

# SYNCHRO-CYCLOTRON EJECTOR

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The second ejection system proposed by Le Couteur<sup>1)</sup> has been analysed in greater detail. Only a summary of this work will be given as a detailed publication is being prepared.

The ejection starts near the edge of the cyclotron, at a point where the field index  $n$  changes rapidly with radius. The phase lag per revolution  $\alpha$  of the radial oscillations determines the sensitivity of these oscillations to the regenerator field. For small amplitudes this phase lag is given by

$$\alpha = 2\pi(1-\Omega) = 2\pi(1-\sqrt{1-n}) \approx n\pi$$

For large amplitudes  $n$  may vary along the orbit from 0.04 to 1.5, so that this equation can no longer be used. An exact and relatively simple equation for the radial oscillations in the median plane can be obtained, however, from the fact that both the momentum  $p$  and the angular momentum  $p_\theta = m r^2 \dot{\theta} + e r A_\theta$  are constants of motion. The following equation results

$$(dr/d\theta)^2 + F(r) = U^2$$

The constant  $U$  determines the amplitude,  $F(r)$  follows from the field shape and is shown in fig. 1. The radius  $r_1$  corresponds to the point where  $rB(r)$  has returned to the original value  $r_0B(r_0)$ . If  $r < r_1$  we can solve this equation by the transition to new variables according to

$$r \rightarrow u \quad u = \sqrt{F(r)}$$

$$\theta \rightarrow \psi \quad d\psi/d\theta = du/dr \text{ along the orbit yielding}$$

$$(du/d\psi)^2 + u^2 = U^2$$

$$u = U \sin(\psi - \psi_0)$$

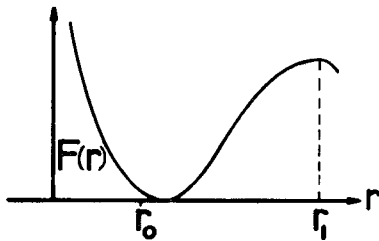


Fig. 1.

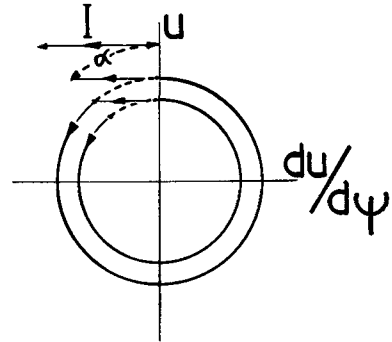


Fig. 2.

The relation between  $\psi$  and  $\theta$  can now be found by integration. The required phase lag  $\alpha$  follows as

$$\alpha = 2\pi - (\psi(\theta + 2\pi) - \psi\theta)$$

The regenerator field changes the direction of the particle as

$$\Delta dr/d\theta = \Delta du/d\psi = -r \int \Delta B/B \cdot d\theta = -J.$$

From this we get a simple picture of the orbit in the  $u, du/d\psi$  phase plane (fig. 2). The horizontal line of length  $J$  corresponds to the passage through the regenerator, the circular path of angular length  $2\pi - \alpha$  to one revolution in the cyclotron field. Such a graph gives the gain in amplitude and the phase of the oscillation. For small amplitudes the phase just before or after the regenerator

$$\alpha = (1-\Omega)2\pi \approx n\pi$$

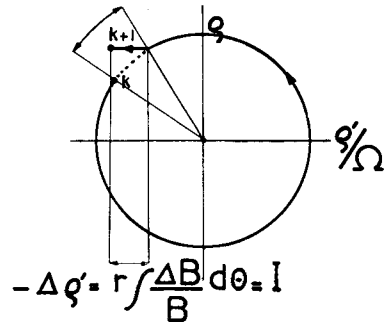


Fig. 3.

may be kept constant. In that case one can use the matrix analysis as it is used in the linear theory of the original ejection method<sup>2)</sup>.

The energy spread due to the horizontal oscillations can be estimated from the following analysis. For small amplitudes the horizontal frequency  $\Omega$  is given by  $\Omega = \sqrt{1-n}$  and the phase lag per revolution by  $\alpha = 2\pi(1-\Omega) \approx n\pi$ . The motion in the  $(r-r_0)$ ,  $dr/d\theta$  phase plane is shown in fig. 3. For relatively small values of  $\alpha$  we may describe the two successive motions corresponding to one complete revolution by a single set of differential equations

$$\begin{cases} d(r-r_0)/dk = \alpha/\Omega (dr/d\theta) \\ d(dr/d\theta)/dk = \alpha \Omega (r-r_0) - J(r) \end{cases}$$

The independent variable  $k$  is the number of the revolution. These equations may be transformed as follows

$$d^2(r-r_0)/dk^2 + \alpha^2(r-r_0) - \alpha/\Omega J(r) = 0$$

$$[d(r-r_0)/dk]^2 + 2\alpha^2 \int [r-r_0 - J/\alpha\Omega] dr = C$$

$$(dr/d\theta)^2 + 2\Omega^2 \int [(r-r_0) - J/\alpha\Omega] dr = C$$

The last equation is of the Hamiltonian type and describes an oscillation in a potential well. We assume that the tail of the regenerator field has a shape as given in fig. 4. Before the particle comes under the influence of the regenerator field the motion is sinusoidal. The change of the shape of the potential well as  $r_0$  increases may be

considered to be adiabatic. If we assume that  $C$  remains constant instead of using the fact that the area in the phase plane is constant we find that the motion becomes unstable at a value of  $r_0$ , determined by the original oscillation amplitude\*. The difference between this value of  $r_0$  and the critical value  $r_{cr}$  where a stable motion is no longer possible, is smaller than the original oscillation amplitude. A small curvature of the regenerator field shape  $J(r)$  at the point where its slope becomes critical helps to reduce the energy spread.

