

Black hole entropy, universality, and horizon constraints

Steven Carlip

Department of Physics, University of California, Davis, CA 95616, USA

E-mail: carlip@physics.ucdavis.edu

Abstract. To ask a question about a black hole in quantum gravity, one must restrict initial or boundary data to ensure that a black hole is actually present. For two-dimensional dilaton gravity, and probably a much wider class of theories, I show that the imposition of a “stretched horizon” constraint modifies the algebra of symmetries at the horizon, allowing the use of conformal field theory techniques to determine the asymptotic density of states. The result reproduces the Bekenstein-Hawking entropy without any need for detailed assumptions about the microscopic theory. Horizon symmetries may thus offer an answer to the problem of universality of black hole entropy.

1. Introduction

In the continuing search for a quantum theory of gravity, the Hawking temperature and Bekenstein-Hawking entropy of the black hole are perhaps the nearest things we have to “experimental” data. While Hawking radiation has not been observed, it has been derived by so many independent means—from quantum field theory in a fixed background [1] to gravitational instanton calculations [2] to thermal Greens functions [3] and constructive quantum field theory [4,5] to the analysis of model particle detectors [6,7] to tunneling calculations [8,9]—that it would seem perverse to consider a quantum theory of gravity that did not allow us to recover the standard results.

Since the Bekenstein-Hawking entropy is inherently quantum gravitational, involving both Planck’s constant \hbar and Newton’s constant G , we might hope that black hole thermodynamics could give us important clues to the mystery of quantum gravity. The fundamental problem is to understand the statistical mechanics underlying black hole thermodynamics—what microscopic states are responsible for the thermal properties of black holes? Ten years ago, at the time of the First Meeting on Constrained Dynamics and Quantum Gravity, the answer was simple: we don’t know. Today, that has changed: many people will tell you with great confidence exactly what microscopic physics leads to black hole thermodynamics.

The trouble is that while many people know the answer, they do not all agree. Black hole entropy may come from weakly coupled string and D-brane states [10,11]; from nonsingular string “fuzzballs” [12]; from states in a dual conformal field theory “at infinity” [13]; from spin network states crossing the horizon [14] or perhaps inside the horizon [15]; from “heavy” degrees of freedom in induced gravity [16]; or from nonlocal topological properties of the black hole spacetime [17]. None of these pictures has yet given us a complete model of black hole thermodynamics. But each can be used to count states for at least one class of black holes, and

within its realm of applicability, each seems to give the correct Bekenstein-Hawking entropy. While the existence of competing models may be healthy, the existence of competing models that all *agree* cries out for a deeper explanation.

2. Symmetry and entropy

One way to explain this “universality” would be to identify a feature of classical general relativity inherited by any quantum theory of gravity. The obvious candidate for such a feature is a symmetry. At first sight, this seems an odd idea: we do not usually think of a classical symmetry as being strong enough to determine such an intrinsically quantum mechanical characteristic as a density of states. In one case, though, a classical symmetry does just that.

Consider a two-dimensional conformal field theory, that is, a theory invariant under diffeomorphisms and local rescalings (Weyl transformations). Choose a system of complex coordinates z, \bar{z} . Then any such theory includes generators $L[\xi]$ and $\bar{L}[\bar{\xi}]$ of holomorphic and antiholomorphic diffeomorphisms, which necessarily satisfy a Virasoro algebra [18]

$$\begin{aligned} [L[\xi], L[\eta]] &= L[\eta\xi' - \xi\eta'] + \frac{c}{48\pi} \int dz (\eta'\xi'' - \xi'\eta'') \\ [L[\xi], \bar{L}[\bar{\eta}]] &= 0 \\ [\bar{L}[\bar{\xi}], \bar{L}[\bar{\eta}]] &= \bar{L}[\bar{\eta}\bar{\xi}' - \bar{\xi}\bar{\eta}'] + \frac{\bar{c}}{48\pi} \int d\bar{z} (\bar{\eta}'\bar{\xi}'' - \bar{\xi}'\bar{\eta}''). \end{aligned} \quad (1)$$

The first terms on the right-hand sides are the standard commutators for the diffeomorphism group. The second terms give the unique central extension of the group, characterized by central charges (or “conformal anomalies”) c and \bar{c} whose values depend on the particular model. Any such two-dimensional conformal theory has two conserved charges, $L_0 = L[\xi_0]$ and $\bar{L}_0 = \bar{L}[\bar{\xi}_0]$, which can be thought of as “energies” with respect to constant holomorphic and antiholomorphic transformations.

