

# STRUCTURE FUNCTIONS, LOW- $x$ and DIFFRACTION \*

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The highlights and principle issues discussed during the group A sessions on structure functions, low- $x$  and hard diffraction are summarized.

## 1 Introduction

Working Group A had the largest number of talks and as a consequence this report can only give a broad overview and highlight the important issues - the reader should refer to individual contributions for details and plots. It is not possible to do justice to the wide range of excellent talks given during the parallel sessions and the convenors apologize for inevitable personal bias.

This report follows the structure of the summary talk, which was in two parts:

- Experimental Topics: Inclusive DIS data; QCD fits and parton densities; RHIC heavy-ion collisions and low- $x$ ; GVDM and colour dipole models; DVCS and vector mesons; Hard Diffraction.
- Theoretical Topics: Low- $x$  both linear and non-linear aspects; Structure Functions; Vector mesons at low- $x$ ; Vector mesons and DVCS; Hard Diffraction.

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\*Summary of Working Group A, DIS03, St Petersburg

One of the major themes of the working group, and indeed of the Workshop, was the ‘universality of low- $x$ ’ physics. It appears in many of the talks reported here. The importance of taking a global view was also emphasized strongly in the plenary talks by Bartels and Ryskin and in the workshop summary by Mueller.

## 2 Experimental Results and Phenomenology

### 2.1 Inclusive DIS data

Final results on the high  $Q^2$  NC and CC cross-sections and the extraction of proton structure functions  $F_2$  and  $xF_3$  were presented by Dubak [1] and Rauthenberg [2] for H1 and ZEUS respectively. The data sets now comprise the full  $e^+p$  and  $e^-p$  luminosity from HERA-I running. NC scattering above  $Q^2 \sim 1000 \text{ GeV}^2$  and CC processes are both statistics limited, but basic features such as the electroweak  $\gamma - Z^0$  interference in NC processes and the helicity structure of the CC processes are clear. Improved measurements were presented by both groups.

Radiative DIS events from HERA were used for two studies. Lendermann [3] reported on an H1 measurement of  $F_2$  for  $Q^2 < 10 \text{ GeV}^2$  and  $0.01 < x < 0.1$  from QED Compton events, giving a good overlap with fixed target measurements. Jo Cole [4] reported on a ZEUS measurement of  $F_2$  and  $F_L$  using ISR events. Although with very large errors, it is the first time  $F_L$  has been measured directly at HERA. The spread in centre-of-mass energy of the ISR events allows one to disentangle  $F_2$  and  $F_L$  from the double differential cross-section. A good understanding of both the detector and the beam conditions is required for accurate subtraction of the large background from overlaid DIS and Bethe-Heitler events ( $ep \rightarrow e p \gamma$ ). Lobodzinska [5] emphasized the importance of  $F_L$  as a discriminant of low- $x$  models in the course of reporting on new indirect measurements of  $F_L$  at lower  $Q^2$  by H1. The H1 methods use the excellent description of  $F_2$  data at moderate  $y$  by NLO QCD to extrapolate  $F_2$  to high  $y$  where its contribution is then subtracted from the double differential cross-section.

Naples [6] outlined the ongoing work by NuTeV to improve the precision of neutrino structure function measurements from their high statistics data samples. To extract  $F_2$ , careful treatment of the contribution of charm and

beauty to  $\Delta xF_3 = xF_3^\nu - xF_3^{\bar{\nu}}$  is needed and for this the variable flavour number scheme of Thorne and Roberts [7] is used. Preliminary results for  $F_2$  and  $xF_3^{AV} = (xF_3^\nu + xF_3^{\bar{\nu}})/2$  were presented. The NuTeV data agree well with those from CCFR for  $x < 0.45$ , but are somewhat higher for larger values of  $x$ .

## 2.2 QCD fits and parton densities

New NLO QCD fits to determine parton densities were presented by H1 and ZEUS. In the case of H1, additional data from the BCDMS experiment are used (Reisert [8]) whereas ZEUS (Rauthenberg [2]) take a more global approach with data from NMC, E665, CCFR in addition to BCDMS. There are other differences of detail in the combination of parton densities fit and whether  $\alpha_s$  is fixed or fit, but in both cases a good description of the data for  $Q^2 > \sim 3 \text{ GeV}^2$  is obtained. Although the data may be described down to values somewhat below  $1 \text{ GeV}^2$ , there are doubts about the validity of the NLO DGLAP framework at such low  $Q^2$  and implicitly low  $x$ . The signs are well known and have been discussed in many papers and previous DIS Workshops: the gluon density rises less steeply than that for the sea quarks at low  $x$  and becomes negative; potentially large  $\ln(1/x)$  terms have been ignored;  $F_L$  may become negative. The problem has become more acute because of the increase in precision of the DIS data at low and moderate  $Q^2$ .

The stability and reliability of global fits at low  $x$  and  $Q^2$  was discussed further in by Martin [9]. Using the MRST global fit as an example he showed how removing data with  $x < 0.005$  and  $Q^2 < 10 \text{ GeV}^2$  improved the stability of the fit, as measured by the incremental change in  $\chi^2$ . He also showed that inclusion of NNLO effects, as far as they are known, gives a significant improvement in the stability of key results, such as  $\alpha_s$  and  $\sigma \cdot B$  for  $W$  production at the Tevatron or LHC. The importance of including NNLO effects and other effects such as nuclear corrections and target mass, was emphasized by Alekhin [10]. It is interesting to note that when the NNLO terms are included the value of  $\alpha_s$  from the global fits decreases slightly, MRST find  $0.1153 \pm 0.002(\text{exp.}) \pm 0.003(\text{theory})$ , Alekhin  $0.1143 \pm 0.0014(\text{exp.}) \pm 0.009(\text{theory})$ , compared with values around 0.117 from most global and recent experimental group fits at NLO. Tung [11] in his talk on the CTEQ6 partons covered the technical advances in handling experimental systematic uncertainties and how these may be propagated effi-

ciently to measurable quantities by diagonalizing the error matrix. As an example the comparison between the preliminary CDF Run-II data on  $d\sigma/dE_T$  for jets and the CTEQ6 prediction shows impressive agreement over 8 orders of magnitude, out to jet energies of 550 GeV. He went on to outline how various new data are helping to improve the flavour decomposition of partons and how, more generally, accurate parton densities are - and will be - a crucial ingredient for ‘beyond the standard model’ searches at colliders.

