

SIMULATION OF 4D EMITTANCE MEASUREMENT AT THE SPALLATION NEUTRON SOURCE

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Abstract

Current experiments at the Spallation Neutron Source (SNS) seek to paint a beam in the accumulator ring which is highly correlated in transverse phase space. It is desirable to know the four-dimensional (4D) emittance of such a beam. To measure the 4D emittance, we will utilize a standard method of reconstructing the covariance matrix using various optics settings in conjunction with beam profile measurements. We present the results of preliminary simulations which aim to optimize this measurement scheme for the SNS Ring-Target-Beam-Transport (RTBT) line.

INTRODUCTION

The Danilov distribution is self-consistent, i.e., it gives rise to linear space charge forces as long as all external forces are linear [1]. Its transverse distribution function is characterized by linear relationships between the particle positions and slopes:

$$f \propto \delta(x' + e_{11}x + e_{12}y) \delta(y' + e_{21}x + e_{22}y). \quad (1)$$

Figure 1 shows an example of the phase space projections of the beam. Space charge linearly couples the x and y dynamics; as a consequence, the 4D emittance ε_{4D} , although still invariant, is no longer the product of the horizontal and vertical emittances. It is instead given by

$$\varepsilon_{4D} = \varepsilon_1 \varepsilon_2 = |\Sigma|, \quad (2)$$

where Σ is the covariance matrix and $\varepsilon_{1,2}$ are called the *intrinsic* emittances as opposed to the *apparent* emittances $\varepsilon_{x,y}$. The relationships in Eq. (1) cause ε_{4D} , and therefore at least one of the intrinsic emittances, to be to zero.

It was proposed in [1] that the Danilov distribution could be realized in practice using phase space painting. Simulations predict that the beam can be stable and retain its key features to a reasonable degree when injected into the SNS accumulator ring [2]. Efforts to paint the beam in reality are underway. One issue is how to quantify the level to which the painted beam approximates a true Danilov distribution; the 4D emittance is ideal for this purpose since it is zero in the case of perfect agreement and climbs to the product of the x and y emittances in the worst (uncorrelated) case. As shown in Eq. (2), measuring this emittance requires the reconstruction of all ten second-order moments of the distribution.

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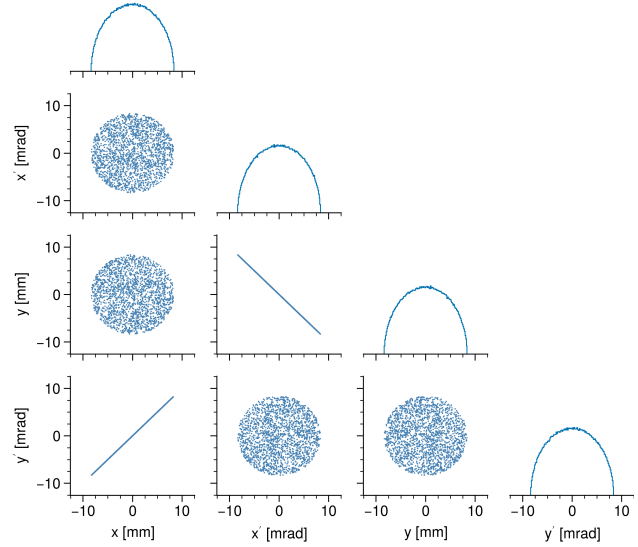


Figure 1: The transverse phase space projections of the Danilov distribution.

MEASUREMENT METHOD

We plan to utilize the well-established quadrupole scan technique [3] to reconstruct the covariance matrix Σ_A at position s_A . The method is as follows. Neglecting space charge, the covariance matrix Σ_B at position s_B downstream of s_A is

$$\Sigma_B = M \Sigma_A M^T, \quad (3)$$

where M is the linear transfer matrix from s_A to s_B . Equation (3) can be expanded to write $\langle x^2 \rangle$, $\langle y^2 \rangle$, and $\langle xy \rangle$ at s_B as a linear combination of the ten moments at s_A , the coefficients of which are determined by M . Changing the transfer matrix, either by changing the lattice optics or by changing s_B , gives three new equations. Taking n measurements gives $3n$ equations which we write in the following form:

$$\mathbf{A}\mathbf{x} = \mathbf{b}, \quad (4)$$

where $\mathbf{x}$ is a ten-element vector of the moments at s_A , A is a $3n \times 10$ matrix determined by the transfer matrix elements, and $\mathbf{b}$ is a $3n$ element vector determined by the real-space moments at s_B . We can then search for the moment vector $\mathbf{x}$ which minimizes $|\mathbf{A}\mathbf{x} - \mathbf{b}|^2$.

