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THE EFFECT OF SOURCE MISMATCH IN THE DECREMENT METHOD OF MEASURING Q

Introduction

For the measurement of very high Q's of superconducting microwave cavities, the decrement method is convenient.[1] The loaded Q is determined from the decay rate of the discharging cavity, and the coupling coefficient β can be determined from a variety of formulations involving the incident, reflected and emitted power during a square wave rf pulse. Then the unloaded Q is determined from $Q_0 = (1+\beta)Q_L$.

Weaver[2] has presented formulae to determine β, and Loew[3] has derived these formulae on a rigorous basis. The formulae, however, are valid only for the case of a generator matched to a lossless transmission line. In the case of generator mismatch, it is impossible to obtain consistent values of β from the various formulae.

The problem solved here is that of a lossless transmission line of characteristic impedance Z_0, driven by a source of impedance Z_S, and terminated by a load of impedance Z_L. The line length is arbitrary, as are the impedances and corresponding reflection coefficients, which may, in general, be complex. Certain simplifications are obtained in the latter part of the analysis because in practice, the reflection coefficient of the source is small, the steady-state reflection coefficient of the cavity is always made to be real by an adjustment of driving frequency, and the reflection coefficient of the unexcited cavity is always -1.

The model upon which the analysis is based is shown in Fig. 1.

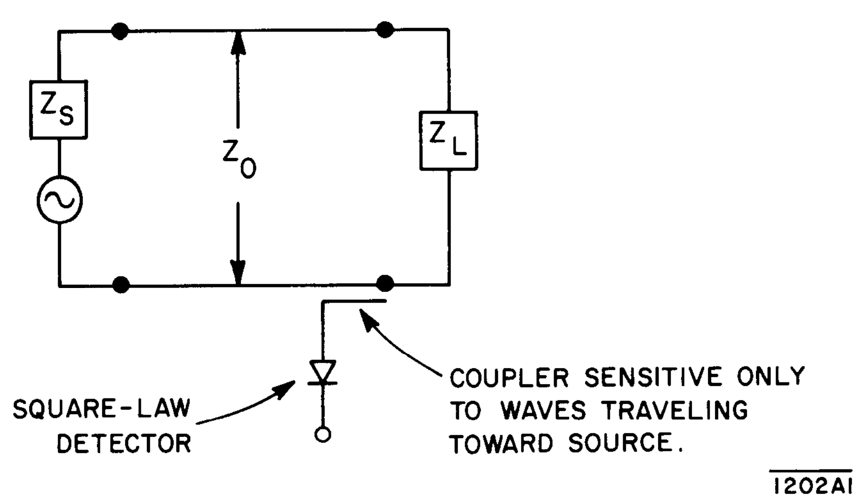

FIG. 1--Circuit model

The generator is a voltage source, capable of delivering a constant voltage regardless of load impedance. The coupler is assumed to introduce negligible loss and reflection, and to have infinite directivity. The detector is assumed to be perfectly square-law.

Notation

The following notation is used:

V_i = voltage at the coupler when rf excitation is first applied

V_r = voltage at the coupler during steady state

V_e = voltage at the coupler when rf excitation is first removed

$P_i = \frac{V_i^2}{Z_0}$ = incident power

$P_r = \frac{V_r^2}{Z_0}$ = reflected power

$P_e = \frac{V_e^2}{Z_0}$ = emitted power

ρ_0 = reflection coefficient of Z_L when rf excitation is first applied

ρ_1 = reflection coefficient of Z_L when steady state is reached

ρ_2 = reflection coefficient of source

V_0 = incident voltage supplied by the source to the line

Z_s = source impedance

Z_L = load impedance

Z_0 = line characteristic impedance

Analysis

When rf excitation is first applied to the unexcited cavity, the cavity appears to be a short circuit. The incident wave is totally reflected, sampled by the coupler, partially reflected again by the mismatched source, reflected again by the short circuit load, and so on.

Thus

$$V_i = V_0\rho_0 + V_0\rho_0^2\rho_2 + V_0\rho_0^3\rho_2^2 + \dots$$

$$= V_0\rho_0 \; [1 + \rho_0\rho_2 + (\rho_0\rho_2)^2 + \dots] \qquad (1)$$

Similarly, during steady state

$$V_r = V_0\rho_1 \; [1 + \rho_1\rho_2 + (\rho_1\rho_2)^2 + \dots] \qquad (2)$$

And after removal of excitation,

$$V_e = [V_0 \,(1 + \rho_1)] \, [1 + \rho_1\rho_2 + (\rho_1\rho_2)^2 + \dots] \qquad (3)$$

From (1), (2), and (3) we obtain

$$P_i = \frac{V_i^2}{Z_0} = \frac{(V_0\rho_0)^2}{Z_0} \; [1 + \rho_0\rho_2 + (\rho_0\rho_2)^2 + \dots]^2 \qquad (4)$$

$$P_r = \frac{V_r^2}{Z_0} = \frac{(V_0\rho_1)^2}{Z_0} \; [1 + \rho_1\rho_2 + (\rho_1\rho_2)^2 + \dots]^2 \qquad (5)$$

$$P_e = \frac{V_e^2}{Z_0} = \frac{[V_0(1 + \rho_1)]^2}{Z_0} \; [1 + \rho_1\rho_2 + (\rho_1\rho_2)^2 + \dots]^2 \qquad (6)$$