### 2.3 RHIC: Heavy Ions and low- $x$

Both low- $x$  physics at HERA and central heavy-ion collisions are dominated by the dynamics of gluons. Various groups of theorists have argued strongly for many years that the connections can and must be made - see the summary talk by Mueller. One of the central questions in low- $x$  physics - posed well before HERA started - is when will gluon saturation occur and how will it be recognized? The existence of an  $x$ -dependent saturation scale,<sup>1</sup>  $Q_s^2(x)$ , is a crucial feature. Do data support these conjectures? The HERA low- $x$  data are tantalizing - they are consistent with many features of saturation models, but, as has been discussed in earlier DIS workshops, the data may be described as well by models without saturation. Low- $x$  effects in heavy-ion collisions should be amplified by factors of the form  $A^p$  with  $p \sim 1/3$ , but the connection between what can be measured and gluon dynamics is much more complicated than in the relatively clean environment of  $ep$  scattering.

The talk by Brian Cole [12] gave a lucid account of what can be done by the RHIC detectors and how the results might be interpreted in saturation and other models. Using a combination of ‘zero degree counters’, very close to the beam line, and activity counters at somewhat larger angles gives a handle on the ‘centrality’ of an  $A - A$  collision. Centrality is a measure of the nuclear overlap and is often translated into the ‘number of participating nucleons’,  $N_{part}$ , for comparison of data with models. The RHIC detectors allow transverse energy and the charged particle multiplicity to be measured over a reasonable range of rapidity. The features of the data can then be studied as a function of  $N_{part}$ . Having been convinced that the experimentalists have good control over what

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<sup>1</sup>Defined roughly by the condition that  $\rho\sigma \sim 1$ , where  $\rho$  is the gluon density and  $\sigma$  the  $gg$  interaction cross-section.

they are measuring, how far does the data support the idea of ‘universality’ of low- $x$  dynamics and give evidence for saturation? Cole showed a number of approaches, with a variety of assumptions, but the bottom line is that using the gluon density measured at HERA in models with  $Q_s^2 \sim 2 \text{ GeV}^2$ , one can describe the features of the data very well. Indeed the KLN [13] model *predicted* the change in multiplicity measured at RHIC as  $\sqrt{s}$  increased from 130 to 200 GeV. However non-saturation models, e.g. the pQCD+Glauber model of [14] (which also uses a gluon density constrained by HERA data), can also describe the data so one should be cautious. A lot more work needs to be done on both the experimental and theoretical sides, but this is a subject that should feature large in future DIS Workshops.

## 2.4 GVDM and colour dipole models

At low- $x$  and low  $Q^2$  a partonic approach may not make sense and an older view of  $\gamma^* p$  interactions may provide a better physical basis – vector meson dominance. The talks by Ingelman [15] and Schildknecht [16] reviewed this approach and compared it to the data from HERA. Schildknecht also emphasized the wide applicability of generalized vector dominance, it gives a consistent picture of inclusive  $\gamma^* p$ ,  $\gamma^* p \rightarrow V p$  and deeply virtual Compton scattering. It also has a close connection to colour dipole models. Szczerba [17] outlined his combined VDM and colour dipole model which gives a good description of the low  $Q^2$  transition region.

Colour dipole models have become very popular in recent years for the reasons given in the plenary talk by Golec-Biernat at DIS02 [18] - they provide a flexible and powerful framework for inclusive and diffractive low- $x$  physics in which saturation is easily included. A lot of attention over the last year or two has been directed at providing a better understanding of their relationship to more formal QCD approaches. Another development discussed in the talk by Kowalski [19] is the extension of the dipole model to include impact parameter dependence. This is an important step in the extension of the dipole model to cover both  $\gamma^* A$  and  $\gamma^* p$  interactions. Kowalski showed how the ‘target’ profile function could be determined or constrained by the  $t$ -dependence of diffractive cross-sections. Knowledge of the profile functions may also give new insights into the approach to saturation.

## 2.5 DVCS and vector mesons

Some impressive new data from the complete HERA-I luminosity was presented under this heading, together with a first look at results from COMPASS and an interesting contribution on  $\gamma\gamma \rightarrow \rho\rho$  from L3 at LEP.

First, deeply virtual Compton scattering (DVCS). This offers many interesting challenges for QCD and for the experimentalist. It is, potentially, a source of information about the transverse degrees of freedom in deep inelastic scattering, generalized parton densities and colour dipoles. Abramowicz [20] outlined a recent ZEUS measurement of DVCS with  $5 < Q^2 < 100 \text{ GeV}^2$  from the full HERA-I luminosity. Experimentally the challenge is to separate the DVCS signal ( $\gamma^* p \rightarrow \gamma p$ ) from the very large Bethe-Heitler radiative process ( $ep \rightarrow ep\gamma$ ), good tracking and electromagnetic calorimetry are vital. The energy dependence of the cross-section,<sup>2</sup>  $\sigma(\gamma^* p \rightarrow \gamma p) \sim W^\delta$ , with  $\delta = 0.75 \pm 0.17$  is typical of a hard process and  $\sigma$  decreases as  $(Q^2)^{-n}$  with  $n = 1.54 \pm 0.09$ . The data agree with a range of models and one looks forward to more precise measurements with the increased luminosity from HERA-II.

