SLAC-TN-63-38
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SIMPLE SHIELDING CALCULATIONS

I. Summary

We describe a simple method for calculating radiation levels in the case of a thick shield protecting against high energy neutrons.

For giant resonance neutrons we obtain the useful figure of 1 neutron/5 BeV of absorbed electron energy.

II. Comparison of Usual and Simple Calculations for High Energy Neutrons

Denote by $d^2n/dTd\Omega$ the yield of high energy neutrons (differential in energy and solid angle) at an angle θ arising from the absorption of a high energy electron. In a shield the neutrons are attenuated with an energy dependent removal mean free path, $\lambda(T)$. The flux of neutrons penetrating a shield of thickness H is

$$\frac{dn}{d\Omega} = \int_0^{T_{max}} \left(\frac{d^2n}{dTd\Omega}\right) \left[\exp - H/\lambda(T)\right] dT \qquad (1)$$

In general the effective value of H is a function of θ because of the geometry. Below $T \sim 150$ MeV the absorption is very strong, and we usually begin the integration at $T = 100$ MeV.

The calculation of $d^2n/dTd\Omega$ is complicated, and it is described in detail in SLAC-9. We make the approximation that all neutrons are produced by two-body reactions and we use two-body kinematics on free nucleons. For a single incident electron, we have

$$\frac{d^2n}{dTd\Omega} = \sum_i \left(\frac{0.57\,E_0}{k^2}\right) X_0 N_0 \,\frac{K\sigma_i}{4\pi}\, \frac{\partial(k,\theta^*)}{\partial(T,\theta)} \qquad (2)$$

E_0	Electron energy
k	Photon energy
X_0	Radiation length
N_0	Avogadro's number
$K\sigma_i$	Total effective single nucleon cross section for i-th reaction (it can be a function of k and θ^*)
$\partial(k,\theta^*)/\partial(T,\theta)$	Jacobian to transform from k, Ω^* to T, Ω.

Neutrons can arise from several reactions, so the flux in Eq. (2) is a sum over all reaction leading to neutrons with the same (T, θ). We consider the reactions

1. $\gamma + N \rightarrow N + \pi$ } with subsequent neutron ejection by the pion
2. $\rightarrow N +$ "di pion" }
3. $\gamma + N \rightarrow \pi + n$

4. $\longrightarrow$ "di pion" + n

5. $\gamma + d \longrightarrow p + n$ pseudo deuteron model

(N is a general nucleon, n is a neutron.)

In general we use the experimental total cross section and assume that it is isotropic in the center of mass.

In our previous shielding calculations we have carried out the integrations in Eq. (1). Following Moyer we convert neutron fluxes into radiation levels by using a factor 1.25×10^{-7} rem/n cm^{-2}, or equivalently 0.45 (mrem/hr)/(n/cm^2-sec).

Moyer approximates Eq. (1) by

$$\frac{dn}{d\Omega}(\theta, H) \approx (\exp - H/\lambda_1) \int_{150\ \mathrm{MeV}}^{T_{max}} \left(\frac{d^2n}{dTd\Omega}\right) dT \qquad (3)$$

In order to compare Eq. (1) and Eq. (3) we calculate the radiation levels in the two ways for several typical geometries.

In Fig. 1 we show

$$\frac{dn}{d\Omega} = \int_{\epsilon}^{T_{max}} \left(\frac{d^2n}{dTd\Omega}\right) dT \qquad (4)$$

