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A. Siebert, K. Baillié, M. Béthermin, F. Cantalloube, E. Josselin, N. Lagarde, J. Malzac, J. Richard, L. Selliez, O. Venot (eds)

LITMUS TESTS OF THE FLAT ΛCDM MODEL AND MODEL-INDEPENDENT MEASUREMENT OF $H_0 R_\mathrm{D}$ WITH LSST AND DESI

B. L'Huillier[1]

Abstract. In order to quantify the ability of Stage-IV surveys to test the concordance model in a model-independent way, we reconstruct the cosmic expansion history using simulated type Ia supernovae from a one-year LSST observation. We then combine these reconstructions with simulated DESI 5-year baryon acoustic oscillation data to perform litmus tests of the concordance model, including the FLRW metric and the curvature, and dark energy as the concordance model. We show that the combination of DESI+LSST will allow to constrain the curvature parameter to a few percent without making any assumption regarding the dark energy model.

Keywords: Cosmology, Statistical Methods, Type Ia Supernovae,

1 Introduction

The ΛCDM concordance model has been remarkably successful in explaining a wide range of observations, yet it rests on assumptions that warrant independent tests. Among these assumptions is the Friedmann-Lemaître-Robertson-Walker (FLRW) metric, which encodes homogeneity and isotropy, and the flatness of the Universe is a key prediction tested by the cosmic microwave background (CMB). At the same time, recent tensions — most prominently on the Hubble constant, or the recent hints by the dark energy spectroscopic instrument (DESI) of departure from the cosmological constant (DESI Collaboration et al. 2024; Lodha et al. 2025) question the validity of these hypotheses. Therefore, it is important to apply model-independent approaches, in particular ones that do not assume a particular dark energy parameterisation (e.g. Perivolaropoulos & Skara 2022).

In an FLRW universe, the comoving distance $\mathcal{D}(z)$ and the expansion history $h(z) = H(z)/H_0$ are related by

$$\mathcal{D}(z) = \frac{1}{\sqrt{-\Omega_{k0}}} \sin\left(\sqrt{-\Omega_{k0}} \int_0^z \frac{dz'}{h(z')}\right), \tag{1.1}$$

$$h^2(z) = \Omega_{\mathrm{m}0}(1+z)^3 + \Omega_{k0}(1+z)^2 + (1 - \Omega_{\mathrm{m}0} - \Omega_{k0}) f_{\mathrm{DE}}(z), \tag{1.2}$$

where $\Omega_{\mathrm{m}0}$ and Ω_{k0} are the present matter and curvature densities, and $f_{\mathrm{DE}}(z)$ encodes dark energy evolution.

From these relations, one defines the $\mathcal{O}k$ diagnostic (Clarkson et al. 2008; Shafieloo & Clarkson 2010; L'Huillier & Shafieloo 2017):

$$\mathcal{O}k(z) = \frac{[h(z)\mathcal{D}'(z)]^2 - 1}{\mathcal{D}^2(z)} = \Omega_{k0} \quad \text{(FLRW)}. \tag{1.3}$$

A constant $\mathcal{O}k(z)$ across redshift is thus a litmus test of the FLRW metric, and provides the curvature density parameter. Departures from constancy thus signal either breakdown of the FLRW metric or the presence of unaccounted for systematics in either data set.

[1] Department of Physics and Astronomy, Sejong University, 05006 Seoul, Korea

2 Data and Method

2.1 Simulated data

We employ forecasts of LSST three-year supernovae from the DESC Time-Domain pipeline (Mitra et al. 2023), comprising nearly 6000 SNe binned into 14 redshift intervals, with statistical and systematic covariance included. We combine this with DESI five-year BAO forecasts (DESI Collaboration et al. 2016), including both radial and transverse distance measures with realistic covariances.

To illustrate the methodology, we consider four fiducial cosmologies: flat ΛCDM, curved ΛCDM with $\Omega_{k0} = 0.1$, phenomenent emergent dark energy (PEDE, Li & Shafieloo 2019) with no curvature, and curved PEDE. PEDE is simple phenomelological model where the density of DE is zero in the distant past, and emerges, to tend towards a cosmological constant in the asymptotic future. This yields a phantom behaviour. We stress here that this particular model is only used here as a proof-of-concept that our method can recover the curvature regardless of the fiducial DE model.

2.2 Iterative smoothing

In order to reconstruct the expansion history without assuming a particular model for dark energy, we employ the iterative smoothing method (Shafieloo et al. 2006; Shafieloo 2007; Shafieloo et al. 2018). to reconstructs the supernova Hubble diagram in a non-parametric way. This method works as follows. Given SNIa observations $\{z_i, \mu_i\}$ and their inverse covariance matrix $\boldsymbol{C}_{\mathrm{SN}}^{-1}$, and starting from an initial guess $\mu_0(z)$, we estimate the distance modulis at each iteration $n+1$ as

$$\hat{\mu}_{n+1}(z) = \hat{\mu}_n(z) + \frac{\boldsymbol{\delta\mu}_n^{\intercal} \cdot \boldsymbol{C}_{\mathrm{SN}}^{-1} \cdot \boldsymbol{W}(z)}{\mathbb{1}^{\intercal} \cdot \boldsymbol{C}_{\mathrm{SN}}^{-1} \cdot \boldsymbol{W}(z)}, \tag{2.1a}$$