H1 presented data on  $J/\psi$  production at both small (Fleischmann, [21]) and large (Beckingham, [22]) values of  $|t|$ . The data at small  $|t|$  and  $Q^2$  values (both less than  $1 \text{ GeV}^2$ ) are well fit by an exponential,  $e^{-bt}$ , with  $b = 4.65 \pm 0.17 \text{ GeV}^{-2}$ . The  $J/\psi$  cross-section rises steeply with energy,  $\sigma \sim W^\delta$  with  $\delta \sim 0.7$ , again characteristic of a hard process and comparable to the value of  $\delta$  for  $\gamma^* p \rightarrow \rho\rho$  with  $Q^2 > 15 \text{ GeV}^2$ . At large values of  $|t|$  vector meson production provides useful tests of pQCD beyond DGLAP. The new H1  $J/\psi$  data extends to  $|t|$  of  $30 \text{ GeV}^2$  and for large  $|t|$   $d\sigma/dt \sim (-t)^{-n}$ , with  $n \sim 3$ . The energy dependence of  $d\sigma/dt$  for  $|t| > 5 \text{ GeV}^2$  is described by a BFKL model but not by DGLAP.

The vector meson data from ZEUS shown by Tandler [23] were also on  $\rho$  and  $J/\psi$  production. The energy dependence of  $J/\psi$  electroproduction does not change with  $Q^2$  and gives a value of  $\delta \sim 0.7$ , much the same as that found for photoproduction. The idea of  $Q^2 + M_V^2$  as a universal scale for vector meson production with naïve SU(4) weightings does not appear to work for the  $J/\psi$ . Using the high statistics for  $\rho$  production, the ZEUS data show that the transverse cross-section falls faster with  $Q^2$  than the longitudinal. A

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<sup>2</sup>Here and below,  $W$  is the  $\gamma^* p$  center of mass energy.

puzzle remains - why is the ratio  $\sigma(\gamma^* p \rightarrow \rho p)/\sigma^{tot}(\gamma^* p)$ , in bins of  $Q^2 + M_V^2$ , independent of  $W$ ?

Korzenev [24] gave a brief account of the COMPASS experiment at CERN and showed some of the first data for  $\rho$  and  $\phi$  production ( $10^6$   $\rho$  events from just 1/6 of the 2002 data!). With such statistics and polarized beam and target, some very detailed studies will be forthcoming. Fedin [25] reported on a detailed study by L3 of  $\gamma\gamma \rightarrow 4\pi$  both at  $Q^2 = 0$  and for  $1.2 < Q^2 < 30$  GeV $^2$ . The final state is dominated by  $\rho\rho$  channels, but higher mass states such as the  $f_2(1270)$  and  $\rho'(1700)$  are also observed. The  $Q^2$  dependence of  $\rho^0\rho^0$  is well described by GVDM and by QCD – the latter with a form  $[Q^n(Q^2 + \bar{W}^2)]^{-1}$  where  $n$  is found to be  $2.4 \pm 0.3$  in agreement with the QCD expectation of  $n = 2$ .

## 2.6 Hard Diffraction

From HERA there was a satisfying increase in the precision and coverage of inclusive data on hard diffraction presented by H1 (Coppens [26]) and ZEUS (Lim [27]), together with new H1 data on aspects of the diffractive final state –  $D^*$  and dijets (Schäfzel [28]). The inclusive data from H1 extends the measurement of  $F_2^{D_3}$  to larger and smaller values of  $Q^2$  and will improve the precision of the DGLAP fit to determine the parton content of the Pomeron. The ZEUS data exploits the plug calorimeter in the forward (proton) direction to increase the rapidity coverage from 4 to 5 units and thus to extend the  $M_X$  range to 35 GeV. Both experiments showed results on the energy dependence of  $\sigma^{diff}$ , as functions of  $Q^2$  and  $M_X$ . It increases with  $Q^2$  at a similar rate to  $\sigma^{tot}(\gamma^* p)$ , giving  $\sigma^{diff}/\sigma^{tot}$  roughly constant as a function of  $W$ , for fixed  $M_X$  and with little  $Q^2$  dependence for  $M_X > 2$  GeV.

Diffraction provides tests of Regge factorization and QCD factorization. The understanding of both, within  $ep$  and  $p\bar{p}$  and between the processes, continues to benefit from better data and from the ongoing theoretical effort. There is a fair degree of consensus that the large suppression of hard diffraction in  $p\bar{p}$  may be explained by the effects of multiple Pomeron exchange reducing the ‘gap survival’ probability in  $p\bar{p}$  compared to that in  $ep$  (e.g. [29]). There may be some hints in the same direction within  $ep$  from the results presented by Schäfzel. Using H1  $ep$  diffractive dijet data and the 2002 H1 diffractive

partons, he showed that for large  $Q^2$  there is factorization, but it is broken for diffractively photoproduced dijets - where the hadronic component of the photon needs to be taken into account.

On a related theme, Soares [30] presented a ZEUS study of final states with a leading neutron (LN) both as a function of  $Q^2$  and those with a  $D^*$ . The data suggests that particle exchange or Regge factorisation is satisfied if there is a hard scale present, so either LN/ALL at large  $Q^2$  or  $(LN+D^*)/D^*$  but not for  $(LN\ Q^2 = 0)/(LN\ DIS)$ .

For the Tevatron, Stevenson [31] gave a summary of Run-I diffractive production of  $W$  and  $Z$  at  $D_0$ , the latter seen for the first time. Terashi [32] speaking for CDF showed some preliminary Run-II results on diffractively produced dijets. Both speakers discussed briefly the improvements made to the detectors for Run-II diffractive physics. Schlein [33] presented a new test of factorization for diffraction in  $ep$  and  $pp$  scattering that does not require a universal Pomeron flux. He showed evidence of this modified form of factorization (to better than 20% or so) for  $1 < Q^2 < 6\text{ GeV}^2$ . At larger  $Q^2$  factorization is broken which may indicate a slow onset of perturbative QCD effects due to the small size of the Pomeron.

### 3 Theory

Theoretical talks were distributed among the sessions: Low- $x$ , Structure Functions and PDFs, Vector mesons at Low- $x$ , Vector mesons and DVCS, Hard Diffraction; with Low- $x$  taking the largest share.