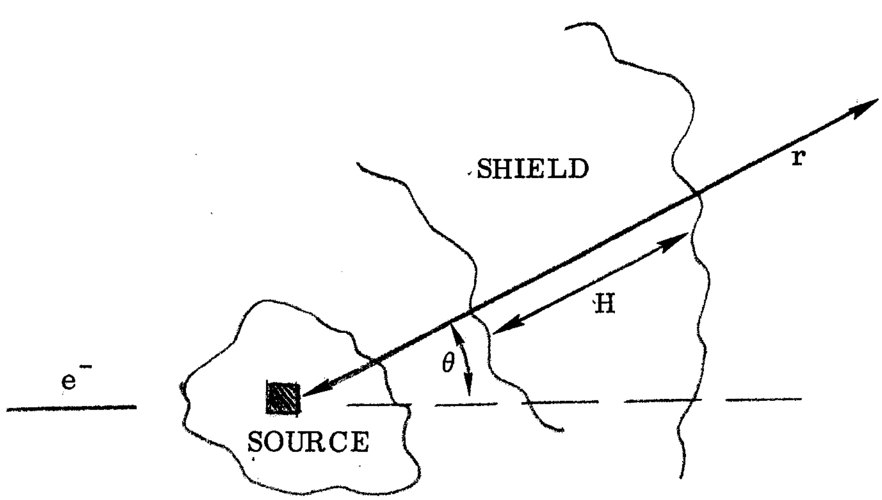
for a 2.7 MW electron beam incident on copper with ϵ = 100, 150, 200 MeV. In the forward directions the differences are not great, but in the backward directions the differences are about a factor of 8. In Fig. 2 for ϵ = 150 MeV, we show the relative contribution of the various reactions. These yields have been checked in the range $50^{\circ} \leq \theta \leq 95^{\circ}$ with 875 MeV bremsstrahlung (see SLAC-9). The pseudo-deuteron model, reaction 5, contributes a modest fraction; single π production followed by interaction of the π in the same nucleus, reaction 1, dominates beyond about 45°; at small angles the recoil neutrons from pion production, reactions 3 and 4, dominate.

In Fig. 3 we compare the radiation levels calculated using Eq. (1) ("usual calculation") and Eq. (3) ("simple calculation") with ϵ = 150 MeV for a 1 MW point source on copper in the forward shielding geometry. The agreement is pretty good. In Fig. 4 we compare the peak radiation level for a point source and a cylindrical shield, and the agreement is fair. The "simple calculation" is conservative in that it gives neutron fluxes that are larger than those calculated using the "usual calculation." In Fig. 4 a value of λ_i about 9% less would give very good agreement; however, the value used gives satisfactory agreement at smaller angles.

Generally, the simple calculations agree well enough with the usual ones so that one can use the simple method for shielding estimates. The procedure is to calculate the neutron flux outside of the shield from a knowledge of the beam power loss and shield geometry using Fig. 1 for $dn/d\Omega$ (with ϵ = 150 MeV). For λ_1 use 158 g/cm^2 for earth and light concrete. For heavier elements λ_1 is greater because $\sigma \propto A^m$. For a black nucleus $m = 2/3$, but for real nuclei m is somewhat greater than 2/3. For neutrons in the energy range 0.3 to 3 BeV the reaction cross section is constant and has $m = 0.75$ (see data summary in UCRL-8966). For pions, an optical model calculation shows that the absorption cross section has m varying from 0.70 to 0.85 for pion energies from 100 to 500 MeV E. M. Sternheimer, Phys. Rev. 101, 384 (1956) . For fission neutrons m is 0.42 (Price, Horton, and Spinney, Radiation Shielding, p. 181). For λ_i we use an m value of 0.75 (note that in SLAC-9 we used $m = 2/3$, see pp. 48-51) and find the following valuse of λ_i:

	A	ρ g/cm^3	λ_1 g/cm^2	λ_1 feet
Earth	20	1.8	158	2.88
Light Concrete	20	2.3	158	2.23
Heavy Concrete	35	3.7	182	1.61
Iron	55.8	7.8	204	0.86

Assume that the beam power loss per unit volume is dP/dv (P in MW).



Then the radiation level is

$$D(\text{mrem/hr}) = \frac{0.45}{2.7}\, a \int_{\text{source}} \frac{dP}{dv} \frac{dn(\theta)}{d\Omega} \frac{e^{-H/\lambda_1}}{r^2}\, dv \tag{5}$$

a	Ratio of radiation length in target to radiation length in copper
r	Distance from source element in cm
0.45	$(\text{mrem/hr})/(\text{n/cm}^2 \cdot \text{sec})$
2.7	Gives $dn/d\Omega$ per MW

For reference, radiation worker tolerance is 0.75 mrem/hr (30 mrem/40 hour week; 1.5 rem/50 week year).

