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COMPREHENSIVE MODELING OF SIBERIAN SNAKES IN BNL'S AGS: SYMPLECTIC TRACKING AND OPTICAL COMPENSATION

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Abstract

Meaningful prediction and enhancement of spin-polarization in the RHIC/EIC accelerator complex relies on accurate modeling of each sub-component. Here we describe a symplectic field approximations of both Siberian Snakes in the AGS, enabling practical long-term tracking calculations. Without such symplectic representations, particle motion destabilizes very quickly close to injection energy. This optical instability manifests in $O(10^3)$ turns, and makes dynamic aperture smaller than realistic emittances. Combined with optimization using the Bmad toolkit, we implement steering and optical corrections of the snake effects at 80 distinct energies from injection to extraction, mimicking the measured lattice conditions at each energy. This process unveils unforeseen snake distortions of the vertical dispersion near injection energy, which are addressed. By interpolating between such optimized lattice configurations, Bmad's tracking capabilities allow advanced simulation of polarization transmission through the full AGS cycle.

INTRODUCTION

The Alternating Gradient Synchrotron of Brookhaven National Laboratory was the first major accelerator of its kind, taking advantage of strong focusing. It also produced the highest energy polarized proton beams of its time. Over the years, different strategies for mitigating depolarization have been developed and deployed at BNL's AGS. The first of these solutions was a solenoid that provided small alterations to the spin tune [1], and later were a pair of helical dipole magnets called Siberian snakes [2]. These snakes had a focusing strength larger than the old solenoids and generated significant orbital kicks with their fields. Such focusing and kicks generated optical defects such as beta beating and dispersion waves, especially strong near injection energy. While these effects can be compensated in design, as has been shown here [3], they have remained difficult to tame in a practical long-term tracking simulation [4]. It has certainly been possible to use dipole correctors and quadrupole magnets to compensate the orbital trajectory and twiss functions around the ring with snakes, even while also fixing the betatron tunes [5]. Nevertheless, particle loss is observed via orbital resonances of particles with realistic betatron amplitudes after they are injected [4]. This has also been reproduced in Bmad modeling of the AGS, as shown in section [6]. The layout of this report will be as follows: in the first section, titled *Non-symplectic Emittance Growth*, we lay out evidence of modeling failure and address the sources. In the next section, titled *Symplectic Maxwellian Solutions*, we describe the new model, estimate uncertainties and we show how this model corrects the failures of prior models. Finally, we use this new model for fitting the entire accelerator lattice model of the AGS to real-time measurement data using the Bmad/Tao toolkit in section (4). We conclude by discussing how this brings us one step closer to modeling a digital twin for the accelerator complex.

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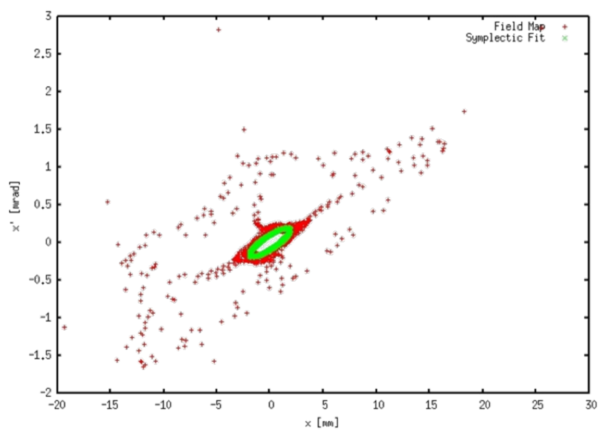

Figure 1: Phase space plots showing the violation of Liousville's theorem for the field map, and apparent conservation of horizontal phase space area for the symplectic fits.

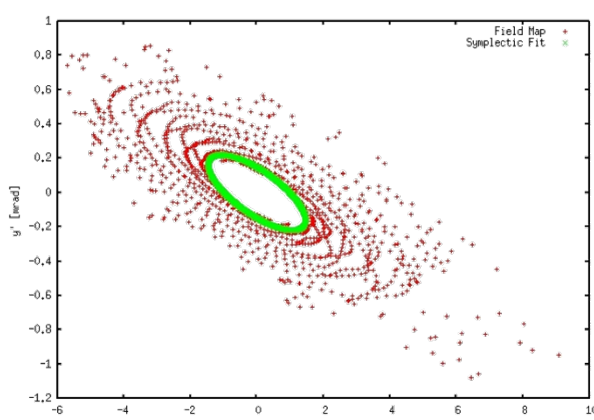

Figure 2: Symplectic fits conserving vertical phase space area, as opposed to field map tracking.