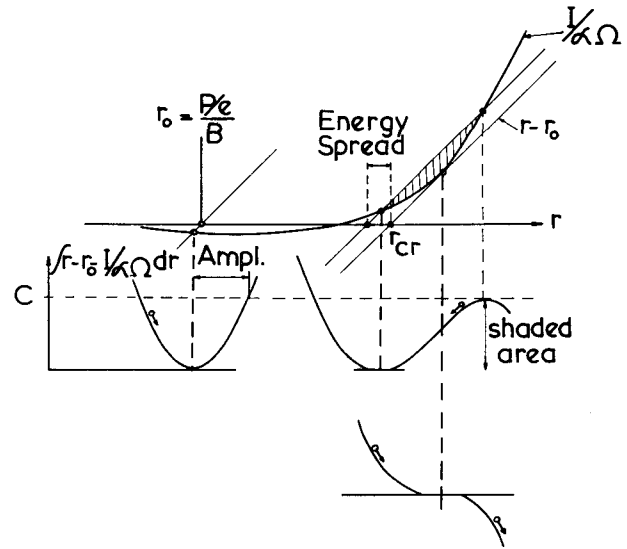


Fig. 4.

#### LIST OF REFERENCES

1. Le Couteur, K. J. and Lipton, S. Non-linear regenerative extraction of synchrocyclotron beams. *Phil. Mag.*, 46, p. 1265-80, 1955.
2. Le Couteur, K. J. The regenerative deflector for synchro-cyclotrons. *Proc. phys. Soc., Lond., B*, 64, p. 1073-84, 1951.

\* As shown in fig. 4.

## DISCUSSION

*A. Roberts* (to *N. F. Verster*): Uniform field inside the channel was shown by *N. F. Verster*. A Le Couteur design made at Rochester contained intentionally a gradient, with reversed sign in the middle which provides AG focusing, so that the beam at the output is well focused vertically and horizontally.

*N. F. Verster*: We have also tried measurements on channels with only 1 wall and with shims, and a linear field inside channel of 2.5 kG/cm. We have also shims with 1 kG/cm, giving similar results.

*A. V. Crewe*: Both these types of channel were considered in Chicago and Liverpool. They do not deflect the beam as quickly as 2 wall channels, but this is a mere geographical difficulty. We also considered methods for reducing energy spread in final beam. We found no mechanism for reducing the spread to less than 8 Mev. We could not explain the reduction in pulse length.

*G. Salvini*: Can we use this in a 1000 Mev electron synchrotron?

*A. V. Crewe*: Certainly.

*G. Salvini*: Is this better than *D. Piccioni* and *B. T. Wright*'s method, due to energy spread in the target?

*A. V. Crewe*: There are 2 points to be mentioned:

1. In some synchrotrons designed some time ago there are no straight sections. This makes *Piccioni*'s scheme difficult.
2. The effective source seems to be small in regenerative systems, in view of the small image that can be obtained from an external beam in a synchro-cyclotron.

*D. Harting*: In the injection scheme of Chicago and Liverpool, the beam losses are 85% till the entry of the magnetic channel, 80% of those 85% in the magnetic channel, which gives a final efficiency of 3%. *A. V. Crewe* pointed out that losses in magnet channels are not of nature and can be overcome. *V. P. Dmitrievski* spoke about a final efficiency of about 5%. These are concurring results. Where are the particles lost in this case?

*V. P. Dmitrievski*: The losses are not in the channel, but at the entry orbit, because we have a smaller channel than in Chicago. The channel is closer to the final ion orbit than in Chicago because the oscillation amplitudes are somewhat smaller.

*G. K. O'Neill*: Beam extraction devices using energy loss targets are already in use and presumably will continue to be valuable. However, storage rings would require much more precisely collimated beams than can be made by such devices except possibly in the Cosmotron's energy range. Systems using forced betatron oscillations based on magnetic field bumps seem more suitable for storage ring applications. The advantages obtainable by the use of storage rings make it worthwhile to put considerable effort into the development of high-efficiency ejection techniques.

*A. V. Crewe*: In order to use the resonance methods for injection we must first understand exactly how the ejection process occurs, in terms of the short pulse length and small energy spread. These factors will have a strong influence on the amount of injected beam one can obtain.

*R. Wideröe*: In our weak focusing betatron we use an ejection system which seems to be quite different from that of *V. V. Vladimirov*.

In our system we increase the orbit radius and simultaneously produce a superimposed magnetic bump over about 10% of the circumference. We thus create a force directed towards the central axis of the orbit, whereby the  $n$  value of the resulting magnetic field at this part exceeds 1. This will build up forced oscillations of the particles with an outwardly directed peal amplitude at the bump, thus causing ejection of the particles after a few turns.

This scheme is very simple and works well.

*D. W. Kerst*: This looks like the 1946 betatron regenerative peeler system with  $n$  slightly less than  $3/4$ . According to the analysis of *Hammer* and *Buro* if a little  $n$  bump is produced by the peeler short circuiting field locally, you move right into an unstable region and the beam is thrown out. This was not calculated in those days.