III. Giant Resonance Neutrons

We calculate the yield of giant resonance neutrons in various materials using Approximation A for the differential photon track length. For one incident electron the number of neutrons produced is (assuming one neutron per interaction)

$$n = \int .57 \frac{E_0}{k^2} X_0 \frac{N_0}{A} \sigma(k)\, dk \tag{6}$$

$$n \approx .57 \frac{N_0}{A} X_0 \frac{E_0 \int \sigma dk}{k_0^2}, \text{ neutrons/electron} \tag{7}$$

E_0	Incident electron energy
k_0	Photon energy at peak, of giant resonance
N_0	Avogadro's number
A	Atomic weight of target
X_0	Radiation length of target
$\sigma(k)$	Giant resonance cross section

From the dipole sum rule one would expect approximately

$$\int \sigma dk = 60 \frac{Z(A - Z)}{A} \text{ MeV - mb} \tag{8}$$

(See, for example, J. S. Levinger, Nuclear Photo-disintegration, Oxford, 1960.)

From the definition of radiation length,

$$\frac{1}{X_0} = r\alpha \frac{N_0}{A} Z^2 r_e^2 \ln 183/Z^{1/3} \tag{9}$$

we have

$$n = \frac{.57}{4\alpha} \frac{(A - Z)}{AZ \ln 183/Z^{1/3}} \frac{E_0}{d_0} \frac{60}{k_0 r_e^2} \tag{10}$$

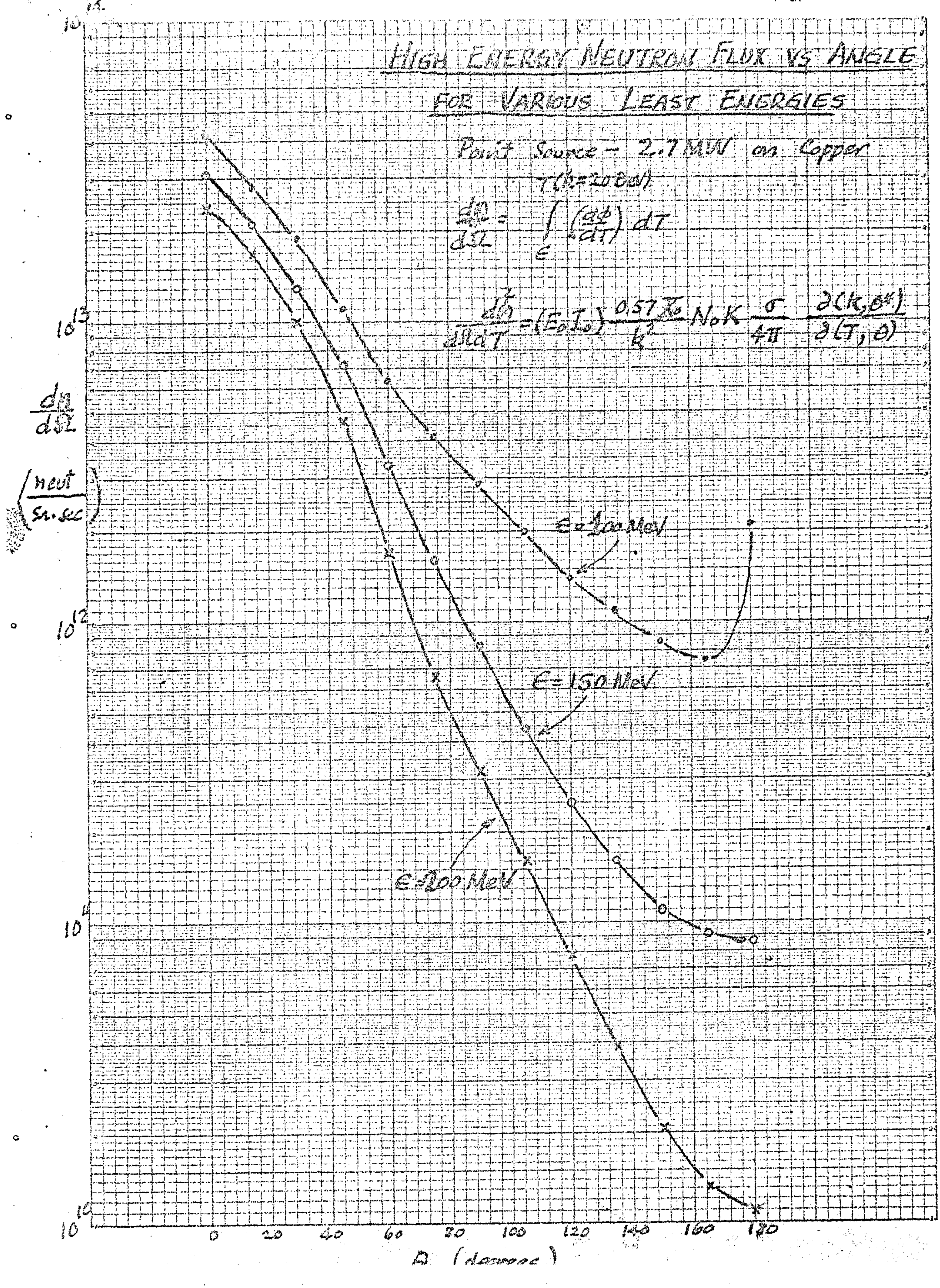
Typically $k_0 = 20$ MeV, so $k_0 r_0^2 = 1600$ MeV-mb, and for $E_0 = 1$ BeV we have,