If we assume $\rho_0 = -1$ and that ρ_2 is small, (0.1 for example), equations (4), (5) and (6) reduce to:

$$P_i \approx \frac{(V_0)^2}{Z_0} \; [1 - \rho_2]^2 \qquad (7)$$

$$P_r \approx \frac{(V_0\rho_1)^2}{Z_0} \; [1 + \rho_1\rho_2]^2 \qquad (8)$$

$$P_e \approx \frac{[V_0(1 + \rho_1)]^2}{Z_0} \; [1 + \rho_1\rho_2]^2 \qquad (9)$$

From equations (7), (8) and (9) we obtain:

$$\left(\frac{P_r}{P_i}\right)^{1/2} = \rho_1 \left(\frac{1 + \rho_1\rho_2}{1 - \rho_2}\right) \tag{10}$$

$$\left(\frac{P_e}{P_i}\right)^{1/2} = (1 + \rho_1) \left(\frac{1 + \rho_1\rho_2}{1 - \rho_2}\right) \tag{11}$$

$$\left(\frac{P_r}{P_e}\right)^{1/2} = \left(\frac{\rho_1}{1 + \rho_1}\right) \tag{12}$$

The expressions for β are as follows:[2]

$\beta < 1$	$\beta > 1$	
$\beta = \dfrac{1 - \left(\frac{P_r}{P_i}\right)^{1/2}}{1 + \left(\frac{P_r}{P_i}\right)^{1/2}}$	$\beta = \dfrac{1 + \left(\frac{P_r}{P_i}\right)^{1/2}}{1 - \left(\frac{P_r}{P_i}\right)^{1/2}}$	(13a, b)
$\beta = \dfrac{\left(\frac{P_e}{P_i}\right)^{1/2}}{2 - \left(\frac{P_e}{P_i}\right)^{1/2}}$	$\beta = \dfrac{\left(\frac{P_e}{P_i}\right)^{1/2}}{2 - \left(\frac{P_e}{P_i}\right)^{1/2}}$	(14a, b)
$\beta = \dfrac{1}{1 + 2\left(\frac{P_r}{P_e}\right)^{1/2}}$	$\beta = \dfrac{1}{1 - 2\left(\frac{P_r}{P_e}\right)^{1/2}}$	(15a, b)

Results

When Eq. (10), (11), and (12) are substituted in the expressions for β, inconsistent values of β are obtained. Only the use of Eq. 15 yields the correct result, since ρ_2 does not appear in this expression. For example, if $\rho_1 = 0.5$ and $\rho_2 = 0.1$, the following results are obtained:

$$\beta = \frac{1 + \left(\frac{P_r}{P_i}\right)^{1/2}}{1 - \left(\frac{P_r}{P_i}\right)^{1/2}} = \frac{1 + \frac{\rho_1 (1 + \rho_1 \rho_2)}{1 - \rho_2}}{1 - \frac{\rho_1 (1 + \rho_1 \rho_2)}{1 - \rho_2}} = 3.8 \qquad (13b)$$

$$\beta = \frac{\left(\frac{P_e}{P_i}\right)^{1/2}}{2 - \left(\frac{P_e}{P_i}\right)^{1/2}} = \frac{(1 + \rho_1)\left(\frac{1 + \rho_1 \rho_2}{1 - \rho_2}\right)}{2 - (1 + \rho_1)\left(\frac{1 + \rho_1 \rho_2}{1 - \rho_2}\right)} = 7 \qquad (14b)$$

$$\beta = \frac{1}{1 - 2\left(\frac{P_r}{P_e}\right)^{1/2}} = \frac{1}{\frac{\rho_1}{1 + \rho_1}} = 3 \qquad (15b)$$

If ρ_2 is made equal to zero, then Eqs. (10), (11), and (12) reduce to the form associated with the condition of a matched generator, and may be substituted into the expressions for β with consistent results. A double-stub or slide-screw tuner can be used to produce such a matched generator.

Note that when $\rho_1 = 0$, Eqs. (13a, b) and (15a, b) yield consistent results ($\beta = 1$), and that the source mismatch can be determined from Eq. (11) or from

$$\beta_2 = \frac{2 - \frac{P_i}{P_e}^{1/2}}{\frac{P_i}{P_e}^{1/2}} \qquad (16)$$

Series loss can also produce erroneous results. For example, in the photographs shown by Loew,[3] a negative value of β is obtained using Eq. (15b). The case of series loss has been analyzed by several authors,[4,5] and since it is known that the transmission line used in the SLAC experiments has a loss of about 1db, this case will be treated next in order to obtain a proper interpretation of the photographic data.

References

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