Consider now a conformal field theory with central charges c, \bar{c} and with lowest eigenvalues Δ_0 and $\bar{\Delta}_0$ of L_0 and \bar{L}_0 . In 1986, Cardy [19] made the remarkable discovery that for large values Δ and $\bar{\Delta}$ of L_0 and \bar{L}_0 , the density of states has the asymptotic form

$$\ln \rho(\Delta, \bar{\Delta}) \sim 2\pi \left[\sqrt{\frac{c_{eff}\Delta}{6}} + \sqrt{\frac{\bar{c}_{eff}\bar{\Delta}}{6}} \right], \quad \text{with } c_{eff} = c - 24\Delta_0, \bar{c}_{eff} = \bar{c} - 24\bar{\Delta}_0. \quad (2)$$

The entropy is thus fixed by the symmetry, independent of any details of the theory! In particular, two conformal field theories with very different field content will have the same asymptotic density of states, provided that their effective central charges are equal.

A result as profound as the Cardy formula (2) ought to have a fundamental physical explanation. I do not know of one. Some insight may be obtained, though, by noting that a central charge represents a (weak) breaking of the conformal symmetry. In particular, in Dirac quantization, one usually demands that physical states be annihilated by the generators of a gauge symmetry; for holomorphic diffeomorphisms, for instance, one would require that

$$L[\xi]|phys\rangle = 0. \quad (3)$$

But if $c \neq 0$, such a condition is incompatible with the commutators (1), and must be relaxed, for instance by requiring that only the positive frequency components of $L[\xi]$ annihilate physical states. We thus obtain a new set of physical states (in conformal field theory language, “descendant” states) that contribute to the density of states (2). As Kaloper and Terning have pointed out [21], this is reminiscent of the Goldstone mechanism [22], in which a broken symmetry gives rise to new physical states.

Now, general relativity is not a conformal field theory, and it is certainly not two-dimensional, so the relevance of the Cardy formula to black hole entropy is not obvious. But there is reason to believe that black hole dynamics may be *effectively* described by a two-dimensional conformal field theory near the horizon. For instance, it is known that near a horizon, matter can be described by a two-dimensional conformal field theory, with fields depending only on t and the “tortoise coordinate” r_* : the mass and transverse excitations are essentially red-shifted away relative to excitations in the r_*-t plane [23, 24]. Jacobson and Kang have pointed out that the surface gravity and temperature of a stationary black hole are conformally invariant as well [25]. And Medved et al. have recently shown that a generic stationary black hole metric always has an approximate conformal symmetry near the horizon [26, 27].

3. The BTZ black hole

The first concrete evidence that conformal symmetry can determine black hole thermodynamics came from the study of the (2+1)-dimensional black hole of Bañados, Teitelboim, and Zanelli [28–31]. The BTZ black hole is the (2+1)-dimensional analog of the Kerr-AdS geometry, with a metric of the form

$$ds^2 = N^2 dt^2 - N^{-2} dr^2 - r^2 \left(d\phi + N^\phi dt \right)^2$$

$$\text{with } N = \left(-8GM + \frac{r^2}{\ell^2} + \frac{16G^2 J^2}{r^2} \right)^{1/2}, \quad N^\phi = -\frac{4GJ}{r^2}, \quad (4)$$

where the cosmological constant is $\Lambda = -1/\ell^2$ and M and J are the anti-de Sitter analogs of the ADM mass and angular momentum. Like all vacuum spacetimes in 2+1 dimensions, the BTZ geometry has constant curvature, and can in fact be expressed as a quotient of anti-de Sitter space by a discrete group of isometries. Nevertheless, it is a genuine black hole:

- It has an event horizon at $r = r_+$ and, if $J \neq 0$, an inner Cauchy horizon at $r = r_-$, where

$$r_\pm^2 = 4GM\ell^2 \left\{ 1 \pm \left[1 - \left(\frac{J}{M\ell} \right)^2 \right]^{1/2} \right\}; \quad (5)$$

- it occurs as the end point of gravitational collapse of matter;
- its Carter-Penrose diagram is identical to that of an ordinary Kerr-AdS black hole;
- most important for our purposes, it exhibits standard black hole thermodynamics, with an entropy given by $S = 2\pi r_+/4\hbar G$, one-fourth of the horizon “area.”

With a little thought, one can appreciate how truly mysterious the entropy of the BTZ black hole is. We would like to understand Bekenstein-Hawking entropy in terms of microscopic quantum states. But in 2+1 dimensions, general relativity has no local degrees of freedom [32]: the number of constraints is equal to the number of components of the spatial metric, and only a finite number of global degrees of freedom remain. It seems that there is no room for enough states to account for what can be an arbitrarily large entropy.