$$n = 37 \frac{A - Z}{AZ \ln 183/Z^{1/3}} \tag{11}$$

This is shown in Fig. 5 as a function of Z. Also shown in Fig. 5 is a curve using values of $\bar{\sigma}$dk derived from a smooth curve based on a number of measured values of the integrated (γ,n) cross section. Figure 5 should be good to about a factor of 2.

The yield is fairly constant and a value of 1/5 neutron/BeV of absorbed energy is reasonable to use for all materials. The giant resonance neutrons are approximately isotropic, and have a typical "boil-off" energy spectrum which peaks around 1 MeV and decreases rapidly at higher energies. In light concrete these are attenuated with $L_{1/2} \approx 2$ inches.

FIG 1



HIGH ENERGY NEUTRON FLUX VS ANGLE
FOR VARIOUS LEAST ENERGIES
Point Source – 2.7 MW on Copper
$\frac{dn}{d\Omega} = \int_{\epsilon}^{T(k=20\,BeV)} \left(\frac{d\phi}{dT}\right) dT$
$\frac{d^2n}{d\Omega dT} = (E_0 I_0) \frac{0.57 X_0}{k^2} N_0 K \frac{\sigma}{4\pi} \frac{\partial(k,\theta^*)}{\partial(T,\theta)}$
ϵ = 100 MeV
ϵ = 150 MeV
ϵ = 200 MeV
$\frac{dn}{d\Omega}$ $\left(\frac{neut}{sr \cdot sec}\right)$
10^{14}
10^{13}
10^{12}
10^{11}
10^{10}
0 20 40 60 80 100 120 140 160 180
θ (degrees)