#### 3.1 Low- $x$

The standard DGLAP approach to DIS fails in the small- $x$  region, in particular because of the necessity to sum the terms of the perturbation series enhanced by powers of  $\log(1/x)$  and the increasing importance at low  $x$  of higher twist contributions. D. Haidt [34] presented evidence that the failure happens already in the HERA kinematical region: the DGLAP kernel leads to significant positive curvature in the small- $x$  region in contrast to the flat behaviour of the HERA data. The BFKL approach gives a resummation of  $\log(1/x)$ -terms, not restricted to twist-two contributions, but it has its own problems. The first is how to take the approach beyond leading order. Resummation of the leading

$\log(1/x)$  ( $LL_{1/x}$ ) terms alone cannot provide reliable accuracy, since neither energy scale nor argument of  $\alpha_s$  can be fixed in this approximation. Although a procedure for resumming the next-to-leading  $\log(1/x)$  ( $NLL_{1/x}$ ) terms is known, it is not yet completed. Moreover, the NLO correction to the kernel of the BFKL equation calculated several years ago in the  $\overline{MS}$  scheme is very large, implying that an improvement to the BFKL series is necessary. Another problem, known a long time ago, is violation of the Froissart bound. It cannot be solved by calculation of radiative corrections at any fixed  $NNN...NL$  order and requires other methods. The most popular now are non-linear generalizations of the BFKL equation, related to the idea of saturation of parton densities [35]. Accordingly, the low- $x$  talks are naturally divided into two groups.

### 3.1.1 Linear evolution: BFKL and DGLAP

At leading order (LO) the BFKL kernel has the remarkable properties of conformal invariance and holomorphic separability [36], which are very important for the solution of the BFKL equation and its generalization for a composite state of  $n$  reggeized gluons. These properties are evidently violated at NLO by the running coupling.

The absence of coupling constant renormalization in  $N = 4$  supersymmetric gauge theory gives hope that they are not violated in this model and has stimulated the investigations reported by L. Lipatov [37]. The characteristic function<sup>3</sup>  $\omega(|n|, \gamma)$  of the BFKL kernel was calculated at NLO in both SUSY and conventional QCD. Contrary to the QCD case at this order, where  $\omega(|n|, \gamma)$  contains terms non-analytic in  $|n|$ , in  $N = 4$  SUSY these terms disappear. It opens the intriguing possibility of restoring the anomalous dimension  $\gamma(j)$  not only at  $j$  near 1, where it is found from the relation  $j - 1 = \omega(0, \gamma(j))$ , but for arbitrary  $j$ , in turn restoring the DGLAP splitting functions from the BFKL kernel. Although for  $N = 4$  SUSY there are several multiplicatively renormalizable twist-2 operators, the eigenvalues of the anomalous dimension matrices can be obtained from the universal function  $\gamma^{uni}(j)$  by a shift of its argument  $j \rightarrow j + k$  with  $k = 0, 1, 2, 3, 4$ . It was noticed that at LO the singularities of  $\gamma^{uni}(j)$  at  $j = 1 + \omega + |n| \rightarrow -r, r = 0, 1, 2, \dots$ , can be obtained

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<sup>3</sup>The Fourier transform on the azimuthal angle and Mellin transform on the ratio of squared transverse momenta of the BFKL kernel.

by the analytic continuation of the anomalous dimensions  $\gamma(|n|, \omega)$ , defined by equation  $\omega = \omega(|n|, \gamma)$ , to negative integer values  $-r - 1$  of the conformal spin  $|n|$ . In particular,  $\gamma^{uni}(j) = g^2 N_c / (4\pi^2) (\psi(1) - \psi(j-1))$  obtained at LO coincides with the result of direct calculations. Thus, for  $N = 4$  SUSY the BFKL equation presumably contains information sufficient for restoring the kernel of the DGLAP equation.

Moreover, in the multi-color limit of the  $N = 4$  SUSY model, the BFKL and DGLAP dynamics in the LL approximation are integrable for an arbitrary number of particles. Unfortunately, at NLO these remarkable properties are partly lost even in the  $N = 4$  model. The holomorphic separability of the BFKL kernel is violated, being substituted by the property of hermitian separability. The correspondence between the BFKL and DGLAP equations cannot be checked completely, because at NLO  $\gamma^{uni}(j)$  has multiple poles  $\sim \alpha^2 / (j+r)^3$  at even  $r$ , which are related to double-logarithmic corrections in the Regge limit. Therefore the procedure of analytical continuation of  $\gamma(|n|, \omega)$  to negative  $|n|$  permits only the main singularities of two-loop  $\gamma^{uni}(j)$  to be found <sup>4</sup>, but the NLO accuracy of  $\gamma(|n|, \omega)$  is not sufficient to find residues of subleading poles. To verify completely the hypothesis, that in  $N = 4$  SUSY the DGLAP splitting functions at NLO can be obtained from the BFKL kernel, one should calculate the NLO corrections to kernel in the region, where  $|n| + r + 1 \rightarrow 0$  and  $\gamma \sim \omega$ .

It is interesting that the all order resummation for  $\gamma^{uni}(j)$  by a method similar to Padé approximation on the basis of the first two terms of perturbative expansion is in rather good agreement with the prediction for large- $j$  behavior of  $\gamma(j)$  ( $\gamma(j) = a(\alpha_s N_c / \pi) \ln j$ , where  $a(z) = -z^{1/2} + \frac{3 \ln 2}{8\pi} + \mathcal{O}(z^{-1/2})$ ) in the strong-coupling limit  $\alpha_s N_c \rightarrow \infty$  based on the AdS/CFT correspondence [38], where this limit is described by a classical supergravity in the anti-de Sitter space  $AdS_5 \times S^5$ .

