Influence of the magnetic interactions on the mass spectrum of elementary particles

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Abstract. Different approaches to the problem of mass quantization are discussed. The Barut ideas of crucial influence of magnetic forces for explaining the properties of the strong interaction are considered in details. It is shown that this approach gives the possibility to understand the enormous number of elementary particles (about 400) as the excited states of stable fundamental particles (e, p, ν) , bounded by magnetic interactions.

1. Introduction

The description of the mass spectrum of the observed elementary particles is included in the Ginzburg list of 30 most important unsolved problems in theoretical physics [1]. There are numerous approaches to its solution: group methods based on SU (N) - symmetries (Gell-Mann); dynamic (Barut); relational (Vladimirov); geometric (Bolokhov, Vladimirov) and many others. Interesting formulas for the masses of leptons and hadrons are obtained.

One of the people who “laid the foundation” was Nambu [2], whose idea was to connect the masses of all elementary particles known at that time with the fine structure constant. Barut was also a supporter of that idea, and in 1979 obtained a formula in the form of an empirical dependence related to lepton masses [3]:

$$m_n = m_e \left(1 + \frac{3}{2\alpha}\sum_{k=0}^{n} k^4\right) \qquad (1)$$

where m_e is the electron mass, α is the fine structure constant.

This formula is in a good agreement with the observed masses of leptons. For example, for n = 0 we get the electron mass $m_e^{theor.} = 0.510999\ MeV$ ($m_e^{exp.} = 0.510999\ MeV$); for n = 1 - the muon mass $m_\mu^{theor.} = 105.549\ MeV$ ($m_\mu^{exp.} = 105.658\ MeV$); for n = 2 the mass of the tauon $m_\tau^{theor.} = 1786.155\ MeV$ ($m_\tau^{exp.} = 1776.822\ MeV$). With a value of n = 3, the 4th lepton with a mass of $10293.711\ MeV$ is predicted, which has not yet been observed.

A little later, a Japanese physicist Yoshio Koide discovered the following relationship between the masses of leptons [4]:

$$m_e + m_\mu + m_\tau = \frac{2}{3}\left(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\right)^2 \qquad (2)$$

Expression (2) is true with a very high accuracy. Based on experimental data (2016), the ratio of the left side of (2) to the right side (on the right side without taking into account the $\frac{2}{3}$ coefficient) is obtained and is equal 0.6666605 ± 0.0000068 (in theory that ratio is $0.666666(6)$). Despite of this, no reasonable theoretical explanation of formula (2) has yet been obtained. The predicted mass of the τ-lepton from the Koide formula, turns out to be $m_\tau^{theor.} = 1786.968884 \pm 0.000065$, and the experimental value is $m_\tau^{exp.} = 1776.822$.

Varlamov [5] also presented his own formula for the particles masses. If we take into account the principle of equivalence between mass and energy, it can be argued that the mass formula

$$m = m_e\left(l + \frac{1}{2}\right)\left(i + \frac{1}{2}\right) \quad (3)$$

defining the mass (energy) of the state (cyclic Lorenz representation (l, i)), is, in a sense, similar to the well-known relation $E = h\nu$, where the electron mass plays the role of "quantum of mass" m_e.

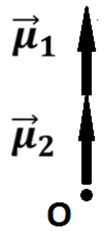
Up to present it is difficult to give preference to any of the existing approaches. In our opinion, Barut approach is more promising, as it allows one to naturally extend it to describe the spectrum of the hadron sector, which is much richer in the number of observed states.

2.Materials and methods

From the very beginning we shall indicate some not well known facts about the magnetic interaction.

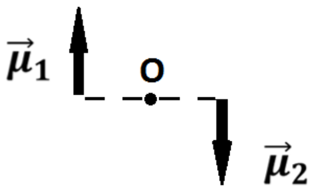
2.1. Unusual (little known) properties of magnetic forces

2.1.1 Attraction is possible at different orientation of magnetic moments.



$$W_{int} = -2\frac{\mu_1\mu_2}{r^3} < 0 \quad (4)$$

Figure 1. Coaxial parallel orientation of magnetic moments



$$W_{int} = -\frac{\mu_1\mu_2}{r^3} < 0 \quad (5)$$

Figure 2. Misaligned antiparallel orientation of magnetic moments

2.1.2. Different dependence upon a distance.

$$W_{int} \sim \pm\frac{b}{r^2} \pm \frac{c}{r^3} + \frac{d}{r^4} \quad (6)$$

2.1.3. Emergence of a repulsive core independently of $\vec{\mu}$ orientation.

$$W_{int} \sim +\frac{[\bar{\mu}\times\bar{r}]^2}{r^6} \quad (7)$$

The presence of terms with different signs in the interaction potential makes it possible to obtain for different particles with different masses and charges a large number of potential wells in which the bound states of particle systems can exist and may be observed experimentally as resonances.