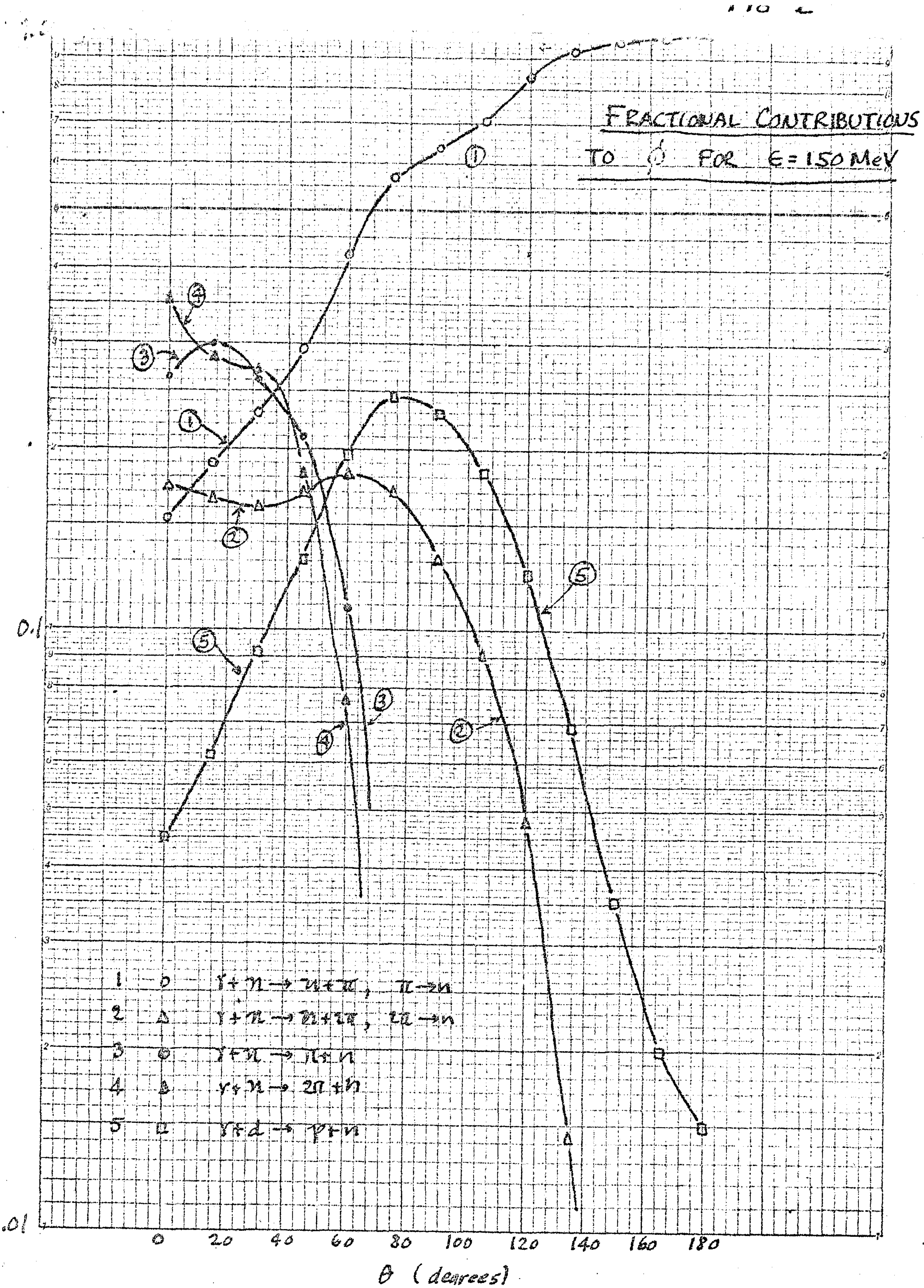