An interesting observation, made by Lipatov, is that the NLO correction to the Pomeron intercept in SUSY is smaller than in QCD. It confirms that most of the QCD correction is related to the running coupling and has therefore strong renormalization scheme and scale dependence. V. Kim reported [39] on a study using NLO BFKL in a non-Abelian physical renormalization schemes with BLM optimal scale setting, where the problems encountered in the  $\overline{MS}$ -scheme are not exhibited, thus allowing applications of NLO BFKL resummation to high-

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<sup>4</sup>Confirmed by direct two-loop calculations of the anomalous dimension matrices

energy phenomenology. It is important that this approach allows one to use many of celebrated features of LO BFKL, such as a conformal invariance. The NLO BFKL predictions, improved by the BLM optimal scale setting, are in good agreement with OPAL and L3 data from LEP2 at CERN.

The renormalization group improved (RGI) approach to the problem of large BFKL corrections was presented by D. Colferai [40]. It is based on the understanding that large corrections are related to collinear contributions which may be resummed using the renormalization group. This resummation is done by inclusion of  $\omega$ -dependence in the BFKL kernel, which takes into account the leading collinear logarithms together with the  $LL_{1/x}$  and  $NLL_{1/x}$  terms, but is model dependent above this level. The approach was used for the calculation of the Green's function and splitting function. The resummation makes growth with energy phenomenologically acceptable and diminishes diffusion and tunnelling into the non-perturbative region, leading to an expansion of the region of validity of perturbative calculations. Strong coupling contributions factorize, so that it is possible to find the resummed splitting function. The function has a dip at  $x \sim 10^{-3}$  and a power increase at smaller  $x$ .

The approach presented by G. Altarelli [41] resembles the RGI one in the aspiration to combine information from both BFKL and DGLAP at low  $x$ , but the realization is different. Whereas in the RGI approach the renormalization group is used to improve the BFKL equation, here the BFKL approach is used to find  $(\alpha_s/(j-1))^n$  corrections to the anomalous dimension  $\gamma(j, \alpha_s)$ , or, equivalently,  $(\alpha_s \ln(1/x))^n/x$  corrections to the splitting functions. For fixed  $\alpha_s$  it can be done without problems using the “duality relation”  $j-1 = \chi(\gamma(j, \alpha_s), \alpha_s)$ , where  $\chi(\gamma(j, \alpha_s), \alpha_s)$  is the characteristic function  $\omega(|n|, \gamma)$  of the BFKL kernel at  $|n| = 0$ , reorganized into the “double-leading” expansion. Allowing the running of the coupling is not trivial; perturbative inclusion of the running coupling terms requires corrections to  $\chi(\gamma, \alpha_s)$  in the “duality relation” with unphysical poles at  $\gamma = 1/2$ , leading to a perturbative instability. These poles are an artifact of the expansion and are absent in the all-order solution, which is proved to be fully compatible with the standard DGLAP evolution of parton distributions. The resummed splitting function remains smooth in the small- $x$  limit, contrary to the naive solution of the BFKL equation with running  $\alpha_s$ , which has oscillatory instabilities. The running coupling effects softens the asymptotic small- $x$  behaviour, as is required by data. It accords

with the results of the RGI approach, although there is a noticeable difference in the splitting functions; in particular, the dip in the region of moderate small- $x$  is absent.

In the BFKL approach scattering amplitudes are given by the convolution of the impact factors of interacting particles and the Green's function determined by the BFKL kernel. For a consistent description of small- $x$  processes one needs to know the impact factors with the same accuracy as the kernel. The impact factor  $\Phi_{\gamma^*}$  of a highly virtual photon is particularly interesting, because it can be calculated "from the first principles" in perturbative QCD. Knowledge of  $\Phi_{\gamma^*}$  at NLO is important, not only at energies at which the BFKL dynamics is completely developed, but also in the case when only the few first terms in the BFKL series contribute. In this latter case the NLO  $\Phi_{\gamma^*}$  gives an estimate of the size of the radiative corrections to the leading contribution in  $s$  of the total cross-section. Unfortunately, the calculation of the NLO  $\Phi_{\gamma^*}$  turns out to be a very complicated problem, which is not yet solved. M. Kotsky [42] discussed the possibility of using the analytical properties of photon-reggeon scattering amplitude for this purpose. They permit a simplification of the calculation; some of the diagrams contributing to the impact factor can be treated without their complete computation. A. Kyrieleis reported the status of calculations of the real gluon production contribution to the impact factor [43]. The production amplitudes have been calculated and the phase-space integration is partially performed analytically. Finite analytic expressions are obtained by combining the result with the singular pieces of the virtual corrections and subtracting the contribution due to the central region. The remaining integration is done numerically.

L. Schoeffel [44] proposed to use the anomalous dimensions, extracted from parametrizations of the data for  $F_2$ , and the "duality relation" to test the resummation schemes for the RGI BFKL kernel with the NLO contributions.

A new method of iterative solution of the BFKL equation for forward scattering was suggested by A. Sabio Vera [45]. The method takes into account all conformal spins, i.e. dependence on the azimuthal angle, that allows one to study spin-dependent observables. The solution is applicable at NLO and is suitable for numerical calculations. The method is based on the introduction of a cut-off in the transverse momentum plane for regularization of virtual and real production contributions separately and resummation of the virtual con-

tribution first. It is analogous to using the Dirac representation to solve the Schrödinger equation, where the role of  $H_0$  is played by the contribution of the gluon Regge trajectory to the kernel.

The description of the small  $x$  data in DIS and hadron-hadron collisions in the framework of the Linked Dipole Chain (LDC) model was discussed by G. Gustafson [46]. The model is of exclusive BFKL type, adjusted to the implementation in MC event generators. An important property of the LDC model is the symmetry between two ends of the parton chain, which makes it applicable to hadron-hadron collisions as well as to DIS. It implies a very strong connection between DIS and  $pp$  scattering. The chain cross section grows as  $s^\lambda$ , implying saturation and unitarization.