2.2. Barut mass formula

To illustrate the effectiveness of the Barut method, let us deduce the above mass Barut formula (1) for leptons [3]. For a particle of mass m with a charge e moving in the field of a magnetic dipole μ we have:

$$\frac{mv^2}{r} = \frac{e\mu v}{r^3} \quad (8)$$

In the nonrelativistic case, the Bohr-Sommerfeld quantization rule can be applied:

$$mvr = n\hbar \quad , \qquad n = 0,1,2,\ldots \quad (9)$$

From (9) we find $r = \frac{n\hbar}{mv}$, then we substitute this expression into (8) and we obtain:

$$v_n = \frac{\hbar^2}{em\mu} n^2 \quad (10)$$

For kinetic energy, we have the expression:

$$E_n = \frac{mv_n^2}{2} = \frac{\hbar^4}{2e^2m\mu^2} n^4 = \lambda n^4 \quad (11)$$

On the other hand, adding the rest mass of the electron to the Nambu formula for the muon [2]:

$$m_\mu = \frac{3}{2\alpha} m_e \quad (12)$$

we get

$$m_\mu = m_e \left(1 + \frac{3}{2\alpha}\right) \quad , \qquad n = 1 \quad (13)$$

Using (11) and (13) we get the Barut formula [3]:

$$m_n = m_e \left(1 + \frac{3}{2\alpha} \sum_{k=0}^{n} k^4\right) \quad (14)$$

Here $n = 0$ for electron; $n = 1$ for muon; $n = 2$ for tauon; $n = 3$ for the 4-th lepton.

2.3. The main ideas of Barut

As it is well known, the quarks show no existence in nature (nobody observed them up to now). Observed elementary particles (more than several hundreds) according to Barut can be described as bounded states of a small number of really stable particles p, e^-, ν .

The features of strong interactions, such as:

1) Short interaction range;
2) Saturation;
3) Charge independence (isotopic invariance);
4) Strong spin dependence;
5) Pairing;
6) Pauli principle;
7) Experimentally observed quark potential;

$$V(r) = \frac{a}{r} + br + c \quad (15)$$

can be explained strictly by electromagnetic forces only.

2.4. Dirac equation with electromagnetic interaction

From the Dirac Equation

$$(\gamma^\mu \partial_\mu + \mathrm{m})\Psi = 0 \quad (16)$$

describing a free particle, one can obtain an extended Dirac equation for a charged particle interacting with an external electromagnetic field in two steps.

1) The extension of derivatives $\partial_\mu \to \partial_\mu - ieA_\mu$ results in the appearance of additional terms

$$\mu_n\, \rho_3(\bar{\sigma}\cdot\bar{H}) + \mu_n\, \rho_1(\bar{\sigma}\cdot\bar{E}) \tag{17}$$

in the Hamiltonian operator, where μ_n is the normal magnetic dipole moment of the charged particle.

2) For a neutral particle with an abnormal (anomalous) magnetic dipole moment (such as a neutron), one has to add on the right hand side of the Dirac Equation , the Pauli coupling term

$$0 \;\rightarrow\; \mu_a F_{\mu\nu}\sigma^{\mu\nu} = \mu_a\, \rho_3(\bar{\sigma}\cdot\bar{H}) + \mu_a\, \rho_1(\bar{\sigma}\cdot\bar{E}) \tag{18}$$

where μ_a is the abnormal magnetic dipole moment.

The above two steps, when applied to the electron moving around a proton, yield a radial equation with an effective potential of the following form:

$$V(r) = \pm\frac{a}{r} + \frac{b}{r^2} \pm \frac{c}{r^3} + \frac{d}{r^4} \tag{19}$$

The coefficients a, b, c, d are obtained automatically and they are fixed in the model and their explicit form will be given below.