where

$$\boldsymbol{W}_i(z) = \exp\left\{\left(-\frac{\ln^2\left(\frac{1+z}{1+z_i}\right)}{2\Delta^2}\right)\right\} \tag{2.1b}$$

is the smoothing kernel,

$$\mathbb{1} = (1, \ldots, 1)^{\intercal} \tag{2.1c}$$

is a column vector,

$$\boldsymbol{\delta\mu}_n|_i = \mu_i - \hat{\mu}_n(z_i). \tag{2.1d}$$

The resulting $\hat{\mu}(z)$ converge towards the solution prefered by the data, independently of the initial guess. We define the goodness-of-fit of the reconstruction at iteration n as

$$\chi^2 = \boldsymbol{\delta\mu}_n^{\intercal} \boldsymbol{C}^{-1} \boldsymbol{\delta\mu}_n. \tag{2.2}$$

We keep all reconstructions with a better χ^2 to the data than the best-fit ΛCDM model : $\chi^2_n < \chi^2_{\Lambda\mathrm{CDM}}$, and end up with a non-exhaustive collection of plausible $\mu(z)$. From the reconstructed distance modulus, we obtain $\mathcal{D}(z)$, $\mathcal{D}'(z)$, and hence $h(z)$ under the assumption of flatness.

Having a non-parametric estimation of $\mathcal{D}(z)$ and $h(z)$ at arbitrary redshifts, we can now evaluate these at the BAO redshifts. This allows us to compare two independent estimates of $c/H_0 r_{\mathrm{d}}$: one from transverse BAO combined with $\mathcal{D}$, and one from radial BAO combined with $\mathcal{D}'$.

$$\frac{c}{H_0 r_{\mathrm{d}}} = \frac{1}{\mathcal{D}(z)} \frac{d_{\mathrm{M}}(z)}{rd} = \frac{1}{\mathcal{D}'(z)} \frac{d_{\mathrm{H}}(z)}{r_{\mathrm{d}}}, \tag{2.3}$$

where the last equality is only true in flat FLRW universe where $\mathcal{D}'(z) = 1/h(z)$. In an FLRW universe, both estimates must agree; discrepancies can be interpreted as evidence for curvature or inconsistency between datasets.

The expansion history can then be obtained from the combination of BAO and SNIa :

$$h(z) = \frac{r_{\rm d}}{d_{\rm H}(z)} \frac{c}{H_0 r_{\rm d}} = \frac{d_{\rm M}(z)/r_{\rm d}}{d_{\rm H}(z)/r_{\rm d}} \frac{1}{\mathcal{D}(z)}, \tag{2.4}$$

which then allows us to construct the $\mathcal{O}k(z)$ diagnostic.

3 Results

We find that LSST+DESI will deliver strong, model-independent constraints:

- The two estimators of $c/H_0 r_{\rm d}$, obtained from transverse and radial BAO modes combined with SNIa reconstructions, reach a precision of about 0.3–0.5%. For flat fiducial models, both estimators agree well with the true value, while in curved universes they become inconsistent, thus providing a direct curvature test.
- The $\Theta(z) = h(z)\mathcal{D}'(z)$ and $\mathcal{O}k(z)$ diagnostics are reconstructed with high fidelity. For $\Omega_{k0} = 0.1$ fiducial models, the diagnostics correctly recover a non-zero curvature, while flat models remain consistent with $\Omega_{k0} = 0$. At redshifts $z \gtrsim 0.5$, the reconstructions are especially accurate, with small uncertainties.
- Overall, the curvature parameter can be constrained at the level of $\sigma(\Omega_{k0}) \sim 0.035$–$0.045$, and Θ to ~ 0.009, without assumptions on the dark energy equation of state. The spread, defined as the difference between the lowest and highest central value of the reconstructed parameter due to SNIa uncertainties, is up to ± 0.1 for $c/H_0 r_{\rm d}$ and ± 0.03 for Θ.
- We performed a hypothesis test to assess type I and type II errors when testing the null hypothesis of flatness ($\Omega_{k0} = 0$). For flat fiducial models, the probability of wrongly rejecting flatness (type I error) is modest, $p_I \simeq 6$–7% at $\alpha = 0.05$ and $\simeq 1$–2% at $\alpha = 0.01$. For curved fiducial models with $\Omega_{k0} = 0.1$, the probability of failing to reject flatness (type II error) is significantly larger, $p_{II} \simeq 36\%$ at $\alpha = 0.05$ and $\simeq 60\%$ at $\alpha = 0.01$. This shows that while LSST+DESI can detect curvature effects, the statistical power is limited when the true curvature is moderate and only low-redshift probes are used.

Representative reconstructions and diagnostic plots are shown in L'Huillier et al. (2025), Figures 2–5.

4 Conclusions

We have demonstrated that combining LSST supernovae with DESI BAO allows stringent, model-independent tests of the ΛCDM model. The iterative smoothing method reconstructs the expansion history without assuming a parametric dark energy form, and BAO provide an absolute calibration. The approach yields percent-level precision on $H_0 r_{\rm d}$ and competitive constraints on curvature.

These results highlight the potential of Stage-IV surveys to go beyond parameter estimation and directly test the fundamental assumptions of cosmology. Future work will extend the method to include weak lensing and clustering, and apply it to real LSST and DESI data.

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