The real-space moments can be measured by scanning horizontal, vertical, and diagonal wires across the path of the beam, and the transfer matrix can be modified by changing the strengths of quadrupoles between the reconstruction point and the wire-scanner. This will need to be done in the Ring-Target-Beam-Transport (RTBT) section of the machine

after the beam has been accumulated in the ring. Figure 2 shows the lattice functions as well as the locations of five wire-scanners in the RTBT. All five wire-scanners can be used to reconstruct the moments at the left-most position in the figure. We present here initial simulations to assess the feasibility of this technique in the SNS.

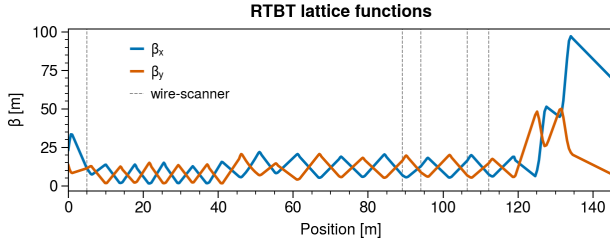


Figure 2: Optics and wire-scanner locations in the RTBT.

SIMULATION RESULTS

We chose to linearly vary the horizontal and vertical phase advances at the last wire-scanner (ws24) in a 180 degree window centered on the default phase advances, as shown in Fig. 3. The measurement was simulated in PyORBIT by varying the quadrupoles to obtain the desired phase advances, launching an ideal, matched Danilov distribution from the reconstruction point, tracking through the RTBT, and recording the real-space moments and transfer matrix elements at the five wire-scanner locations. Twelve steps are taken, which we estimate to take around one hour in reality. The reconstructed emittances are nearly identical to the true emittances if no sources errors are considered in the simulated measurement. There exist, however, various sources of error such as uncertainty in the measured moments, quadrupole field and alignment errors, fringe fields, uncertainty in the beam energy, chromaticity, beam mismatch at the reconstruction point, and space charge forces. A detailed analysis is not performed in this work; instead, we provide rough estimates for a subset of these sources of error and observe their effect on the reconstructed emittances. Future work will refine these estimates and consider all known sources of error.

Figure 4 shows the reconstructed (blue) and true (red) emittances for 200 different trials. In each trial, a random

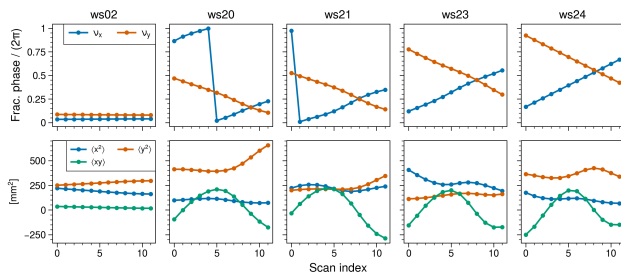


Figure 3: Phase advance (top row) and beam moments (bottom row) at each wire-scanner during a simulated measurement with no sources of error considered.

error was added to the measured beam moments ($\pm 5\%$), quadrupole tilt angles (± 1 mrad), quadrupole field strengths ($\pm 1\%$), beam kinetic energy (± 3 MeV), or beam Twiss parameters at the reconstruction point ($\pm 5\%$), with all other effects turned off, and the beam was tracked using the envelope model [4]. All these effects are considered simultaneously in the last column. Figure 5 then shows the normalized phase space at the reconstruction point, as well as lines obtained by transporting point (σ_x, x') from the measurement point to the reconstruction point, where σ_x is the beam size measured in the simulation and x' is the unknown slope (same for y). These lines are ideally tangent to the phase space ellipse. The reconstructed intrinsic emittances are $\varepsilon_1 = 0.57 \pm 0.63$ and $\varepsilon_2 = 40.66 \pm 1.04$ with true values 0.0 and 40.0, and the apparent emittances are $\varepsilon_x = 21.04 \pm 0.75$ and $\varepsilon_y = 20.31 \pm 0.55$ with true values 20.0 and 20.0, all in units of mm mrad. The errors are tolerable, with the largest being due to beam mismatch at the reconstruction point. It was found in [3] that significant mismatch, especially in the alpha parameter, can have a large effect on the accuracy due to a sub-optimal range of phase advances being covered in the scan.