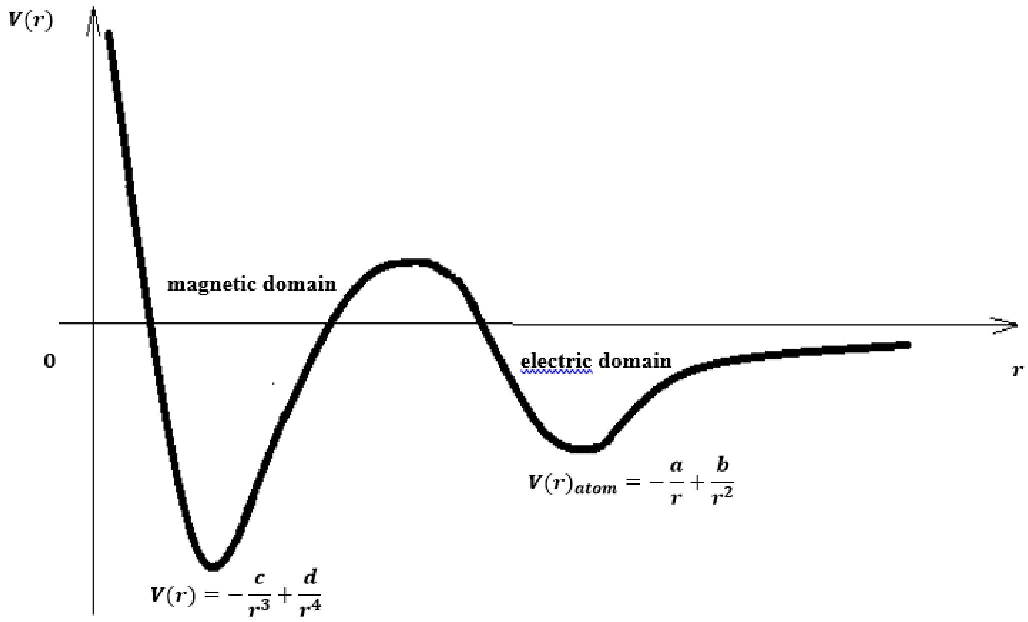


Figure 3. Effective interaction potential of the electron with the proton in the Barut model

Figure 3 shows two potential wells that are obtained for the "electron-proton" system, for given values of the parameters a, b, c, d which are automatically fixed in the model as a result of the above procedure (two steps) to turn on the interaction.

The right potential well has a minimum at $r \approx 10^{-8}$ cm. In this well the terms $-\frac{a}{r} + \frac{b}{r^2}$ play the main role. We call this domain electric. Other terms in (19) related to magnetism give small corrections. It is in this well that a familiar bound state the hydrogen atom arises. In the left potential well with a minimum at $r \approx 10^{-13} - 10^{-14}$ cm, the short-range magnetic forces play the main role, and the electric forces give small corrections.

Here the formation of heavy particles is possible due to magnetism, which in the experiment will look like resonances. In the case of a coupled system of two heavy particles (for example, a neutron-proton), the left well is interpreted in the Barut model as a well that reproduces all the properties of strong interaction with small electromagnetic corrections from the right well.

2.5. Effective potential for two charged particles with normal and anomalous magnetic moments
Let us consider two interacting charged particles with normal and anomalous magnetic moments. The full Hamiltonian of the system in this most general case is as follows [6]:

$$H = \frac{1}{2m_1}\left(\vec{p}_1 - \frac{e_1}{c}\vec{A}_2\right)^2 + \frac{1}{2m_2}\left(\vec{p}_2 - \frac{e_2}{c}\vec{A}_1\right)^2 + \frac{e_1 e_2}{|\vec{r}_1 - \vec{r}_2|} + S_{12}(\vec{r}_1 - \vec{r}_2) \qquad (20)$$

Here $\vec{A}_1 = \frac{\vec{M}_1 \times (\vec{r}_2 - \vec{r}_1)}{|\vec{r}_1 - \vec{r}_2|^3}$, $\vec{A}_2 = \frac{\vec{M}_2 \times (\vec{r}_1 - \vec{r}_2)}{|\vec{r}_2 - \vec{r}_1|^3}$ are vector potentials of the electromagnetic field created by one particle at the point of location of the other particle; $\vec{M} = \frac{e\hbar}{2mc}(1+a)\vec{\sigma}$ is the intrinsic magnetic moment of a charged particle with spin $1/2$, proportional to the Bohr magneton; a is a parameter that determines the magnitude of the intrinsic anomalous magnetic moment of the particle; $\vec{\sigma}$ is the particle spin operator; $e_i, m_i (i = 1,2)$ are charges and masses of particles; the last term describes the spin-spin interaction of the intrinsic magnetic moments of the particles. It is usually written as:

$$S_{12}(\vec{r}_1 - \vec{r}_2) = \frac{1}{r^3}\left[\left(\vec{M}_1\vec{M}_2\right) - 3\left(\vec{M}_1\vec{r}_0\right)\left(\vec{M}_2\vec{r}_0\right)\right] \quad , \qquad \vec{r}_0 = \frac{\vec{r}}{|\vec{r}|} \qquad (21)$$