A partial answer was discovered independently by Strominger [33] and Birmingham, Sachs, and Sen [34]. The conformal boundary of a (2+1)-dimensional asymptotically anti-de Sitter spacetime is a cylinder, so it is not surprising that the asymptotic symmetries of the BTZ black hole are described by a Virasoro algebra (1). It is a bit more surprising that this algebra has a central extension. But as Brown and Henneaux showed in 1986 [35]—and many authors subsequently confirmed [31]—the classical central charge, computed from the standard ADM constraint algebra, is nonzero:

$$c = \frac{3\ell}{2\hbar G}. \quad (6)$$

Furthermore, the conserved charges L_0 and \bar{L}_0 can be computed in the standard ADM formalism, yielding

$$\Delta_0 \sim \frac{1}{16\hbar G\ell}(r_+ + r_-)^2, \quad \bar{\Delta}_0 \sim \frac{1}{16\hbar G\ell}(r_+ - r_-)^2 \quad (7)$$

Inserting (6–7) into the Cardy formula, we find

$$S \sim \frac{2\pi}{8\hbar G}(r_+ + r_-) + \frac{2\pi}{8\hbar G}(r_+ - r_-) = \frac{2\pi r_+}{4\hbar G}, \quad (8)$$

the correct Bekenstein-Hawking entropy. We thus learn that the BTZ entropy is related to symmetries and boundary conditions at infinity.

A further piece of the answer begins with the observation [36, 37] that (2+1)-dimensional gravity with a negative cosmological constant can be written as an $\text{SO}(2, 1) \times \text{SO}(2, 1)$ Chern-Simons theory. The Chern-Simons action is gauge invariant on a compact manifold. But boundaries and the resulting boundary conditions can partially break that invariance, in a manner reminiscent of the symmetry breaking discussed at the end of the preceding section. The resulting Goldstone-like modes can be described by a WZW model [38–40], or, in the case of (2+1)-dimensional gravity, by a Liouville theory with central charge (6) [41].

This result has now been confirmed in a number of different approaches, ranging from Chern-Simons methods to the AdS/CFT correspondence; see [31] for a review. Of particular interest here is the explicit derivation of the Liouville field as a “would-be diffeomorphism” that becomes dynamical because of the need to impose asymptotic boundary conditions [42, 43]. This resolves our earlier paradox: although (2+1)-dimensional gravity on a compact manifold has only a small number of topological degrees of freedom, the presence of a boundary—or an asymptotic boundary at infinity—partially breaks the diffeomorphism invariance, promoting “gauge” degrees of freedom to physical excitations.

Whether the resulting Liouville degrees of freedom can account for the entropy (8) remains an open question [31]. A naive application of the Cardy formula yields the correct entropy. The relatively well-understood “normalizable” sector of Liouville theory has a nonzero minimum value of L_0 , however, lowering the effective central charge c_{eff} in (1) and ruining the correspondence [44]. Chen has recently shown that the more poorly understood “nonnormalizable” sector of the theory may admit a sensible quantization, though, with states that can be explicitly counted and that seem to reproduce the correct entropy [45].

4. Horizons and constraints

The real world, of course, is not 2+1 dimensional, and the derivations of the BTZ black hole entropy rely on features that do not easily generalize. Only in 2+1 dimensions is the asymptotic boundary of spacetime two dimensional, allowing an easy application of the Cardy formula. Moreover, it seems more natural to associate black hole states with the horizon rather than spatial infinity. For a single black hole in 2+1 dimensions, the distinction may be unimportant, since there are no additional dynamical degrees of freedom lying between the horizon and infinity. In 3+1 dimensions, though, the prospect of extracting the degrees of freedom of a single black hole from the asymptotics of a complicated spacetime is daunting.

We can, however, draw a few lessons from the (2+1)-dimensional model. We should look for “broken gauge invariance” from boundary conditions, and hope for an effective two-dimensional picture. But we should also start by looking near horizon rather than at infinity.

There is an immediate objection to this proposal. Unlike spatial infinity, the horizon of a black hole is not a true boundary, and it is not at all clear that the boundary effects seen for the BTZ black hole will still occur. To clarify this matter, it is helpful to take a step back and ask what it means to ask a question about a black hole in a quantum theory of gravity.

This issue is rarely discussed, because in the usual semiclassical treatments the answer is obvious: we simply fix a black hole background, and look at quantum fields (including fluctuations of the metric) in that background. In a full quantum theory of gravity, though, we cannot do this. Such a theory has no fixed background; the metric is an operator, and the uncertainty principle tells us that its value cannot be exactly specified. At best, we can restrict a small portion of the spacetime and ask *conditional* questions: “If a black hole of type X is present, what is the probability of observing phenomenon Y?”

I know of two ways to obtain such a conditional probability. The first, discussed in [46], is to treat the horizon as a “boundary” at which suitable boundary conditions are imposed. In the sum over histories formalism, for example, we can divide spacetime into two regions along a hypersurface \mathcal{H} and perform separate path integrals over fields on each piece, with fields restricted at the “boundary” by the requirement that \mathcal{H} be a suitable black hole horizon. This kind of split path integral has been studied in detail in 2+1 dimensions [47], where it leads to the same WZW model that was discussed in the preceding section. The basic point is that although the horizon is not a true boundary, it *is* a hypersurface upon which we impose “boundary conditions,” and this can be good enough.