### 3.1.2

The idea of parton saturation is that the BFKL growth of parton densities is stopped by interactions between partons in the parton cascade, and some equilibrium (saturated) state of partons is created. The DIS kinematical region is separated into two part by the "saturation line"  $Q^2 = Q_s^2(x) \sim Q_0^2 x^{-\lambda}$ : the low density domain for  $Q^2 > Q_s^2(x)$ , in which the parton evolution is described by the BFKL equation, and the high density domain for  $Q^2 < Q_s^2(x)$ , with large nonlinear effects and parton saturation.

Most conveniently, saturation in DIS is described within the color dipole approach [47], which has a clear physical interpretation in the target rest frame, with  $\gamma^*$  splitting into a  $q\bar{q}$  colour dipole and the interaction of the dipole with a target separated in time. In this approach the cross-sections  $\sigma_{\gamma^*_{T,L}}(x, Q^2) = 4\pi^2\alpha_{em}F_{T,L}(x, Q^2)/Q^2$  are presented as

$$\sigma_{\gamma^*}(x, Q^2) = \int d^2r \int_0^1 dz |\Psi_{\gamma^*}(r, z, Q^2)|^2 \sigma_{dp}(r, x), \quad (1)$$

where  $\Psi_{\gamma^*}(r, z, Q^2)$  is the splitting amplitude,  $z$  is the quark momentum fraction,  $\vec{r} = \vec{r}_1 - \vec{r}_2$ ,  $\vec{r}_1$  and  $\vec{r}_2$  are the  $q$  and  $\bar{q}$  transverse coordinates, and  $\sigma_{dp}(r, x)$  is the dipole cross section,

$$\sigma_{dp}(r, x) = 2 \int d^2b (1 - S(\vec{r}_1, \vec{r}_2; Y)), \quad (2)$$

with  $\vec{b} = (\vec{r}_1 + \vec{r}_2)/2$  the impact parameter and  $Y = \log(1/x)$ .  $S(\vec{r}_1, \vec{r}_2; Y) =$

$\langle \text{tr} (U^+(\vec{r}_1)U(\vec{r}_2)) / N_c \rangle_Y$  is the elastic scattering  $S$ -matrix, which can be presented as the average of the product of two Wilson lines

$$U^+(x_\perp) = P \exp[ig \int dx^- A_a^+(x^-, x_\perp) t^a]. \quad (3)$$

There are two different approaches to the calculation of its variation with  $Y$ . In the first one an infinite hierarchy of coupled equations for products of any given number of Wilson lines is derived [48]. In the limit of large  $N_c$ , for the case when the target is a large nucleus, the equation for the two-line correlator decouples and takes a simple and graceful form [49]:

$$\begin{aligned} \frac{\partial S(\vec{r}_1, \vec{r}_2, Y)}{\partial Y} = & \frac{\alpha_s N_c}{2\pi^2} \int d^2\vec{r} \frac{(\vec{r}_1 - \vec{r}_2)^2}{(\vec{r}_1 - \vec{r})^2(\vec{r}_2 - \vec{r})^2} [S(\vec{r}_1, \vec{r}, Y)S(\vec{r}, \vec{r}_2, Y) \\ & - S(\vec{r}_1, \vec{r}_2, Y)] . \end{aligned} \quad (4)$$

At small  $1 - S = N$ , the equation reduces to the colour-dipole version of the BFKL equation for  $N$ . The second approach, called the Color Glass Condensate (CGC) [50], is based on a model for the small- $x$  hadronic wavefunction. The small- $x$  short-lived gluons are radiated semi-classically by a “frozen” configuration of faster partons (glass) with a random distribution  $W_Y[\rho]$  of the colour charge  $\rho_a(\vec{x})$ . For the gluonic modes with  $|k_\perp| < Q_s(x)$  the occupation numbers are  $\sim 1/\alpha_s$  (condensate), corresponding to strong classical fields  $A_a^\mu[\rho] \sim 1/g$ . Evolution of  $W_Y[\rho]$  with  $Y$  is governed by the functional evolution equation which is derived using the Wilson renormalization group. It is affirmed [51] that both approaches give identical results for the calculation of observables.

There are two contradictory accounts of the large  $b$  behaviour of  $S(\vec{r}_1, \vec{r}_2, Y)$  and the related high energy limit of  $\sigma_{dp}$ . One [52] claims that the BK equation (4) provides the exponential suppression at large  $b$  and the unitarization of the cross-section, therefore non-perturbative corrections must be taken into account only in an initial condition, but not in the kernel of the equation. The other [53] affirms that account must be taken of the non-perturbative corrections to the kernel to achieve the same results.

The results of the semi-classical analysis of the BK equation, focusing on large  $b$  behaviour and methods for incorporating non-perturbative contributions, were covered in the talk of E. Levin [54]. The motivation to use the semi-classical approach, in spite of difficulties on the experimental side, was

the possibility of finding an analytical solution, in this approximation, which could then be used as a basis for the development of numerical methods for solving the BK equation. It also provides a natural definition of the saturation scale  $Q_s$  and the possibility to investigate its  $b$ -dependence. Besides this, the semi-classical approach allows one to solve the simplified BK equation analogously to the BFKL one, without large complications. The result obtained is that it is sufficient to include the non-perturbative contributions in the initial condition, leaving the kernel unchanged. The effect is to provide an exponential decrease of  $Q_s$  at large  $b$ , that leads to the asymptotic behaviour  $\sigma_{dp} \sim \ln^2(1/x)$  with fixed coupling and  $\sigma_{dp} \sim \log(1/x)$  with running  $\alpha_s$ , in accordance with the Froissart bound. In the saturation region the solution has the geometrical scaling property:  $S(\vec{r}_1, \vec{r}_2, Y) = f(r^2 Q_s^2(x; b))$ .