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NON-SYMPLECTIC EMITTANCE GROWTH

Modelling the dynamic process of ramping magnets during polarized proton acceleration can be broken down into several few independent processes: injection, ramping, gamma transition, ramping, extraction. For a well-aligned machine, injection is when trajectories experience the largest betatron amplitudes. This implies that accurate tracking at injection energies is key in tracking simulations to avoid numerical inaccuracies that add up over long integration times. For idealized magnetic fields, these inaccuracies usually remain at machine precision, but using electromagnetic field simulations causes inaccuracies that are much larger and can accumulate rapidly to massive errors over a few thousand of turns. This is relevant in modeling the partial Siberian snakes in the AGS, whose large numerical inaccuracy due to magnetic field simulation errors causes severe particle loss for reasonable emittances at injection as shown in red in Figs. 1 and 2. This type of tracking error is a signature of non-symplectic motion since the phase space area grows over time in the absence of non-conservative processes [7].

Tracking the same ramping process without the inclusion of Siberian snakes or by only using a thin-lens model instead of the field maps is perfectly stable and does not induce emittance growth. Even tracking with zero emittances while using field maps is stable, indicating that the field maps lose accuracy away from the central axis and begin violating Maxwell's equations. Tracking with the same magnetic field description near extraction energy is more stable since adiabatic damping causes the emittances to decrease (the rigidity of the beam is higher).

Tracking for 10^4 turns of injection at constant energy shows the clear difference between field maps and our symplectic fits. This kind of fit allows stable tracking on the true timescale of the AGS ramp, which is on the order of 10^5 turns, as opposed to particle loss occurring at around 10^3 turns with field maps. In the field map models, large amplitude particles were experiencing tune drift and hitting the ¾ orbital resonance, causing emittance growth [4], this effect being much more pronounced at injection energies. The immediate solutions were to either turn off orbital effects, or to approximate an effective orbital transfer matrix. Both solutions do not account for higher-order field effects in the snakes, e.g. chromaticity.

SYMPLECTIC MAXWELLIAN SOLUTIONS

We address this challenge by using magnetic field representations which fully solve Maxwell's equations with each term. By using this more appropriate representation, one has an analytic expression for the field through all space inside the magnet. This ideas builds on the computationally efficient, generalized-gradient representation of fields [8], which is very accurate but unfortunately non-symplectic due to its longitudinally-localized description.

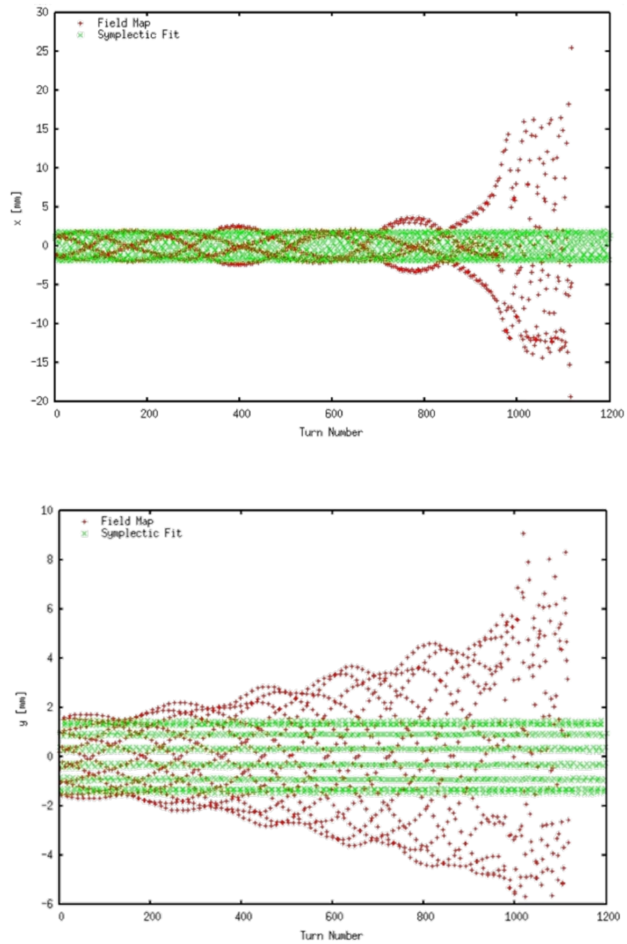

Figure 3: Time evolution of a sample particle trajectory.

This is as opposed to being confident of the field's accuracy at certain points while also having to interpolate between these points. Even more so, it becomes possible to write an analytic expression for the Hamiltonian or Lagrangian of the system, which can then be used in conjunction with a symplectic integration scheme of choice. We've used both cylindrical and cartesian multipole expansions, of which the former is a complete set, to approximate the magnetic field map simulation for a given beam-pipe aperture. Here we present only the form of the cylindrical expansion:

$$B_\rho = \mathrm{Re}\Bigg[\sum_{n=-N/2}^{N/2-1} \frac{1}{2}\, e^{ik_n z} \cos(m\theta - \theta_{0m}) \;\times\; b_m(n)\left[I_{m-1}(k_n\rho) + I_{m+1}(k_n\rho)\right]\Bigg]$$

$$B_\theta = \mathrm{Re}\Bigg[\sum_{n=-N/2}^{N/2-1} \frac{-1}{2}\, e^{ik_n z} \sin(m\theta - \theta_{0m}) \;\times\; b_m(n)\left[I_{m-1}(k_n\rho) - I_{m+1}(k_n\rho)\right]\Bigg]$$