Alternatively, we can impose “horizon constraints” directly, either classically or in the quantum theory. We might, for example, construct an operator ϑ representing the expansion of a particular null surface, and restrict ourselves to states annihilated by ϑ . As we shall see below, such a restriction can affect the algebra of diffeomorphisms, allowing us to exploit the Cardy formula to count states.

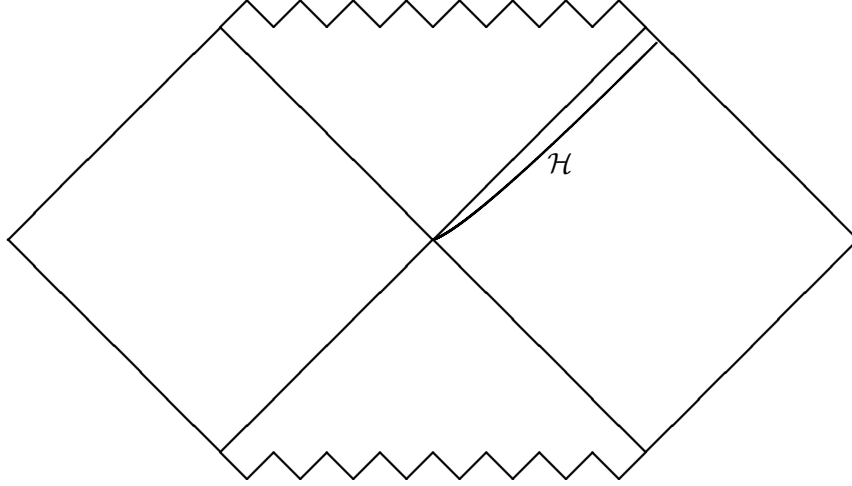
The “horizon as a boundary” approach has been investigated by a number of authors; see, for instance, [48–57]. One naturally finds a conformal symmetry in the r_*-t plane of the sort discussed at the end of section 2, and one can obtain a Virasoro algebra with a central charge that leads to the correct black hole entropy. However, the diffeomorphisms whose algebra yields that central charge—essentially those that leave the lapse function invariant—are generated by vector fields that blow up at the horizon [58–60], and it is not clear whether this is permissible. A related approach looks for approximate conformal symmetry near the horizon [61–64]; again, one finds a Virasoro algebra with a central charge that seems to lead to the correct entropy.

The alternative “horizon constraint” approach is still in the early stages of development. Suppose we wish to constrain our theory of gravity by requiring that some prescribed surface be a black hole horizon. In particular, we can require that \mathcal{H} be an “isolated horizon” [65] or a “dynamical horizon” [66]. Such constraints restrict the allowed data on \mathcal{H} , and in principle we should be able to use the well-developed apparatus of constrained Hamiltonian dynamics [67–69] to study such conditions. A further complication arises, though, because an isolated horizon is by definition a null surface, requiring some version of “constrained light cone quantization.” This is a difficult program, and work has just begun.

As a simpler warm-up exercise, we can impose constraints requiring the presence of a spacelike “stretched horizon” that becomes nearly null, as illustrated in figure 1. On such a hypersurface, it is possible to employ standard methods of constrained dynamics. As I will discuss in the next section, such a stretched horizon constraint leads to a Virasoro algebra with a calculable central charge, and at least in the case of two-dimensional dilaton gravity, yields the correct Bekenstein-Hawking entropy.

5. Horizon constraints and the dilaton black hole

I now turn to a more technical analysis of a particular case, the two-dimensional dilaton black hole. Details can be found in [70, 71]. Just as the Universe is not 2+1 dimensional, it is surely not two dimensional, so this may again seem too great a specialization. As argued in section 2, though, it is reasonable to expect the near-horizon dynamics of an arbitrary black hole to be effectively two dimensional, so a dimensionally reduced model is a reasonable starting point.

Figure 1: A spacelike “stretched horizon” \mathcal{H}

Two-dimensional dilaton gravity [72, 73] is described by an action

$$I = \int d^2x \sqrt{-g} [AR + V(A)], \quad (9)$$

where R is the two-dimensional scalar curvature and A is a scalar field, the dilaton (often denoted as φ). $V(A)$ is a potential whose form depends on the higher-dimensional theory we started with; we will not need an exact expression. As the notation suggests, A is the transverse area in the higher-dimensional theory, in units $16\pi G = 1$. The analog of the expansion—the fractional rate of change of area along a null curve with null normal l^a —is

$$\vartheta = l^a \nabla_a A / A. \quad (10)$$

It is useful to rewrite the action (9) in terms of a null dyad (l^a, n^a) with $l^2 = n^2 = 0$, $l \cdot n = -1$. These determine “surface gravities” κ and $\bar{\kappa}$, defined by the conditions

$$\nabla_a l_b = -\kappa n_a l_b - \bar{\kappa} l_a l_b, \quad \nabla_a n_b = \kappa n_a n_b + \bar{\kappa} l_a n_b, \quad (11)$$

and the action becomes

$$I = \int d^2x \left[\epsilon^{ab} (2\kappa n_b \partial_a A - 2\bar{\kappa} l_b \partial_a A) + \sqrt{-g} V \right]. \quad (12)$$