A brief review of the CGC theory was given by E. Iancu [55]. He pointed out an important result that for  $|k_\perp| < Q_s(x)$  the correlator  $\langle \rho_a(k_\perp) \rho_b(-k_\perp) \rangle_Y \sim k_\perp^2$ , which means gluon colour charge shielding: the gluons look like dipoles for probes with resolution  $\Delta x_\perp > Q_s^{-1}(x)$ . The non-linear effects suppress diffusion into the infrared region and eliminate the infrared problem;  $Q_s(x)$  appears as the infrared cutoff. The extension of geometric scaling and the high energy behaviour of  $\sigma_{dp}$  were discussed in more detail by K. Itakura [56]. Geometrical scaling, natural for the saturation region  $Q^2 < Q_s^2(x) \sim Q_0^2 x^{-\lambda}$ , is valid as well in the region  $Q^2 \sim Q_s^4(x)/\Lambda_{QCD}^2$ , where  $1 - S(\vec{r}_1, \vec{r}_2, Y) \simeq (r^2 Q_s^2(x, b))^\gamma$ , with  $\gamma \simeq 0.64$ . The black-disk radius grows  $\sim Y$ , therefore  $\sigma_{dp}(r, x)$  saturates the Froissart bound (as also found in [54]). This does not agree with the results of the numerical analysis of the BK equation with the  $b$ -dependence presented by A. Stasto [57]. The most important conclusion from this analysis is that, in spite of an exponential decrease at large  $b$  in the initial condition, the power-like tail in  $b$  emerges as a result of evolution due to long range contributions. Therefore the Froissart bound is violated despite the local unitarity in impact parameter space. In order to satisfy the bound the evolution kernel must be changed to include confinement.

M. Lublinsky reported on applications of the model, based on the dipole picture and the solution of the BK equation, to soft processes [58]. The model describes all available low- $x$  data for  $F_2$ , down to  $Q^2 = 0.045 \text{ GeV}^2 \simeq \Lambda_{QCD}^2$  and provides a smooth interpolation between the intercepts of the hard and soft Pomerons. Moreover, it also describes the photoproduction and hadron-proton

cross-sections, although there are no grounds for expecting this.

Saturation and the unitarization in the colour dipole model for  $\gamma^*\gamma^*$  scattering [59] was discussed in the talk of M. Kozlov. The dipole-dipole interaction is considered as an eikonal rescattering with BFKL Pomeron exchanges. The main result is the same as in [54]: to satisfy the unitarity constraints it is sufficient to include the non-perturbative corrections only in initial conditions.

The results of the dipole approach to  $J/\Psi$  photo- and electro-production on nucleons and nuclei were presented by E. Naftali [60]. The wavefunction  $\psi_{J/\Psi}(r, z)$  is approximated by its value at  $r = 0$ ,  $z = 1/2$ , which is considered as a free parameter. The dipole scattering amplitude is taken as the convolution in impact parameter space of the solution of the BK equation with the profile function extracted from the electromagnetic form-factor. A good description of the HERA data is obtained. The predictions for production on nuclei are given using the Glauber approach. The unitarity effects are appreciable already at  $x \leq 10^{-3}$ .

S. Munier [61] advocated the study of the geometric scaling at fixed impact parameter and suggested that the geometric scaling should manifest itself in exclusive processes. He proposed a simple model based on the elementary two-gluon exchange dipole-dipole cross-section, which exhibits geometric scaling and the almost perfect symmetry of  $(Q/Q_s)\sigma_{\gamma^* p}(Q/Q_s)$  under the interchange of  $Q$  and  $Q_s$ .

### 3.2 Structure functions and PDF

Apart from talks of A. Martin, Wu-Ki Tung and S. Alekhin, discussed in Sec. 2.2, the talks of A. Oganesian, S. Moch and H. Kawamura were presented on this session. They deal with rather different items.

A. Oganesian [62] discussed calculations of the structure functions of polarized and unpolarized hadrons using generalized QCD sum-rules [63]. The advantage of the calculation is that it is performed in a model-independent way and is based only on QCD and the operator product expansion. Valence quark distributions in pions, polarized  $\rho$  mesons and nucleons in the region of intermediate  $x$  are obtained. The accuracy of the results is expected to be about 20-30% for pions and longitudinally polarized  $\rho$ -mesons and about 30-50% for transversely polarized  $\rho$ -mesons. The main source of uncertainty is the value

of the quark condensate and possible non-logarithmic perturbative corrections. Polarization effects are very significant and there is strong suppression of quark- and gluon-sea distributions in longitudinally polarized  $\rho$  mesons.

Increasing precision of experimental data requires NNLO corrections to both splitting functions and partonic cross sections. This aim is not yet achieved. The two-loop coefficient functions for  $F_2, F_3$  and  $F_L$  were found some time ago [64], but calculation of the NNLO corrections to the splitting functions is not yet completed. Up to now only a finite number of Mellin moments are available [65]. S. Moch [66] reported on the progress in completing the goal: the calculation of the  $n_f$ -parts parts of the three-loop non-singlet anomalous dimensions and coefficient functions for  $F_2$  and  $F_L$ . The implications of the results for large momentum  $j$  behaviour of the Mellin moment for  $F_2$  at NNLO  $\log j$  accuracy is discussed.

H. Kawamura [67] reported on the calculation of the NLO QED corrections to deep inelastic scattering. The  $O(\alpha^2 \log(Q^2/m^2))$  leptonic corrections are found. The cross-section is presented in terms of splitting and coefficient functions. The  $O(\alpha^2 \log(Q^2/m^2))$  corrections are calculated in analytic form. The problem of corrections from lepton-hadron photon exchanges is not considered.

### 3.3 Vector mesons at low- $x$ and DVCS

The model for  $\gamma_L^* p \rightarrow V_L p$  at large  $Q^2$  and small  $x$  was suggested by S. Goloskokov [68] in the framework of generalized parton densities, modified by the inclusion of  $q\bar{q}$  transverse momenta in the subprocess  $\gamma_L^* g \rightarrow V_L g$  with Sudakov suppressions of the contributions from the end-point regions of  $x_q, x_{\bar{q}}$ . The model gives reasonable agreement with the data, while the leading-twist contribution substantially exceeds experiment.