After the transition to the center of mass system, the Hamiltonian takes the form:

$$H = \frac{\vec{p}^2}{2\mu} + \frac{e_1 e_2}{r} - \frac{1}{r^3}\left[\vec{L}\left(\frac{e_1\vec{M}_2}{m_1 c} + \frac{e_2\vec{M}_1}{m_2 c}\right)\right] + \frac{e_1^2}{2m_1c^2}\left(\frac{\vec{M}_2 \times \vec{r}}{r^3}\right)^2 + \frac{e_2^2}{2m_1c^2}\left(\frac{\vec{M}_1 \times \vec{r}}{r^3}\right)^2 + \frac{M_1 M_2}{r^3}\left[\vec{S}^2 - 3\left(\vec{S}\vec{r}_0\right)^2\right] \qquad (22)$$

Here $\mu = \frac{m_1 + m_2}{m_1 m_2}$ is the reduced mass; $\vec{r} = \vec{r}_2 - \vec{r}_1$; $\vec{p} = \frac{m_1\vec{p}_1 - m_2\vec{p}_2}{M}$; $M = m_1 + m_2$; $\vec{P} = \vec{p}_1 + \vec{p}_2$; $\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{M}$ is the radius of the mass center; $\vec{L} = \vec{r} \times \vec{p}$; $\vec{S} = \frac{1}{2}(\bar{\sigma}_2 + \bar{\sigma}_1)$ is the operator of the total spin of a system of two particles.

Formula (22) gives the effective potential of interaction for the radial function in the form:

$$V(r) = \frac{b_1}{r} + \frac{b_2}{r^2} + \frac{b_3}{r^3} + \frac{b_4}{r^4} \qquad (23)$$

In this expression the centrifugal potential $\sim \frac{1}{r^2}$ appeared as a result of variables separation of the Laplacian inside the term $\frac{\vec{p}^2}{2\mu} = -\frac{\hbar^2 \Delta_{r,\varphi,\theta}}{2\mu}$.

The coefficients b_1, b_2, b_3, b_4 are of the form [6]:

$$b_1 = e_1 e_2$$

$$b_2 = \hbar^2 l(l+1)$$

$$b_3 = \frac{e_1 e_2 \hbar}{2m_1 m_2 c^2}\left[\vec{L}(a_2\bar{\sigma}_2 + a_1\bar{\sigma}_1)\right] + \frac{e_1 e_2 \hbar}{m_1 m_2 c^2}\left[\vec{L} \cdot \vec{S}\right] + \frac{e_1 e_2 \hbar^2 (1+a_2)(1+a_1)}{4m_1 m_2 c^2}\left[\vec{S}^2 - 3\left(\vec{S} \cdot \vec{r}_0\right)^2\right]$$

$$b_4 = \frac{e_1 e_2 \hbar^2}{4m_1 m_2 c^4}\left(\frac{(1+a_1)^2}{m_1} + \frac{(1+a_2)^2}{m_2}\right) \qquad (24)$$

In general case, both the relativistic and nonrelativistic descriptions of two interacting fermions do not allow to completely solve the problem analytically. The main advantage of a relativistic description is a wide range of allowable energies. However, as a result, the final equations are complex and analytically solvable only for a small number of fields, often with a very specific choice of parameters. The scope of nonrelativistic equations is naturally limited by the range of permissible energies. The advantages of a nonrelativistic analysis are the relative simplicity of the equations and the possibility of easy comparison with known results, as well as the ability to use fewer “adjustable” parameters.

3. Conclusion

We presented in a brief form the main ideas of Barut and gave a general view of the potential of interaction of two charged particles with normal and anomalous magnetic moments. Using the general expression (20) for the Hamiltonian, a number of problems were solved earlier. In the case of the “ep” system, it was predicted the possible existence of small Barut-Vigier atoms with a size of $r \approx 10^{-11}$ cm [7]. The use of expression (20) for the bound “neutron-proton” system (deuterium nucleus) turned out to be very effective [8], allowing us to describe the basic properties of the deuteron. For example, the absence of a singlet state in a deuteron on the experiment is easily proved. In this case, the spins of the particles are antiparallel (which means that the magnetic moments are parallel due to the fact that $\mu_p = +2.7\frac{e\hbar}{2M_pc}$, $\mu_n = -1.9\frac{e\hbar}{2M_nc}$) and there will be no potential well due to the repulsion of the magnetic moments. There is hope using the general formula (20) to obtain a more accurate mass spectrum of light and heavy particles.

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