If we now define components of our dyad with respect to coordinates (u, v) as

$$l = \sigma du + \alpha dv, \quad n = \beta du + \tau dv, \quad (13)$$

it is easy to find the Hamiltonian form of the action [70]. The system has three first-class constraints; denoting a derivative with respect to v by a prime, they are

$$\begin{aligned} C_{\perp} &= \pi_{\alpha}' - \frac{1}{2} \pi_{\alpha} \pi_A - \tau V(A) \\ C_{\parallel} &= \pi_A A' - \alpha \pi_{\alpha}' - \tau \pi_{\tau}' \\ C_{\pi} &= \tau \pi_{\tau} - \alpha \pi_{\alpha} + 2A'. \end{aligned} \quad (14)$$

C_{\perp} and C_{\parallel} generate diffeomorphisms orthogonal to and parallel to a line of constant u , while C_{π} generates local Lorentz transformations.

We can now impose “stretched horizon” constraints at the surface $u = 0$. We first demand that \mathcal{H} be “almost null,” i.e., that its normal be nearly equal to the null vector l^a . By (13), this requires that $\alpha = \epsilon_1 \ll 1$. We next demand that \mathcal{H} be “almost nonexpanding.” This condition is more subtle, since the absolute scale of l^a is not fixed, and while the requirement of exactly vanishing expansion is independent of this scale, a requirement $\vartheta \ll 1$ is not. Fortunately, though, the surface gravity κ scales identically under constant rescalings of l^a , so we can consistently require that $l^v \nabla_v A / \kappa A = \epsilon_2 \ll 1$. Rewriting these conditions in terms of canonical variables, we obtain two constraints:

$$\begin{aligned} K_1 &= \alpha - \epsilon_1 = 0 \\ K_2 &= A' - \frac{1}{2}\epsilon_2 A_+ \pi_A + \frac{a}{2}C_{\pi} = 0, \end{aligned} \quad (15)$$

where a is an arbitrary constant and A_+ is the horizon value of the dilaton. By looking at exact black hole solutions, one can verify that these constraints do, in fact, define a spacelike stretched horizon like that of figure 1.

K_1 and K_2 are not quite “constraints” in the usual sense of constrained Hamiltonian dynamics, but they are similar enough that many existing techniques can be used. In particular, observe that the K_i have nontrivial brackets with the momentum and boost generators C_{\parallel} and C_{π} , so these no longer generate invariances of the constrained theory. But we can fix this by a method suggested years ago by Bergmann and Komar [69]: we define new generators

$$\begin{aligned} C_{\parallel} &\rightarrow C_{\parallel}^* = C_{\parallel} + a_1 K_1 + a_2 K_2 \\ C_{\pi} &\rightarrow C_{\pi}^* = C_{\pi} + b_1 K_1 + b_2 K_2 \end{aligned} \quad (16)$$

with coefficients a_i and b_i chosen so that $\{C^*, K_i\} = 0$. Since $K_i = 0$ on admissible geometries, the generators C^* are physically equivalent to the original C ; but they now preserve the horizon constraints as well.

We now make the crucial observation that the redefinitions (16) affect the Poisson brackets of the constraints. With the choice $a = -2$ in (15), it may be shown that

$$\begin{aligned} \{C_{\parallel}^*[\xi], C_{\parallel}^*[\eta]\} &= -C_{\parallel}^*[\xi\eta' - \eta\xi'] + \frac{1}{2}\epsilon_2 A_+ \int dv(\xi'\eta'' - \eta'\xi'') \\ \{C_{\parallel}^*[\xi], C_{\pi}^*[\eta]\} &= -C_{\pi}^*[\xi\eta'] \\ \{C_{\pi}^*[\xi], C_{\pi}^*[\eta]\} &= -\frac{1}{2}\epsilon_2 A_+ \int dv(\xi\eta' - \eta\xi'). \end{aligned} \quad (17)$$

The algebra (17) has a simple conformal field theoretical interpretation [18]: the C_{\parallel}^* generate a Virasoro algebra with central charge

$$\frac{c}{48\pi} = -\frac{1}{2}\epsilon_2 A_+, \quad (18)$$

while C_{π}^* is a primary field of weight one. Different choices of the parameter a in (15) yield equivalent algebras, although with slightly redefined generators.