Studies of the contributions of the quark longitudinal momentum fraction end-points in the  $\gamma^* - V$  impact factor for the diffractive electroproduction amplitude were reported by R. Kirschner [69]. These contributions are important for understanding the ratio  $\sigma_L/\sigma_T$  and the angular-decay distributions. It was shown, that they don't violate the short-distance factorization and can be included in the modified target parton distribution.

M. Diehl [70] argued that the impact parameter representation is quite suitable for description of parton dynamics. Being applied to processes like

DVCS, elastic meson production, and hard diffraction it provides a spatial distribution of partons within a hadron, fully consistent with the principles of relativistic field theory.

Generalized Parton Distributions [71] attracted a lot of attention since they contain the most complete information about parton structure of hadrons. While chiral-even GPDs may be probed in a number of hard exclusive processes, special efforts are required to find a process sensitive to chiral-odd GPDs. L. Szymanowski discussed the electroproduction of mesons  $M_1$  and  $M_2$  with a large gap between them and a much smaller, but still large enough gap, between  $M_2$  and the recoiling nucleon [72]. In the case of  $\rho_L^0 \rho_T^+$  production on a proton this process was suggested for probing chiral-odd GPDs. It was shown that at the Born level it can be described consistently within the collinear factorization approach.

### 3.4 Hard Diffraction

An important prediction of perturbative QCD is the existence of the Odderon. An evident candidate for Odderon searches is the diffractive photoproduction. The results of the calculation of  $\gamma^{(*)} p \rightarrow \eta_c + X$  cross section in the region of large  $M_X$  (triple Regge region) were presented by M. Braun [73]. Due to large value of  $M_X$  proton coupled to the Pomeron, which allows to avoid the uncertainties with the proton-Odderon coupling. The integrated cross section is found to be  $\sim 60\text{pb}$  for photoproduction  $\sim 1.5\text{pb}$  at  $Q^2 = 25(\text{GeV}/c)^2$ .

Studies of charge and single-spin asymmetries due to Pomeron-Odderon interference in hard diffractive electroproduction of a  $\pi^+ \pi^-$  pair were reported by O. Teryaev [74]. The proposal to search for the Odderon at HERA using highly virtual photons instead of quasi-real, as suggested in [75], is motivated by the possibility to use perturbative QCD and so to obtain more reliable predictions. However, one has to pay for this by much lower values of the cross-sections. It is also not possible to remain in the framework of perturbative QCD, since the amplitude includes the 2-pion generalized distribution amplitude and the Pomeron-Odderon proton impact factors. The predicted charge asymmetry is sizable, although it depends crucially on the nature of  $f_0$  meson. The single spin asymmetry turns out to be rather small.

A brief review of the different models for diffractive Higgs boson production

was given by C. Royon [76]. For the inclusive production both the factorizable and non-factorizable models give very low at the Tevatron and quite high at the LHC cross section (although uncertainties are large). The exclusive production has the advantage of very clean events. The problem is that one needs to suppress totally QCD radiation. The price to pay is the suppression of the cross section. It occurs to be extremely small for the Tevatron. However, it is large enough for the LHC.

Saturation effects in the Drell-Yan di-lepton  $p_T$  distribution were looked for [77] in the framework of the color dipole approach, as discussed by M. B. Gay Ducati. At first sight, the appearance of a dipole is surprising; but in the target rest frame the DY process looks like bremsstrahlung with the subsequent decay of the virtual photon into a lepton pair. From the two bremsstrahlung diagrams, the same dipole amplitude as in small- $x$  DIS arises [78]. Two parametrizations of the dipole cross-section were used, one given by the multiple scattering Glauber-Mueller approach and the other by the BGBK saturation model [79]. Adding a Reggeon contribution allows extension of the dipole approach down to lower energies where both models then provide good description of the low energy (ISR) data, where unitarity corrections in the Glauber-Mueller approach are negligible. The unitarity effects are quite sizeable at LHC energies, at both large  $p_T$  and rapidity, but remain small in the central rapidity region.

#### 4 Conclusions and Outlook

Over the last couple of years the improved precision of the data used in global fits to extract parton densities and the development of tools to handle errors ‘properly’ has led to a more conservative view of the region in  $x, Q^2$  in which the standard DGLAP NLO extraction is reliable. At the same time the resulting partons are becoming much more of a precision tool. Over the next year or so the completion of the NNLO calculations and continuing attempts to exploit the NLO BFKL calculations for parton phenomenology will reduce uncertainty further. One may also expect improvements in the flavour decomposition of partons as the data from NuTeV is finalized and data from HERA-II high  $Q^2$  NC and CC become available.

The totality of data on hard diffraction and vector mesons from HERA-I

is impressive in its range and precision. With the expectation of new Run-II measurements on hard diffraction from the upgraded detectors soon, the emerging understanding of where and how factorization is modified by unitarity corrections will be consolidated.

While new data will undoubtedly help to bring issues to a sharper focus, there is much work to be done on the theoretical side. It is well known that any enlargement of our knowledge leads to new problems, so that the significant progress in theory achieved over the last few years has not reduced the number of problems waiting for solution. New problems, such as those concerning the relationship between DGLAP and BFKL, have accrued to such longstanding items as the calculation of the NLO impact factors, the non-forward BFKL kernel and the NNLO anomalous dimension. A lot of problems remain unsolved in the effective theory of high parton density, in spite of the advances in understanding of many properties of non-linear dynamics. The impact parameter dependence of the dipole scattering amplitude in the region of large  $b$  and the account of the non-perturbative corrections are among them. The region of applicability of the dipole picture itself remains undetermined.

Overall at low- $x$ , there is continued excitement over the rapid development of saturation models, stimulated in part by the desire to link gluon dynamics at HERA and RHIC. More precise data over a greater range of energies and observables will certainly help towards answering the question of whether saturation has been seen, but it may be too much to hope that the answer will be unambiguous. One may have to wait for the higher energies available at the LHC, by which time there should have been significant progress on many of the open theoretical questions.

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