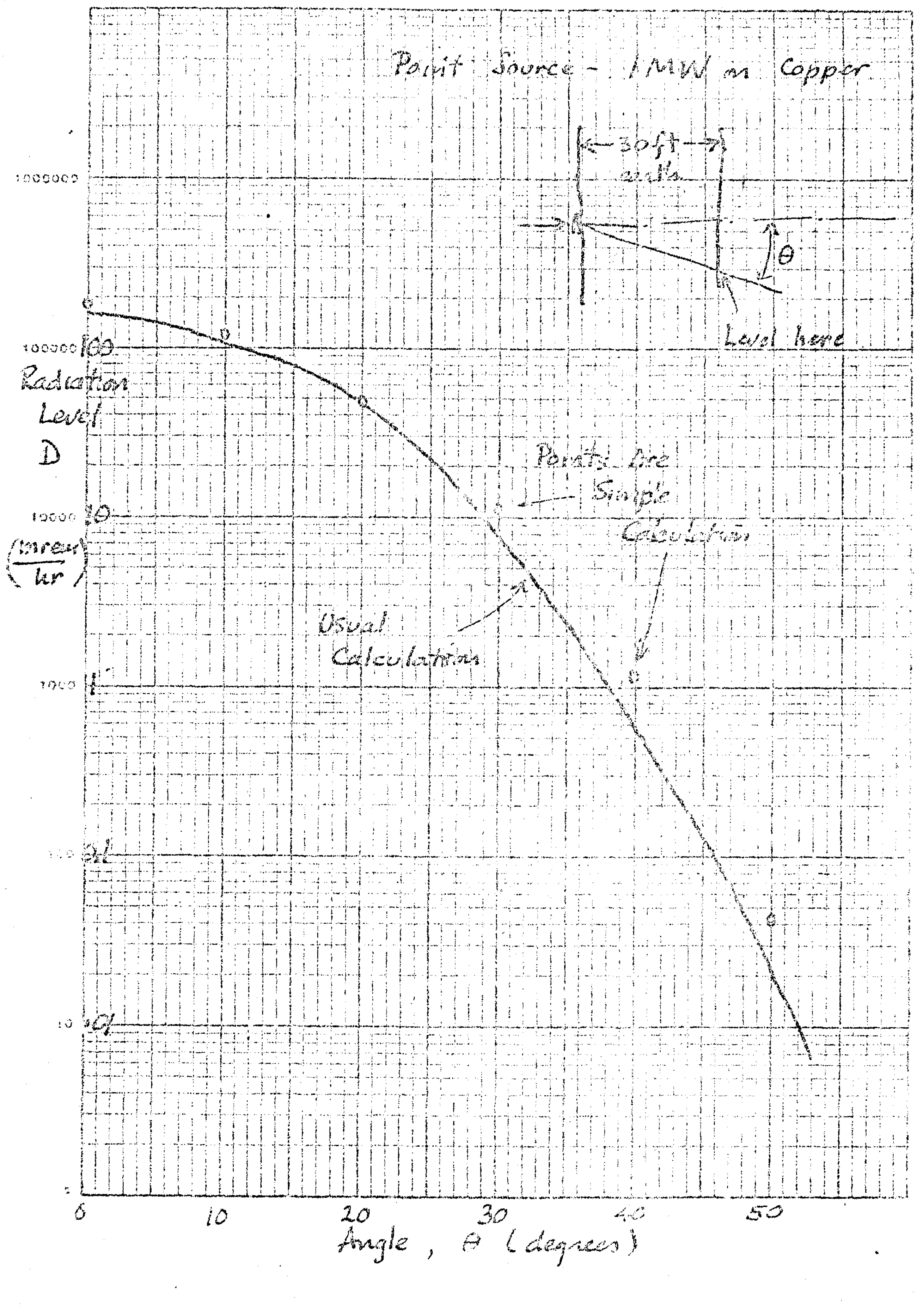