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$$B_z = \mathrm{Re}\left[\sum_{n=-N/2}^{N/2-1} i\, e^{ik_n z}\, \sin(m\theta - \theta_{0m}) \quad I_m(k_n\rho)\right]$$

To balance the trade-off between accuracy of the fit and speed of tracking through the field expansion, we found that 300 terms in the expansion should be sufficient for our purposes. This field representation is tracked through using Bmad's kick-drift-kick symplectic integrator. The RMS deviation of the symplectic fits from field data is less than 1% at an aperture of up to 2 cm x 2 cm, and less than 5% with an aperture of up to 10 cm x 10 cm . The integrated spin-precession map through the closed-orbit agrees within less than 5% of design value. In Figs. 1 and 2, the green data show how phase space area seems to be conserved well. The first-order linear transfer matrix satisfies symplecticity condition to less than one part in 10^{12}.

FITTING TO MACHINE SETTINGS

Now in a position where tracking is feasible, the lattice is tuned to match the accelerator working point. This working point depends on energy, so we perform three optimizations at 100 energy values between 2.5 GeV and 25 GeV. First, the orbital defects due to the snakes are compensated (at all energies below γ-transition) locally before and after each snake using dipole correctors and backleg windings as can be seen in the closed orbit in Fig. 4. Second, we compensate snake optical focusing effects locally via 7 special compensation quadrupoles. Thirdly, we fit the tune quadrupole currents to match the tunes along the AGS user ramp at BNL, which become fixed above γ-transition to $Q_x, Q_y = (8.72, 8.985)$, fitting right above the spin tune gap around the $G\gamma \in \mathbb{Z}$. Now that the lattice is matched to the machine at many discrete steps in energy, by simply interpolating between all 100 settings while ramping, we can ensure the beam remains centered in the pipe as the BPMs read and the betatron tunes are the same as measured. We proceed to simulate a proton with a normalized emittance of 1.7 mm mrad ramping from 10 to 23 GeV through the lattice, obtaining an expected polarization transmission measurement of 97%, as shown in Fig. 5.

CONCLUSION

To simulate polarization transmission through the EIC complex for optimization purposes, it is important to be able to track particles through the AGS from injection to extraction. This requires a stable model injection lattice, which can track a beam through the Ĩs energy ramp to RHIC/HSR (10^5 turns). At a minimum, beam tracking must display adiabatic damping of emittance over the 2.5 GeV – 25 GeV energy range, while reproducing the measured tunes. In this paper, we fix a long-standing issue of particle loss near injection energy due to snake field maps, and improve previous first-order transfer matrix models by fitting a fully nonlinear, differentiable field description that agrees with other models and extends them. Our model allows us to calculate a closed orbit at all energies, accounting for both orbital and nonlinear optical effects of the snakes. With this tool we have generated over 200 lattice configurations to match measured conditions, such as working point and a zero closed orbit, over the entire energy range. Future work entails implementing the gamma transition quadrupole jumps to simulate polarization transmission over the entire energy range.

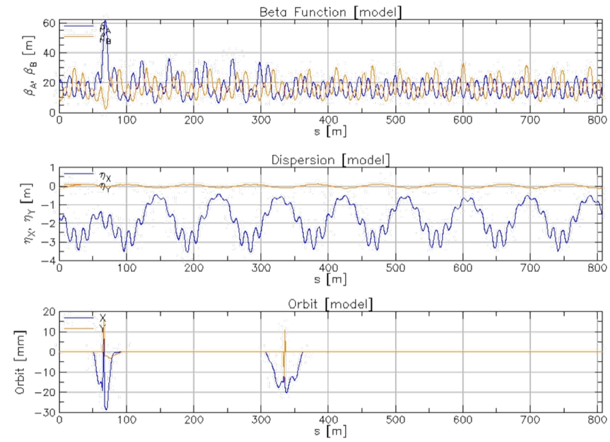

Figure 4: Optics and Orbit at injection energy of 2.5 GeV, showing closed-orbit bumps surrounding both snakes

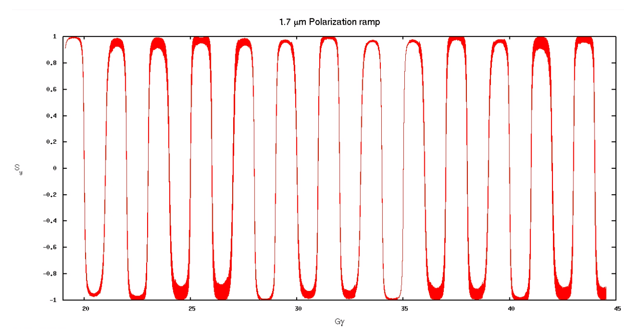

Figure 5: Polarization

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