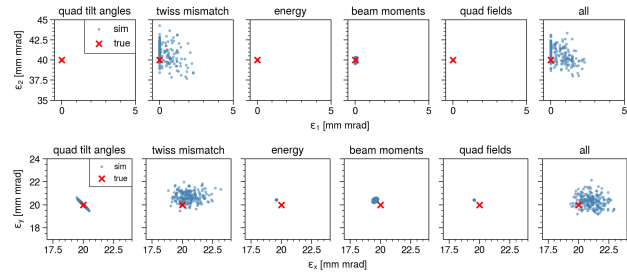


Figure 4: Reconstructed intrinsic (top) and apparent (bottom) emittances from simulation with the inclusion of various sources of error.

Another effect to consider is that of space charge forces which can render the quadrupole scan method invalid for high-perveance beams [5]. Figure 6 shows the measured emittances as a function of the beam intensity at a kinetic energy of 0.8 GeV with no other effects considered. The maximum intensity in the SNS is around 1.5×10^{14} , and it seems that space charge does not greatly affect the motion in this intensity region over the length of the RTBT.

CONCLUSION

Simulations were performed to determine the feasibility of measuring the intrinsic emittances of a painted beam in the SNS using the quadrupole scan technique. The diagnostics and optics in the RTBT section of the SNS were found to be sufficient for this task. No inhibitive sources of error were found among those considered in this work, including uncertainty in quadrupole tilt angles and field strengths, beam energy, measured moments, and beam Twiss parameters at the reconstruction point. It was suggested that matching the beam to the lattice optics is important to ensure an accurate

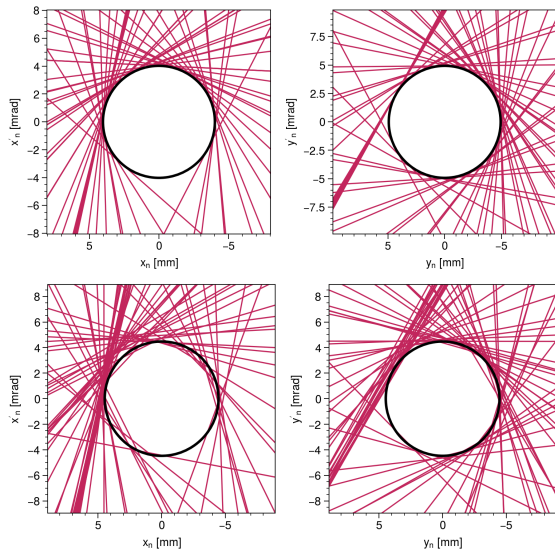


Figure 5: Normalized $x-x'$ and $y-y'$ phase space at the reconstruction point for one of the trials in Fig. 4 with (top row) and without (bottom row) the inclusion of sources of error. Pink lines: obtained by transporting the measured beam backwards from the measurement location with an unknown slope. Black lines: horizontal and vertical phase space ellipses defined by the lattice optics.

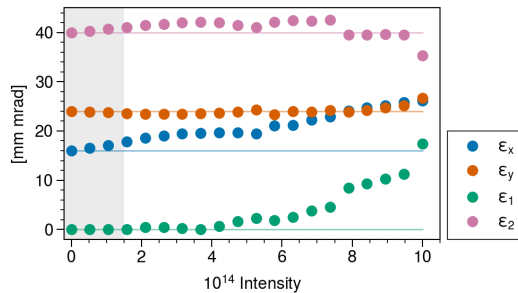


Figure 6: Simulated measured emittances vs. beam intensity. Intensities in the grey region are obtainable in SNS.

reconstruction. The reconstruction accuracy was also examined as a function of the beam intensity, and it was found that the method should remain valid at realistic SNS intensities.

Future work will refine these error estimates and consider a few additional sources of error such as fringe fields, wire-

scanner orientation, and chromaticity [6]; however, it is not expected that these will dramatically change the accuracy of the method. It is therefore concluded that the quadrupole method should be feasible to perform and can be used to measure the level to which a painted beam resembles an ideal Danilov distribution. The method eventually needs to be simulated with a beam which has been produced by a full injection simulation with realistic effects included. Finally, the measurement will be carried out experimentally and compared with simulations.

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