FRACTIONAL CONTRIBUTIONS
TO ϕ FOR ϵ = 150 MeV
1 o γ+n → n+π, π→n
2 Δ γ+n → n+2π, 2π→n
3 ● γ+n → π+n
4 ▲ γ+n → 2π+n
5 □ γ+d → p+n
0.1
.01
0 20 40 60 80 100 120 140 160 180
θ (degrees)

FIG. 3

RADIATION LEVEL VS ANGLE - FORWARD SHIELDING

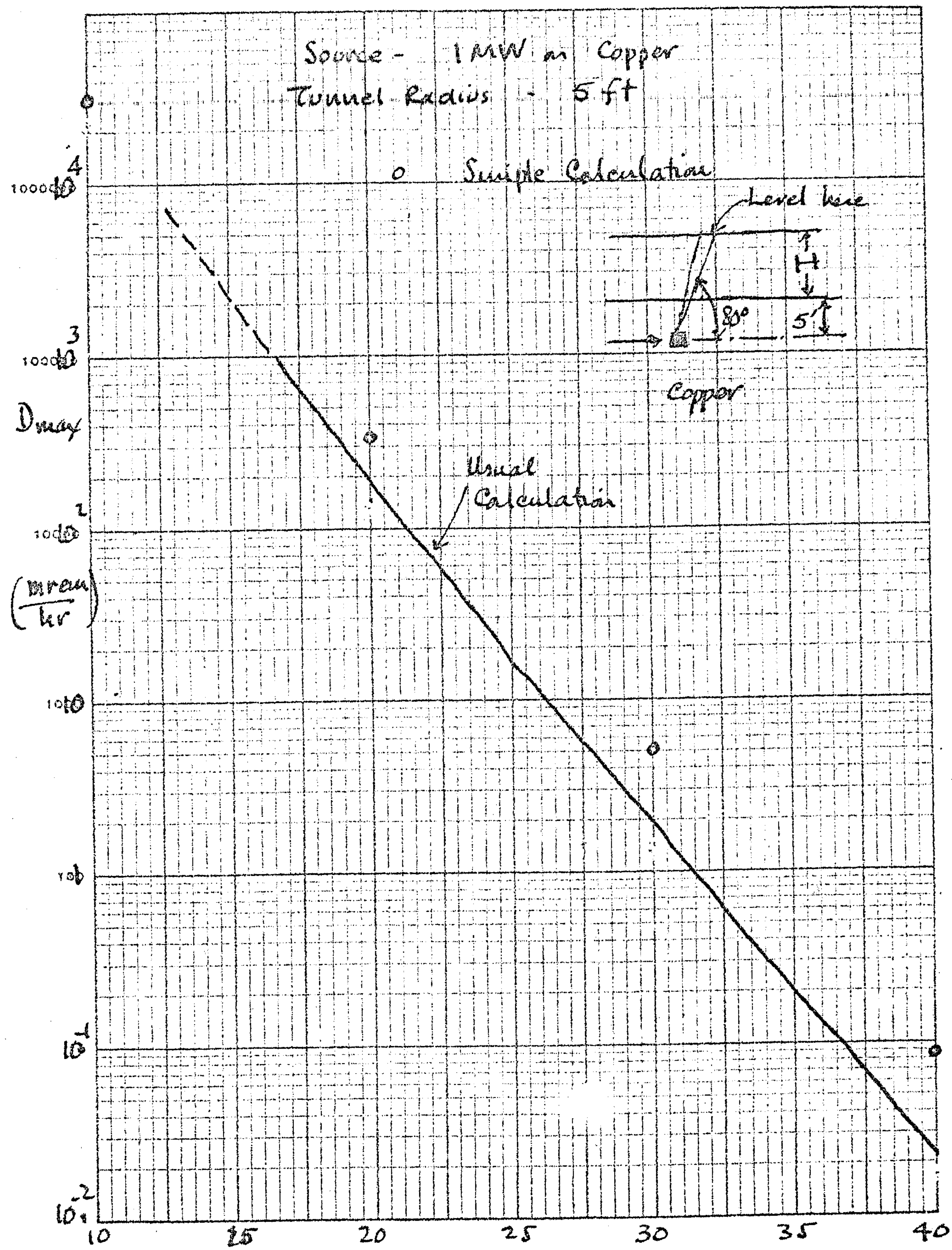
MODEL NEUTRONS ONLY DATE



Peak Dose Rate - Cylindrical Shield and Point Source

