maximum value of the differential cross section is estimated about 7 times larger than that of experimental one (Fig. 1). The maximum values of the cross sections for the other excited states, i. e. $E_{exc}=9.63$ Mev 3^- $T=0$ and 22.3 Mev 1^- $T=1$, are also estimated about twice larger than that of the experimental results (Figs. 2 and 3).

In the case of Gillet and vinh Mau, the sum of terms $X_{\bar{j}j}+Y_{\bar{j}j}$ contribute to the spin nonflip term in transition spin matrix, but the sum of terms $X_{\bar{j}j}-Y_{\bar{j}j}$ to the spin flip term in transition spin matrix. The maximum value of the estimated differential cross section for $E_{exc}=4.43$ Mev 2^+ $T=0$ is twice larger than the experimental value. This fact has also been obtained by Nishida[10] using a form factor which is determined from electron inelastic scattering cross section. The maximum value of cross section for state $E_{exc}=22.3$ Mev 1^- $T=1$ is also estimated somewhat larger than the experimental one, but for state $E_{exc}=9.63$ Mev 3^- $T=0$, the maximum value of cross section is less than the experimental one.

The calculated values of the cross sections are considerably different from the experimental one for the each excited state in both the cases, i. e. Gillet and Vinh Mau's case and Goswami and Pal's case. This facts will suggest us that if P. W. I. A. is a good approximation, one cannot explain the experimental cross sections by the use of one particle-hole and one hole-particle amplitudes only.

The angular distribution of polarization for each excited state is reproduced well at the smaller scattering angular region than 40°, because the form factor in spin nonflip term is very large compared with the one in spin flip term. Therefore, in either case, the calculated polarization distributions have almost similar angular distributions in each other cases and are also similar to that derived by L-S coupling shell model. These polarizations reproduce well experimental one.

Acknowledgements

One of the authors (T. T.) would like to thank Professor M. Kobayasi for his encouragements and Professor Y. Nishida for his useful advices and discussions.

The authors wish to express their appreciation to the member of computing center in Kumamoto University for computer programs.

References

1) S. Fallieros and R. A. Ferrell, Phys. Rev. **116** (1959), 660.
2) M. Kobayashi and T. Marumori, Prog. Theor. Phys. **23** (1960), 387.
M. Baranger, Phys. Rev. **120** (1960), 957.
N. Vinh Mau and G. E. Brown, Nucl. Phys. **29** (1962), 89.
3) V. Gillet and N. Vinh Mau, Nucl. Phys. **54** (1964), 321.
4) A. Goswami and M. K. Pal, Nucl. Phys. **44** (1963), 294.
5) V. Gillet and M. A. Melkanoff, Phys. Rev. **133** (1964), 1190.
6) R. M. Haybron and H. McManus, Phys. Rev. **136** (1964), 1730; **140** (1965), 638.
7) H. K. Lee and H. McManus, Phys. Rev. **161** (1967), 1087.
8) A. K. Kerman, H. McManus and R. M. Thaler, Ann. of Phys. **8** (1959), 551.
9) J. C. Jacmart, J. P. Garron, M. Rion and C. Ruhla, *Direct Interactions and Nuclear Reaction Mechanism*, edited by E. Clemental and C. Villi (Gordon and Breach, Science Publishers, Inc., New York, 1962), p. 1145.
10) Y. Nishida, Nucl. Phys. **43** (1963), 592.