The Cardy formula (2) requires both the central charge and the conserved charge Δ . As in the usual approaches to black hole mechanics, the latter charge comes from a boundary term needed to make the generator C_{\parallel}^* “differentiable” [74]. It was shown in [70] that this term is

$$C_{\parallel}^*{}_{bdry}[\xi] = -\xi\pi_A A|_{v=v_+}, \quad (19)$$

which will give a nonvanishing classical contribution to Δ .

Finally, we also need a mode expansion to define the Fourier component L_0 , or, equivalently, a normalization for the “constant translation” ξ_0 . For a conformal field theory defined on a circle, or on a full complex plane with a natural complex coordinate, this normalization is essentially unique. Here, though, it is not so obvious how to choose the “right” complex coordinate. As argued in [62], however, there is one particularly natural choice,

$$z = e^{2\pi i A/A_+}, \quad \xi_n = \frac{A_+}{2\pi A'} z^n, \quad (20)$$

where the prefactor is chosen so that $[\xi_m, \xi_n] = i(n-m)\xi_{m+n}$.

Equation (19) then implies that

$$\Delta = C_{\parallel bdry}^*[\xi_0] = -\frac{A_+}{2\pi A'} \pi_A A_+ = -\frac{A_+}{\pi \epsilon_2}. \quad (21)$$

Inserting (18) and (21) into the Cardy formula, assuming that Δ_0 is small, and restoring the factors of $16\pi G$ and \hbar , we obtain an entropy

$$S = \frac{2\pi}{16\pi G} \sqrt{\left(-\frac{24\pi\epsilon_2 A_+}{6\hbar}\right) \left(-\frac{A_+}{\pi\epsilon_2\hbar}\right)} = \frac{A_+}{4\hbar G}, \quad (22)$$

exactly reproducing the standard Bekenstein-Hawking entropy.

6. Open questions

While the results of section 5 are intriguing, they are certainly not yet conclusive. There are several straightforward steps that may take us further:

- (i) We should determine how sensitive the result is to the exact definition (15) of the stretched horizon \mathcal{H} . As a first step, a similar computation has now been carried out in radially quantized Euclidean quantum gravity [75]. Here, the constraints have a simpler geometric interpretation—they essentially fix the proper distance of the initial surface from the origin—and it may be possible to relate the constraint analysis to the path integral methods of [76]. Ideally, the analysis should also be repeated in a true light cone quantization; work on this is in progress.
- (ii) We should extend the analysis beyond two dimensions. This is probably not too hard conceptually, but the technical details are complex.
- (iii) We should try to relate the horizon constraint approach of section 5 to the “horizon as boundary” methods described in section 4.

Two other questions are more difficult, but perhaps more profound. First, if it is true that the horizon symmetry described here gives a universal explanation of black hole entropy, then the symmetry should be identifiable in other approaches to black hole microphysics. There is some chance that this connection can be made for a large class of “stringy” black holes. Many of the higher dimensional black holes whose entropy can be computed in string theory have near-horizon geometries that look like that of the BTZ black hole [77], allowing thermodynamic properties to be computed by the methods of section 3. If the Virasoro algebra of the BTZ black hole at infinity can be related to a near-horizon algebra, it may be possible to demonstrate the role of horizon constraints in these string theoretical black holes. A similar interpretation may be possible for state-counting in induced quantum gravity, where a conformal field theory description is also possible [78], but again, the issue has not yet been carefully studied. Whether something along the same lines can be done in loop quantum gravity remains completely open.

Second, there is much more to black hole thermodynamics than the Bekenstein-Hawking entropy. In particular, if the proposal I have presented here is correct, then it must be possible to show that the Goldstone-like “would-be diffeomorphism” degrees of freedom couple properly to external fields to produce Hawking radiation. In 2+1 dimensions, there has been one computation of this sort [79], in which the coupling of a classical source to the boundary degrees of freedom was shown to yield a correct description of Hawking radiation. A recent discussion of Hawking radiation in terms of diffeomorphism anomalies at the horizon [80] may also be relevant. But people have barely begun to look at this question, and a far better understanding is needed.