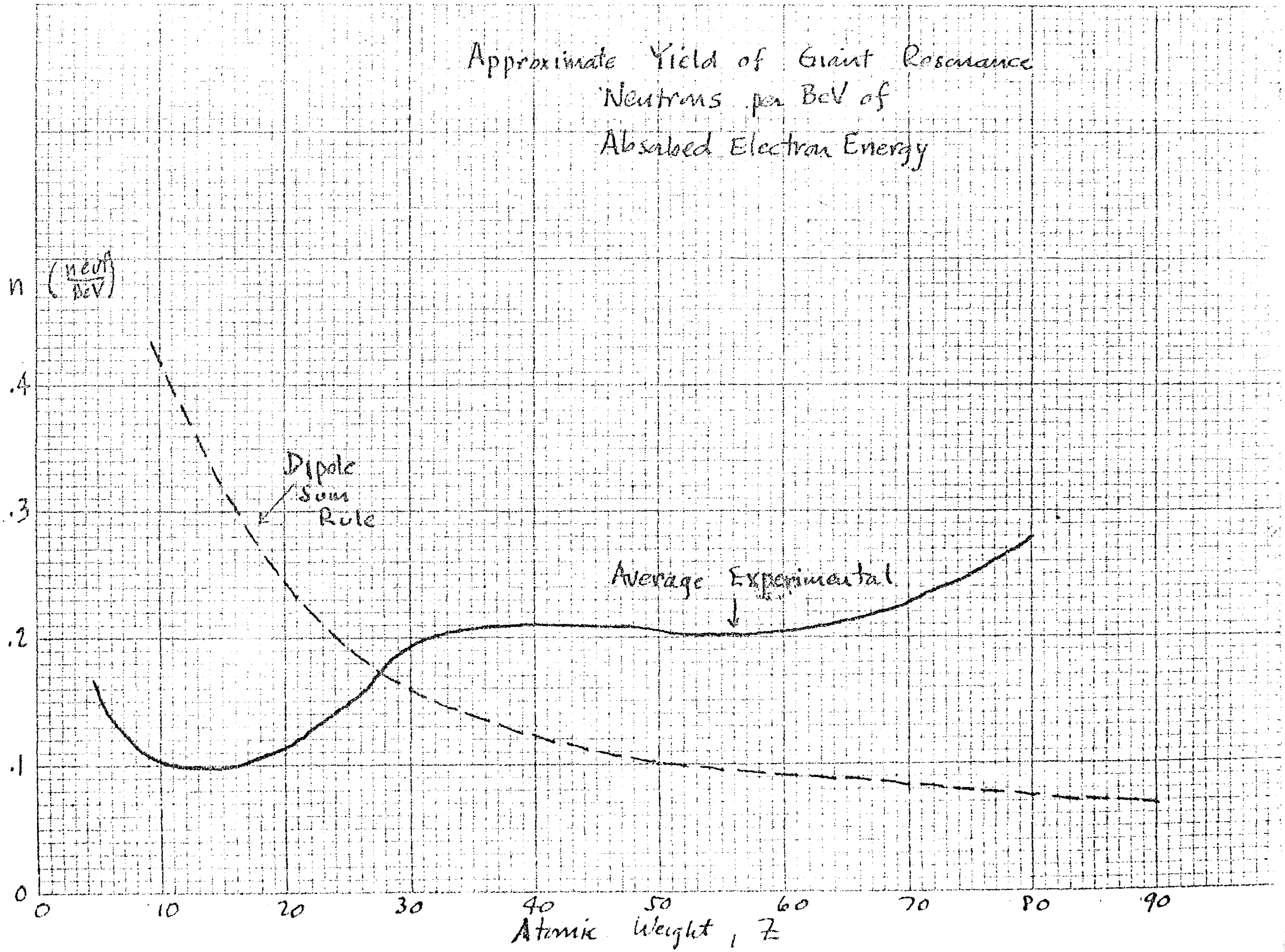
Fig. 4

MODEL DATE



Shield Thickness, H (feet of earth)

3 CYCLES X 50 DIVISIONS



Approximate Yield of Giant Resonance
Neutrons per BeV of
Absorbed Electron Energy
n (neut/BeV)
.4
.3
.2
.1
0
Dipole
Sum
Rule
Average Experimental
0
10
20
30
40
50
60
70
80
90
Atomic Weight, Z