References

- [1] Hawking S W 1974 *Nature* **248** 30
- [2] Gibbons G W and Hawking S W 1977 *Phys. Rev. D* **15** 2752
- [3] Gibbons G W and Perry M J 1978 *Proc. Roy. Soc. Lond. A* **358** 467
- [4] Fredenhagen K and Haag R 1990 *Commun. Math. Phys.* **127** 273
- [5] Kay B S and Wald R M 1991 *Phys. Rept.* **207** 49
- [6] Unruh W G 1976 *Phys. Rev. D* **14** 870
- [7] DeWitt B S 1979 in *General relativity: an Einstein centenary survey* ed Hawking S W and Israel W (Cambridge: Cambridge University Press) chap. 14
- [8] Parikh M K and Wilczek F 2000 *Phys. Rev. Lett.* **85** 5042 (*Preprint hep-th/9907001*)
- [9] Nadalini N, Vanzo L and Zerbini S 2005 *Preprint hep-th/0511250*
- [10] Strominger A and Vafa C 1996 *Phys. Lett. B* **379** 99 (*Preprint hep-th/9601029*)
- [11] Peet AW 1998 *Class. Quant. Grav.* **15** 3291 (*Preprint hep-th/9712253*)
- [12] Mathur S D 2005 *Fortsch. Phys.* **53** 793 (*Preprint hep-th/0502050*)
- [13] Aharony O, Gubser S S, Maldacena J M, Ooguri H and Oz Y 2000 *Phys. Rept.* **323** 183 (*Preprint hep-th/9905111*)
- [14] Ashtekar A, Baez J, Corichi A and Krasnov K 1998 *Phys. Rev. Lett.* **80** 904 (*Preprint gr-qc/9710007*)
- [15] Livine E R and Terno D R 2005 *Preprint gr-qc/0508085*
- [16] Frolov V P and Fursaev D V 1997 *Phys. Rev. D* **56** 2212 (*Preprint hep-th/9703178*)
- [17] Hawking S W and Hunter C J 1999 *Phys. Rev. D* **59** 044025 (*Preprint hep-th/9808085*)
- [18] Di Francesco P, Mathieu P and Sénéchal D 1997 *Conformal Field Theory* (New York: Springer)
- [19] Cardy J A 1986 *Nucl. Phys. B* **270** 186
- [20] Blöte H W J, Cardy J A and Nightingale M P 1986 *Phys. Rev. Lett.* **56** 742
- [21] Kaloper N and Terning J T 2004 personal communication
- [22] Coleman S 1985 *Aspects of symmetry* (Cambridge: Cambridge University Press) chap. 5
- [23] Birmingham D, Gupta K S and Sen S 2001 *Phys. Lett. B* **505** 191 (*Preprint hep-th/0102051*)
- [24] Gupta K S and Sen S 2002 *Phys. Lett. B* **526** 121 (*Preprint hep-th/0112041*)
- [25] Jacobson T and Kang G 1993 *Class. Quant. Grav.* **10** L201 (*Preprint gr-qc/9307002*)
- [26] Medved A J M, Martin D and Visser M 2004 *Class. Quant. Grav.* **21** 3111 (*Preprint gr-qc/0402069*)
- [27] Medved A J M, Martin D and Visser M 2004 *Phys. Rev. D* **70** 024009 (*Preprint gr-qc/0403026*)
- [28] Bañados M, Teitelboim C and Zanelli J 1992 *Phys. Rev. Lett.* **69** 1849 (*Preprint hep-th/9204099*)
- [29] Bañados M, Henneaux M, Teitelboim C and Zanelli J 1993 *Phys. Rev. D* **48** 1506 (*Preprint gr-qc/9302012*)
- [30] Carlip S 1995 *Class. Quant. Grav.* **12** 2853 (*Preprint gr-qc/9506079*)
- [31] Carlip S 2005 *Class. Quant. Grav.* **22** R85 (*Preprint gr-qc/0503022*)
- [32] Carlip S 2005 *Living Rev. Rel.* **8** 1 (*Preprint gr-qc/0409039*)
- [33] Strominger A 1998 *JHEP* **02** 009 (*Preprint hep-th/9712251*)
- [34] Birmingham D, Sachs I and Sen S 1998 *Phys. Lett. B* **424** 275 (*Preprint hep-th/9801019*)
- [35] Brown J D and Henneaux M 1986 *Commun. Math. Phys.* **104** 207
- [36] Achúcarro A and Townsend P K 1986 *Phys. Lett. B* **180** 89
- [37] Witten E 1988 *Nucl. Phys. B* **311** 46
- [38] Witten E 1984 *Commun. Math. Phys.* **92** 455
- [39] Elitzur S, Moore G W, Schwimmer A and Seiberg N 1989 *Nucl. Phys. B* **326** 108
- [40] Carlip S 1991 *Nucl. Phys. B* **362** 111
- [41] Coussaert O, Henneaux M and van Driel P 1995 *Class. Quant. Grav.* **12** 2961 (*Preprint gr-qc/9506019*)
- [42] Manvelyan R, Mkrtychyan R and Müller-Kirsten H J W 2001 *Phys. Lett. B* **509** 143 (*Preprint hep-th/0103082*)
- [43] Carlip S 2005 *Class. Quant. Grav.* **22** 3055 (*Preprint gr-qc/0501033*)
- [44] Martinec E J 1998 *Preprint hep-th/9809021*

- [45] Chen Y-J 2004 *Class. Quant. Grav.* **21** 1153 (*Preprint* hep-th/0310234)
- [46] Carlip S 1997 *Constrained Dynamics and Quantum Gravity 1996*, *Nucl. Phys. Proc. Suppl.* **57** ed de Alfaro V et al. (Amsterdam: Elsevier) p 8 (*Preprint* gr-qc/9702017)
- [47] Witten E 1992 *Commun. Math. Phys.* **144** 189
- [48] Carlip S 1999 *Phys. Rev. Lett.* **82** 2828 (*Preprint* hep-th/9812013)
- [49] Carlip S 1999 *Class. Quant. Grav.* **16** 3327 (*Preprint* gr-qc/9906126)
- [50] Navarro D J, Navarro-Salas J and Navarro P 2000 *Nucl. Phys. B* **580** 311 (*Preprint* hep-th/9911091)
- [51] Jing J and Yan M-L 2001 *Phys. Rev. D* **63** 024003 (*Preprint* gr-qc/0005105)
- [52] Izquierdo J M, Navarro-Salas J and Navarro P 2002 *Class. Quant. Grav.* **19** 563 (*Preprint* hep-th/0107132)
- [53] Park M-I 2002 *Nucl. Phys. B* **634** 339 (*Preprint* hep-th/0111224)
- [54] Silva S 2002 *Class. Quant. Grav* **19** 3947 (*Preprint* hep-th/0204179)
- [55] Cvitan M, Pallua S and Prester P 2003 *Phys. Lett. B* **555** 248 (*Preprint* hep-th/0212029)
- [56] Cvitan M, Pallua S and Prester P 2004 *Phys. Rev. D* **70** 084043 (*Preprint* hep-th/0406186)
- [57] Cadoni M 2005 *Preprint* hep-th/0511103
- [58] Dreyer O, Ghosh A and Wisniewski J 2001 *Class. Quant. Grav.* **18** 1929 (*Preprint* hep-th/0101117)
- [59] Koga J 2001 *Phys. Rev. D* **64** 124012 (*Preprint* gr-qc/0107096)
- [60] Pinamonti N and Vanzo L 2004 *Phys. Rev. D* **69** 084012 (*Preprint* hep-th/0312065)
- [61] Solodukhin S N 1999 *Phys. Lett. B* **454** 213 (*Preprint* hep-th/9812056)
- [62] Carlip S 2002 *Phys. Rev. Lett.* **88** 241301 (*Preprint* gr-qc/0203001)
- [63] Giacomini A and Pinamonti N 2003 *JHEP* **0302** 014 (*Preprint* gr-qc/0301038)
- [64] Solodukhin S N 2004 *Phys. Rev. Lett.* **92** 061302 (*Preprint* hep-th/0310012)
- [65] Ashtekar A, Beetle C and Fairhurst S 1999 *Class. Quant. Grav.* **16** L1 (*Preprint* gr-qc/9812065)
- [66] Booth I(2005 *Preprint* gr-qc/0508107)
- [67] Dirac P A M 1950 *Can. J. Math.* **2** 129
- [68] Dirac P A M 1951 *Can. J. Math.* **3** 1
- [69] Bergmann P G and Komar A B 1960 *Phys. Rev. Lett.* **4** 432
- [70] Carlip S 2005 *Class. Quant. Grav.* **22** 1303 (*Preprint* hep-th/0408123)
- [71] Carlip S 2005 to appear in *The Kerr spacetime: Rotating black holes in general relativity* ed Scott S et al. (Cambridge: Cambridge University Press) (*Preprint* gr-qc/0508071)
- [72] Louis-Martinez D and Kunstatter G 1995 *Phys. Rev. D* **52** 3494 (*Preprint* gr-qc/9503016)
- [73] Grumiller D, Kummer W and Vassilevich D V 2002 *Phys. Rept.* **369** 327 (*Preprint* hep-th/0204253)
- [74] Regge T and Teitelboim C 1974 *Annals Phys.* **88** 286
- [75] Carlip S 2005 in preparation
- [76] Carlip S and Teitelboim C 1995 *Class. Quant. Grav.* **12** 1699 (*Preprint* gr-qc/9312002)
- [77] Skenderis K 2000 *Lect. Notes Phys.* **541** 325 (*Preprint* hep-th/9901050)
- [78] Frolov V P, Fursaev D and Zelnikov A 2003 *JHEP* **0303** 038 (*Preprint* hep-th/0302207)
- [79] Emparan R and Sachs I 1998 *Phys. Rev. Lett.* **81** 2408 (*Preprint* hep-th/9806122)
- [80] Robinson S P and Wilczek F 2005 *Phys. Rev. Lett.* **95** 011303 (*Preprint* gr-qc/0502074)